Analytic Conformal Bootstrap

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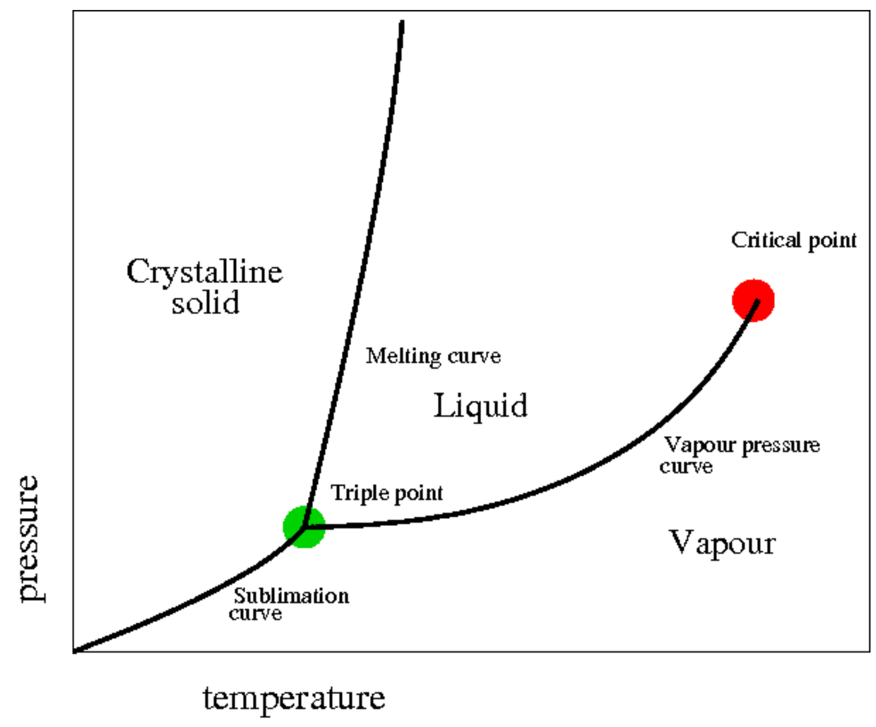
Critical Phenomena



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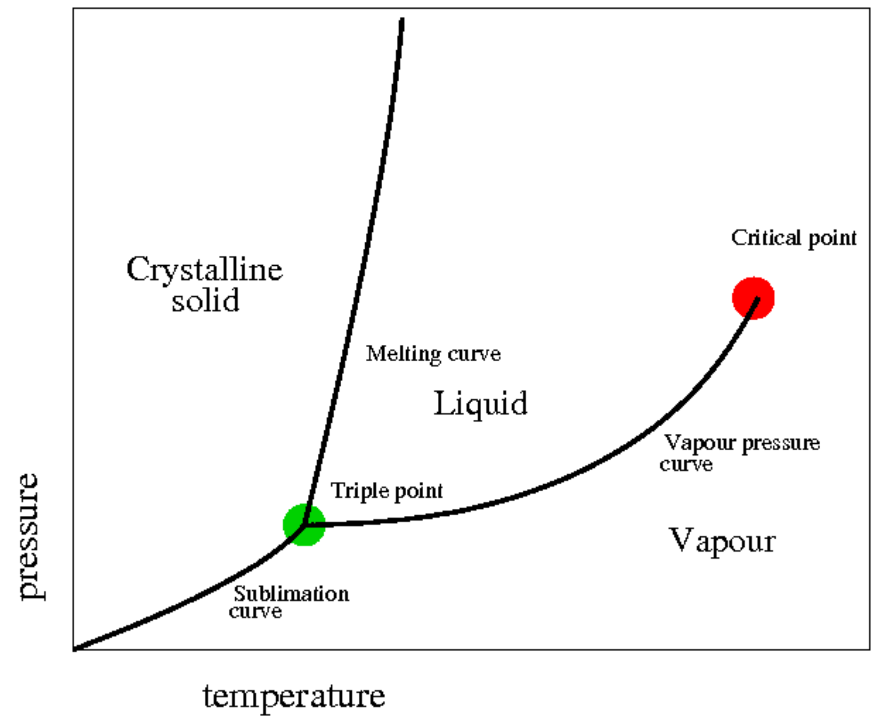
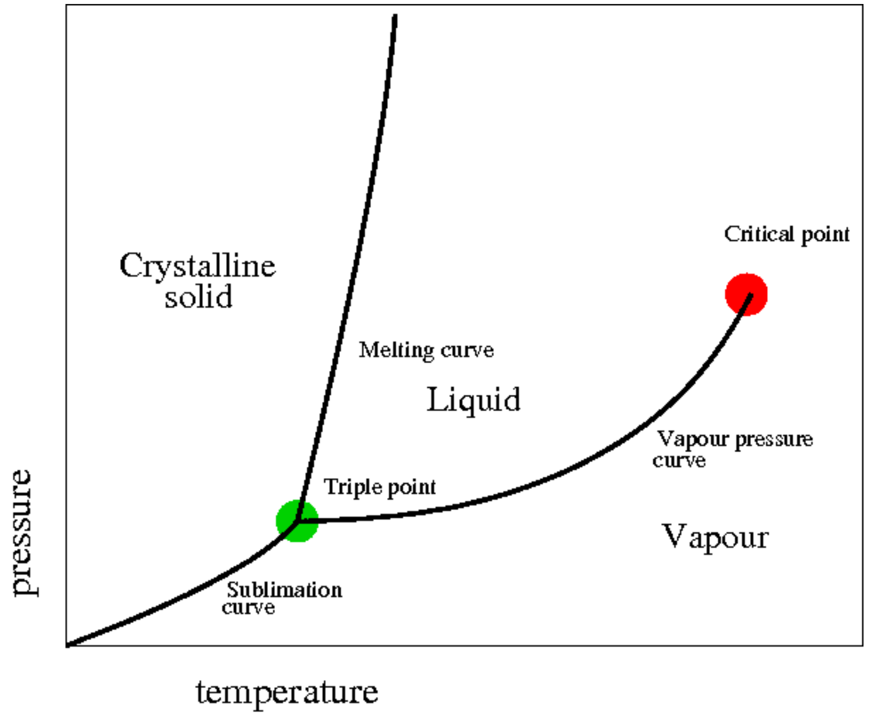


Critical point:

It is the end point of the phase equilibrium curve. In the vicinity of the critical point, the physical properties of the liquid and vapour change dramatically and both phases become even more similar.

Critical Phenomena



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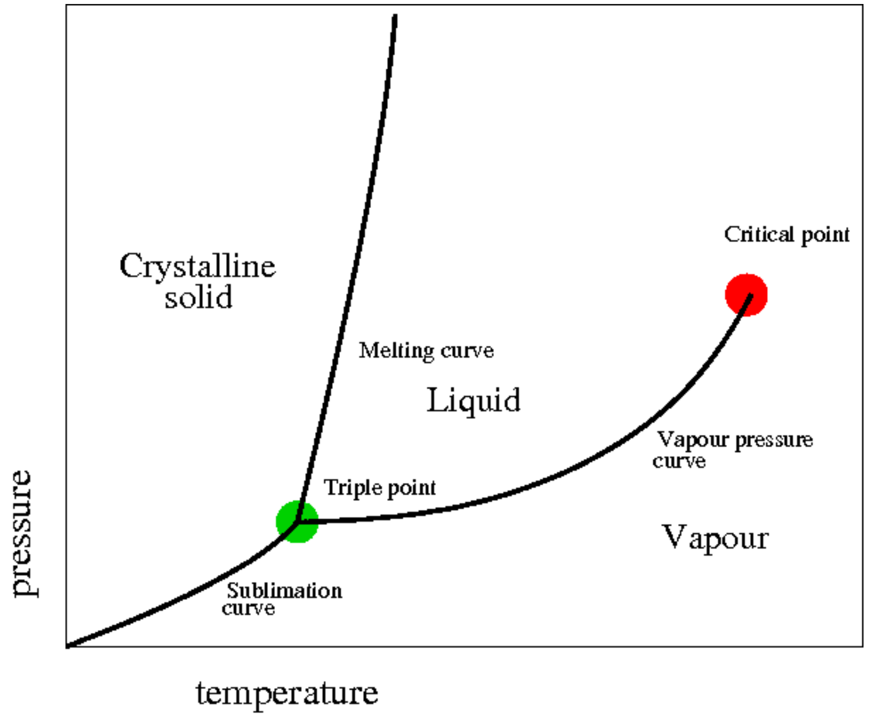
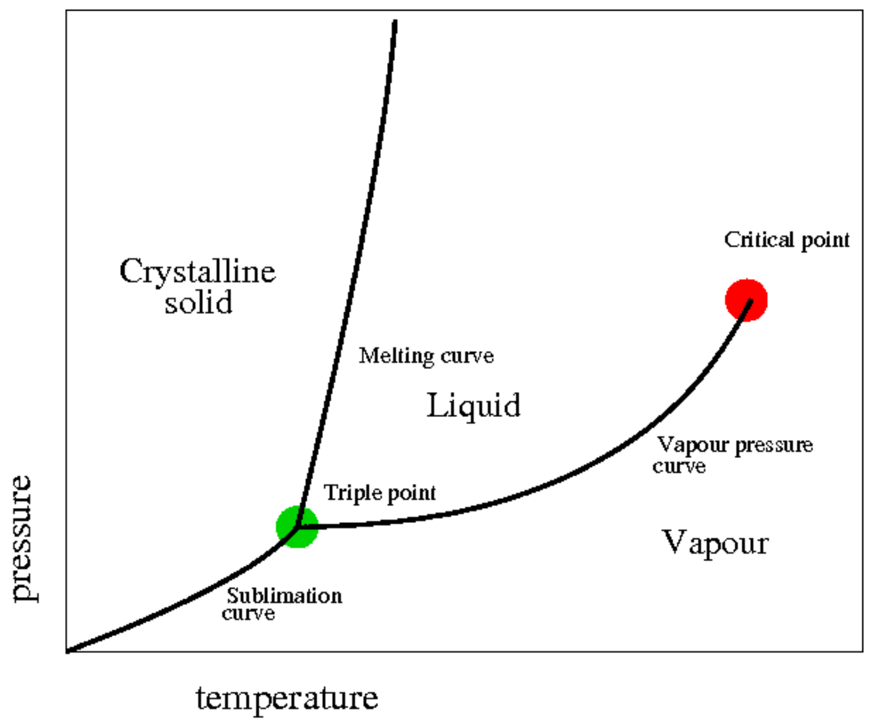


Why is it important?

Critical Phenomena



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There are fluctuations of the fluid density $\delta\rho$ that occur over longer and longer distances measured by the correlation length ξ.

$$\langle \delta\rho(x_1)\delta\rho(x_2)\rangle \sim \begin{cases} e^{-|x_1-x_2|/\xi} & |x_1-x_2| \gg \xi \\ \dfrac{1}{|x_1-x_2|^{1+\eta}} & |x_1-x_2| \ll \xi \end{cases}$$

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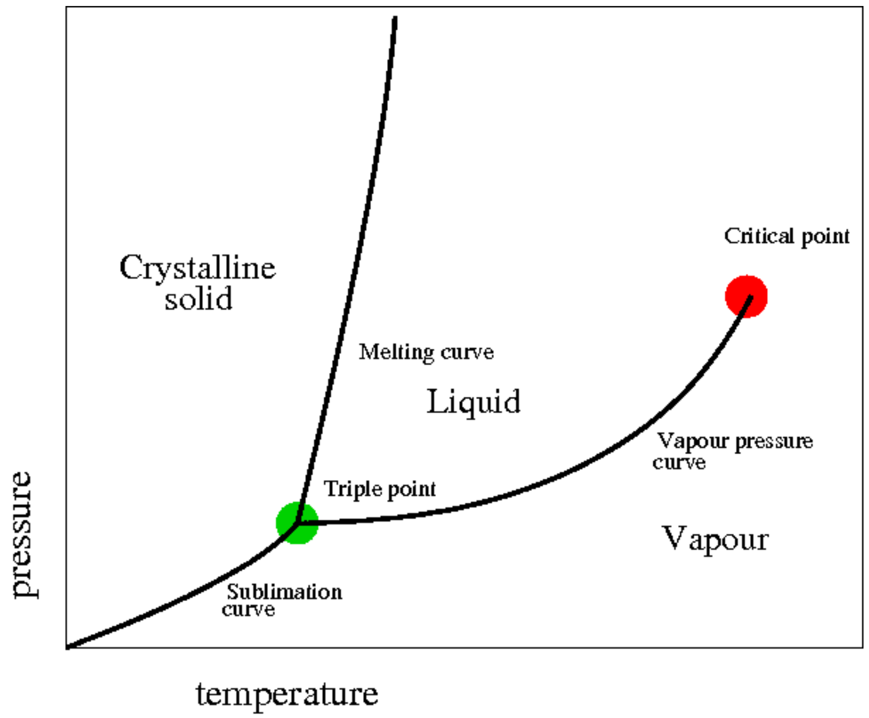

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$\xi \sim (T-T_c)^{-\nu}$, for $T \to T_c \quad \xi \to \infty$

Near the critical point and at fixed pressure, the correlation length diverges.

↓

All scales drop out

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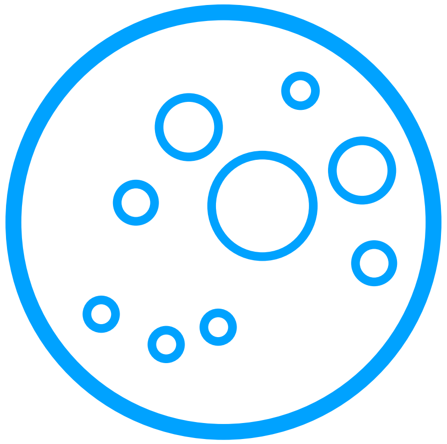
Scale invariance



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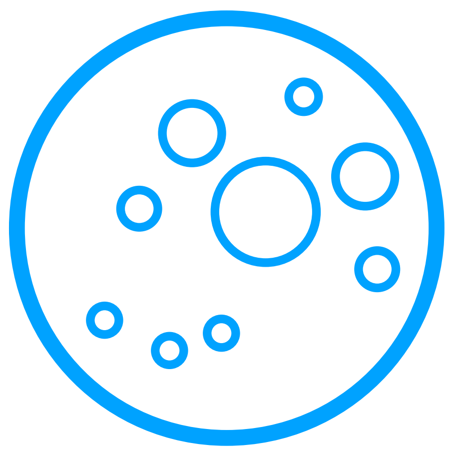

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This means that while the models at finite scales are very different, in the vicinity of the critical point asymptotic phenomena (e.g. critical exponents) are the same in all models falling in the same universality class.

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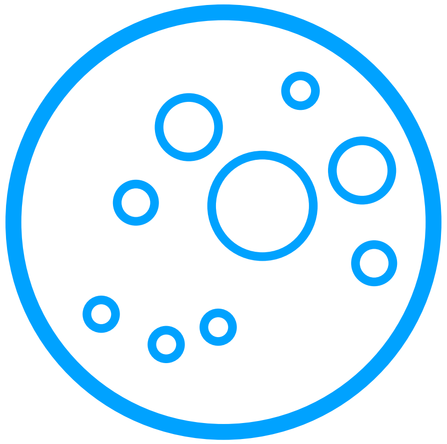

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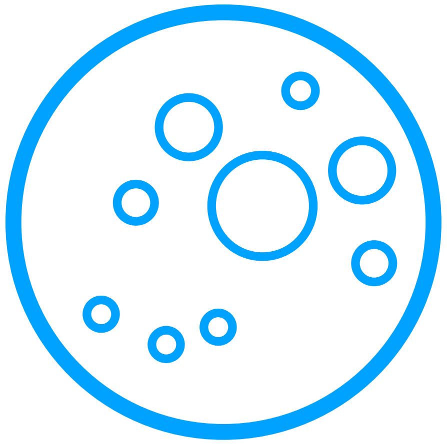

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Scale invariance

invariance under rescaling (dilatation) of all coordinates by a uniform factor $x \to \lambda x$

Conformal invariance

An interesting class of transformations are conformal transformations, which preserve angles.

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locally at each point it is a rotation + a dilatation (with rescaling factor x-dependent)

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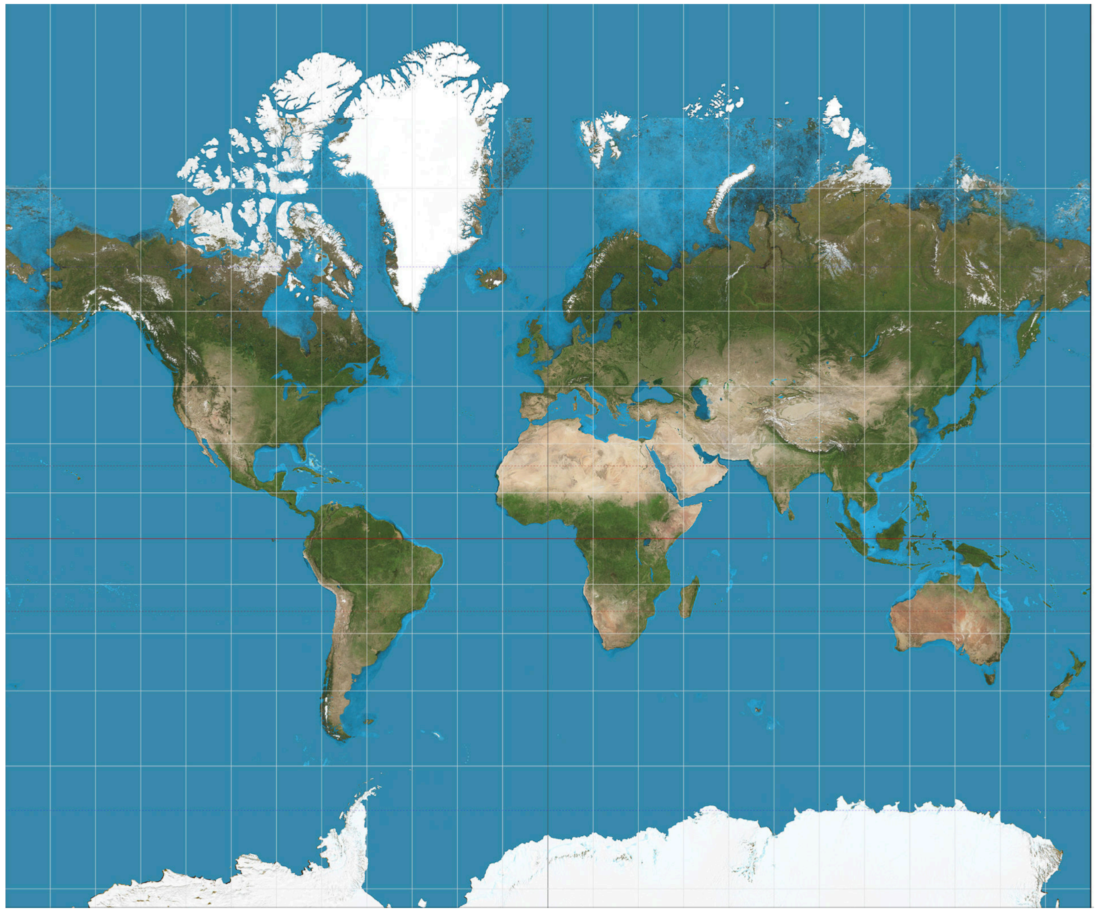

Mercator's map

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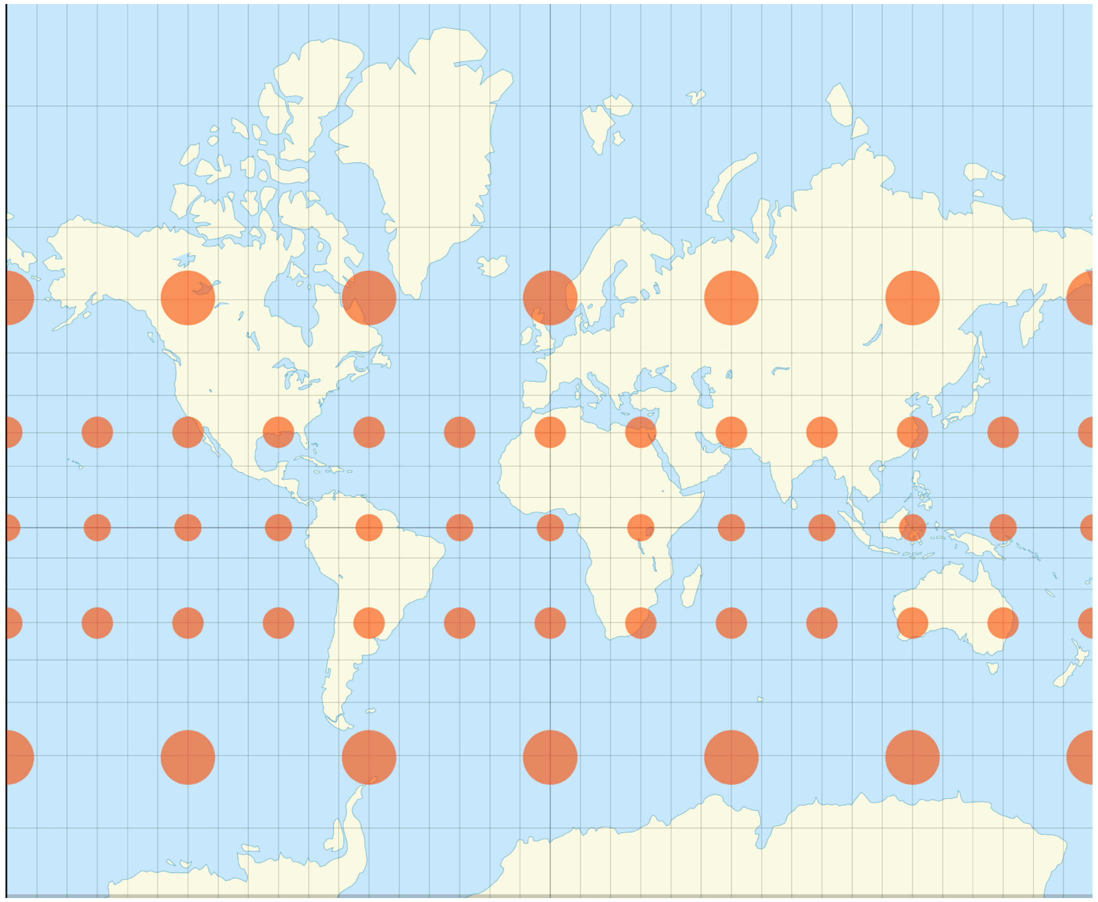

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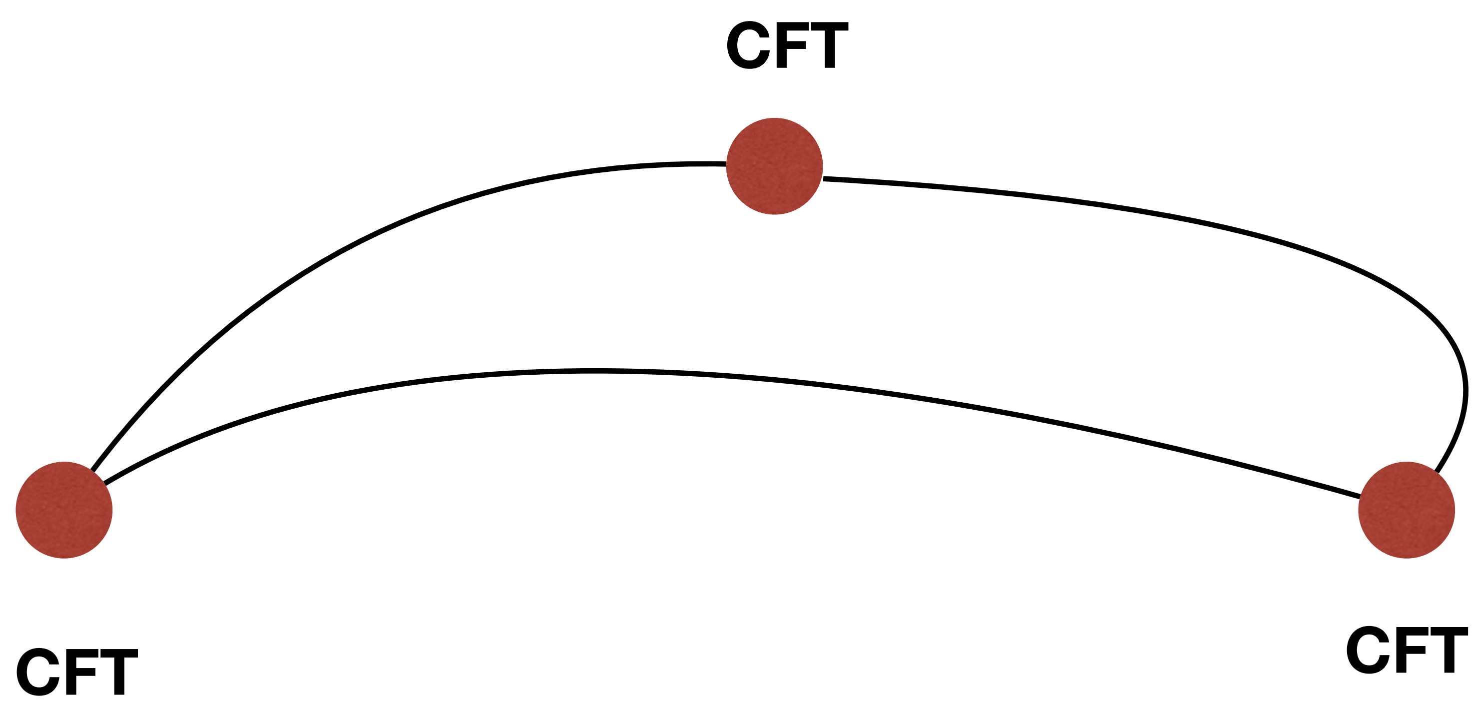
The description of fixed points boils down to classifying conformal field theories.

Centrality of CFTs

Conformal Field Theories (CFTs) are central also in the characterisation of QFTs.

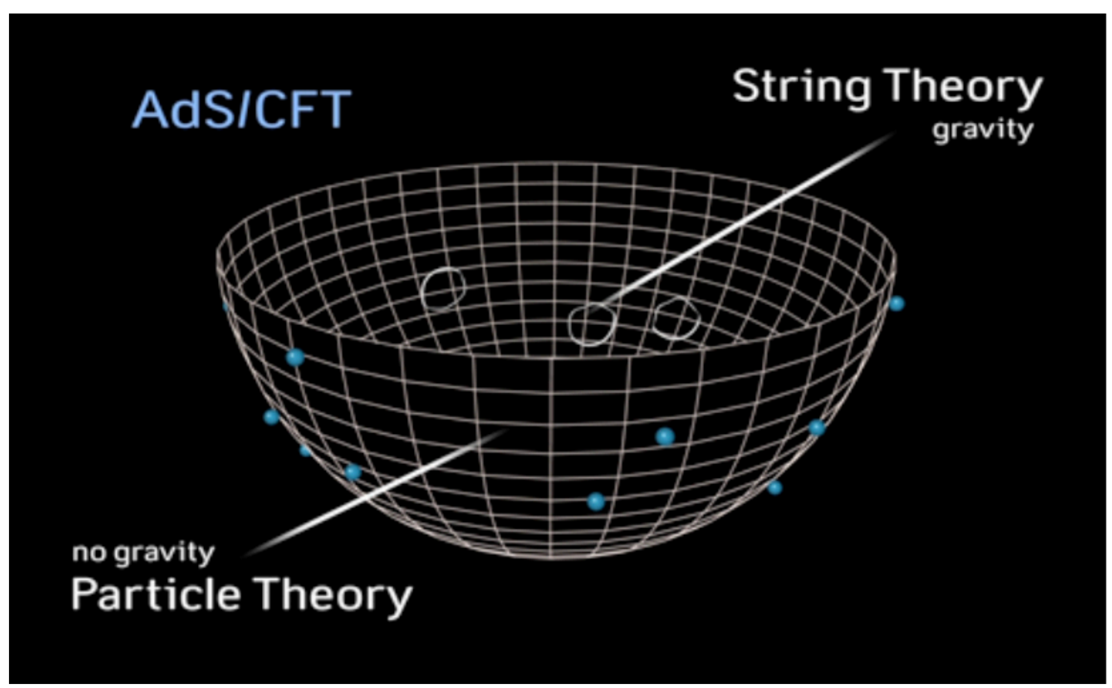


Large classes of QFTs can be seen as RG flows which emerge from a CFT (UV fixed point) and another non trivial CFT (IR fixed point)

Centrality of CFTs

They are related to theories of quantum gravity via the AdS/CFT correspondence

Operative mapping: observables (correlation functions and scattering amplitudes) in both theories are related in a very specific way.



Maldacena 1998

Main aim

Study conformal field theories using an approach which is based on symmetries and with very little/no input from the microscopical description of the theory

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conformal invariance

+

existence of the operator product expansion (OPE)

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Correlators

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Scale invariance

Extend the long distance behaviour to any distance: **continuous limit**

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Here it is shown for scalars, but it is similar for spinning operators.

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cross ratios $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$ $v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$

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Using the OPE inside correlators of *n*-points, with $n \geq 4$, it is possible to reduce them to two point functions.

Conformal blocks

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$$

Dolan Osborn 2002

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$$\frac{\delta_{mn}}{y_{12}^{2\Delta_m}}$$

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conformal blocks

What is m? It denotes the quantum numbers of the exchanged operators, which in this particular case are the conformal dimension and the Lorenz spin (Δ, ℓ).

Dolan Osborn 2002

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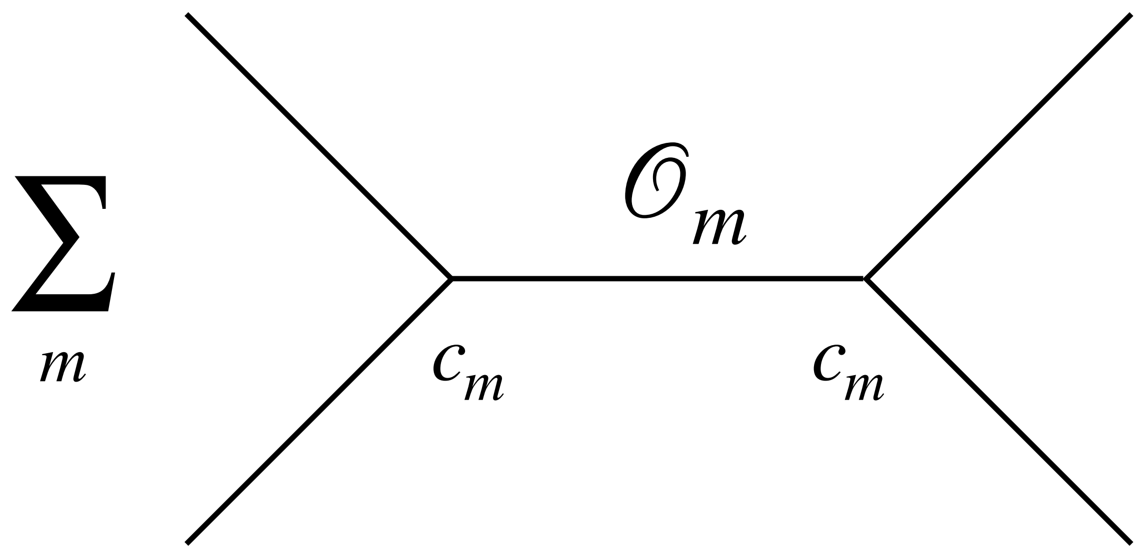

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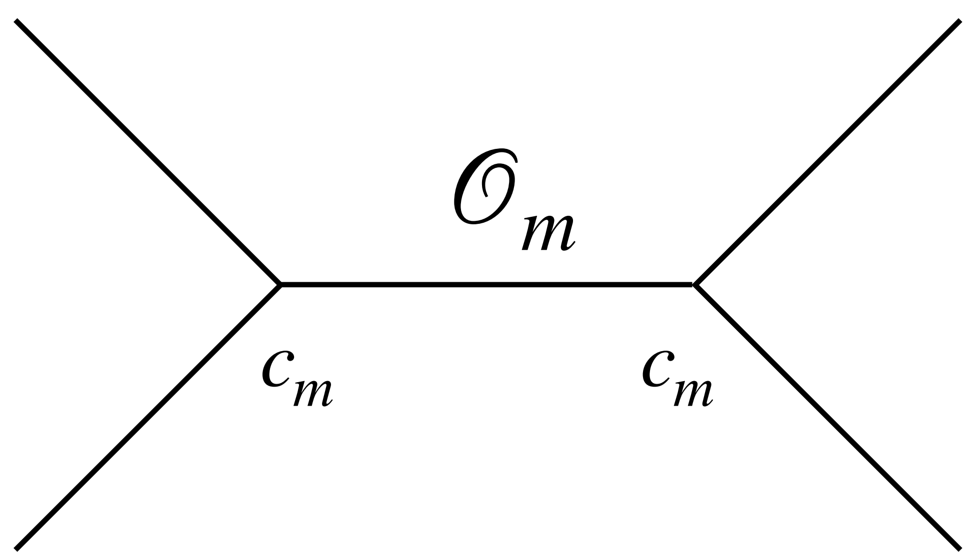

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Polyakov 1974, Ferrara Grillo Gatto 1973

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Crossing relations

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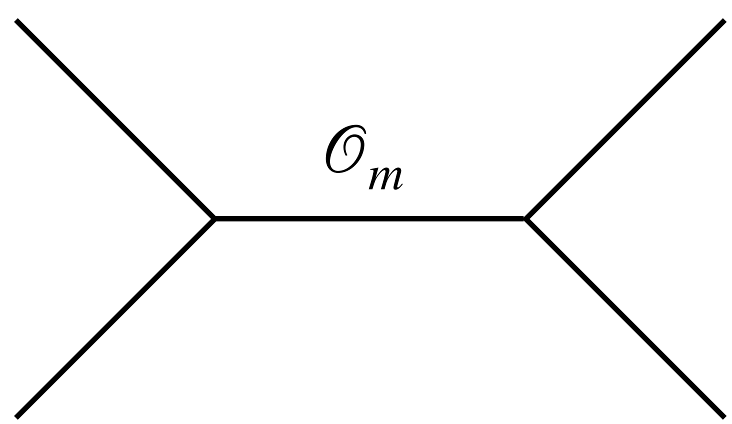
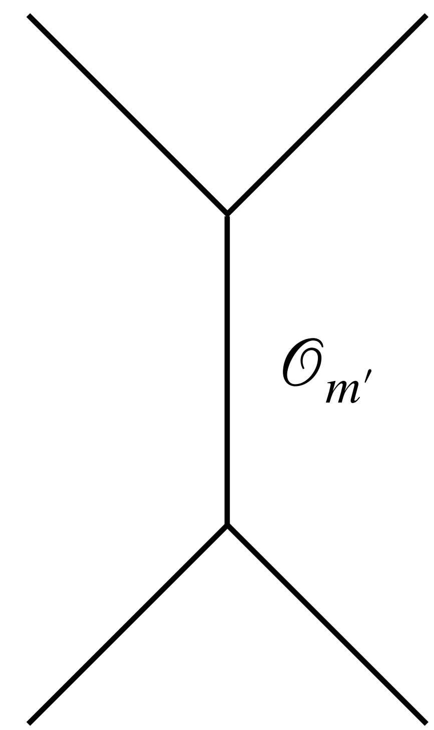
$\sum_m$ 

$=$ $\sum_{m'}$ 

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We won't be able to solve them completely, but we will discuss some approaches to find solutions consistent with the crossing relations.

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It is related to the fact that the norm of a state is positive. In this context it means that $\Delta \geq \dfrac{d-2}{2}$ for scalars and $\Delta \geq d + \ell - 2$ for operators of spin ℓ, and that the $c_m \in \mathbb{R}$

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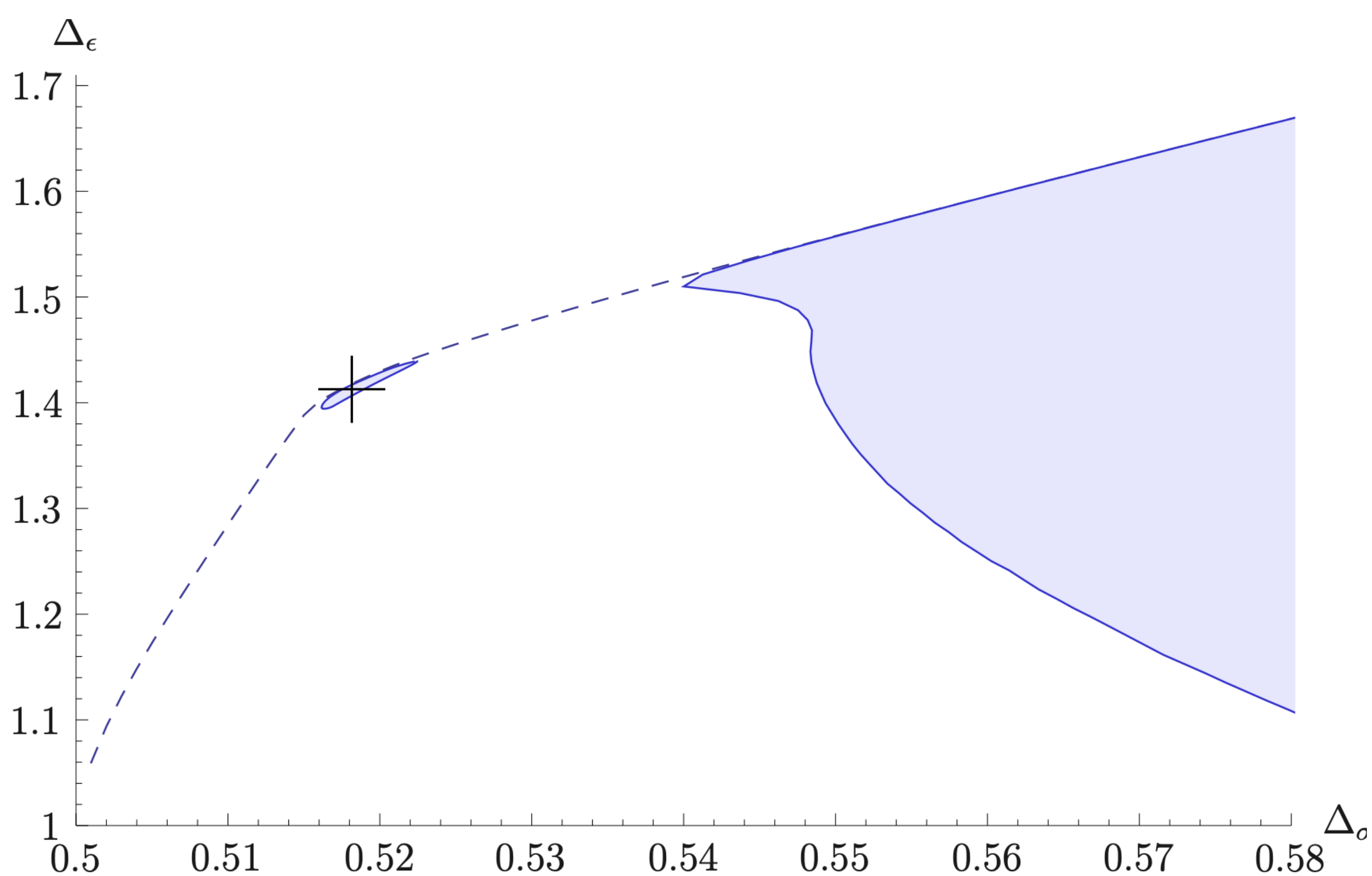

3d Ising Model

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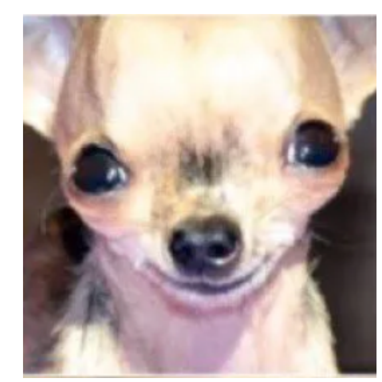
Numerical Bootstrap

The idea is to use the crossing relations as necessary conditions for conformal dimensions and OPE coefficients to belong to a CFT.

Tentative CFT data

crossing relations

MAYBE



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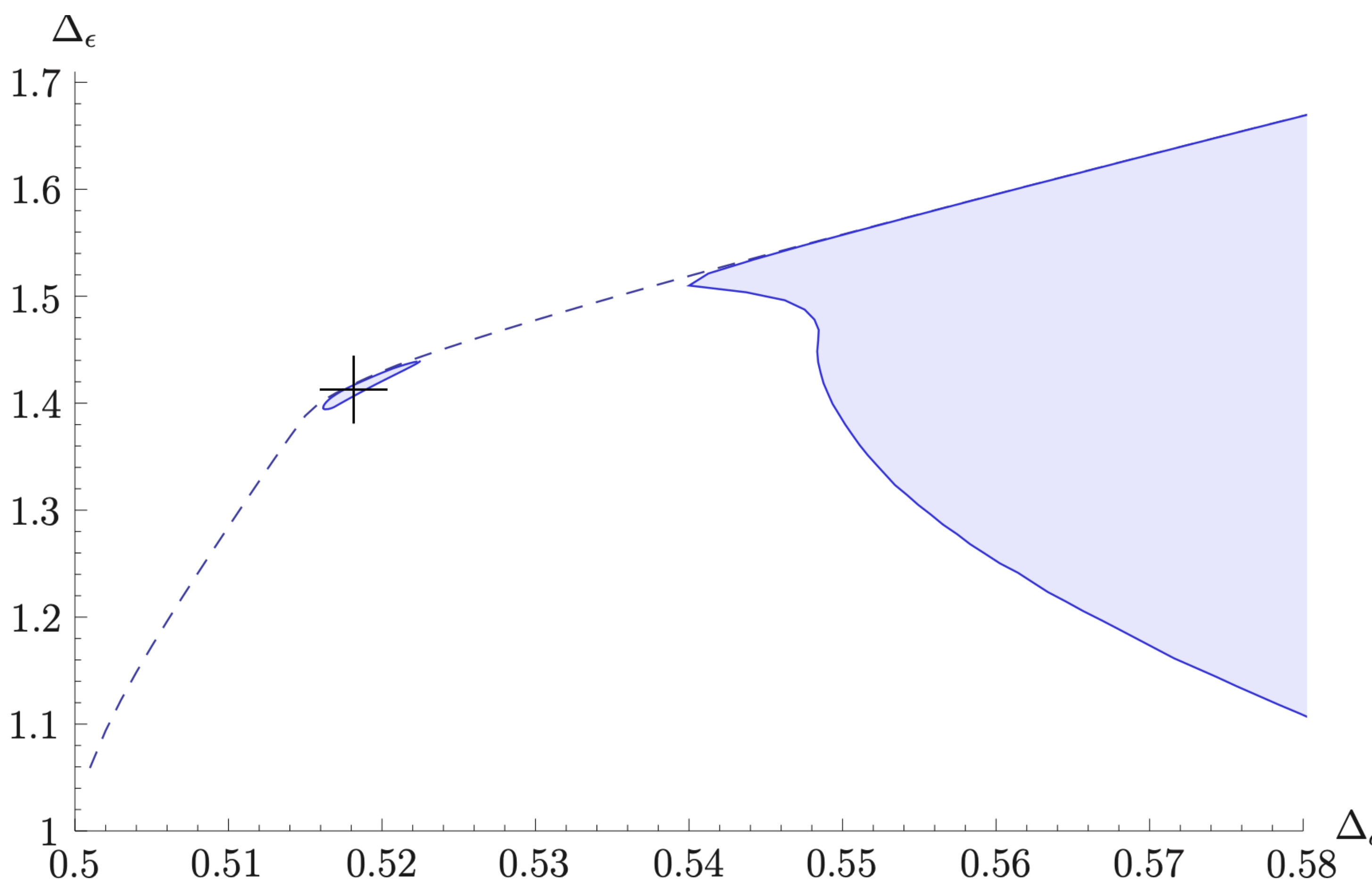

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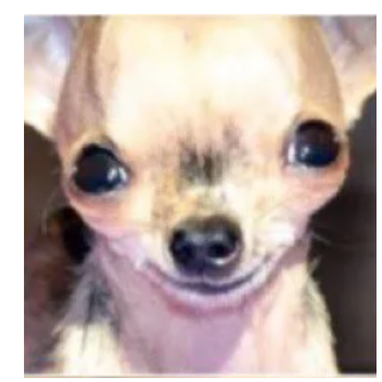

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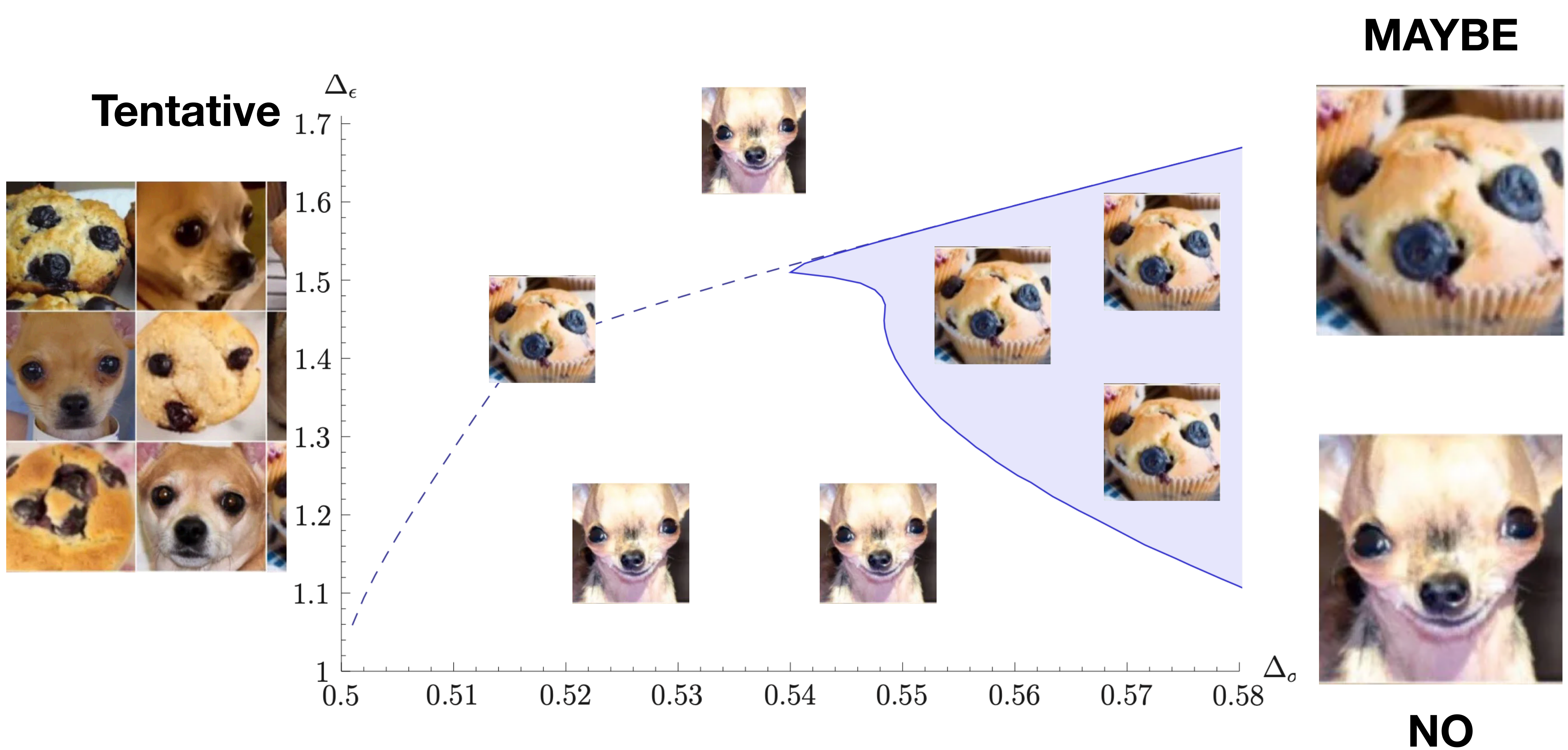

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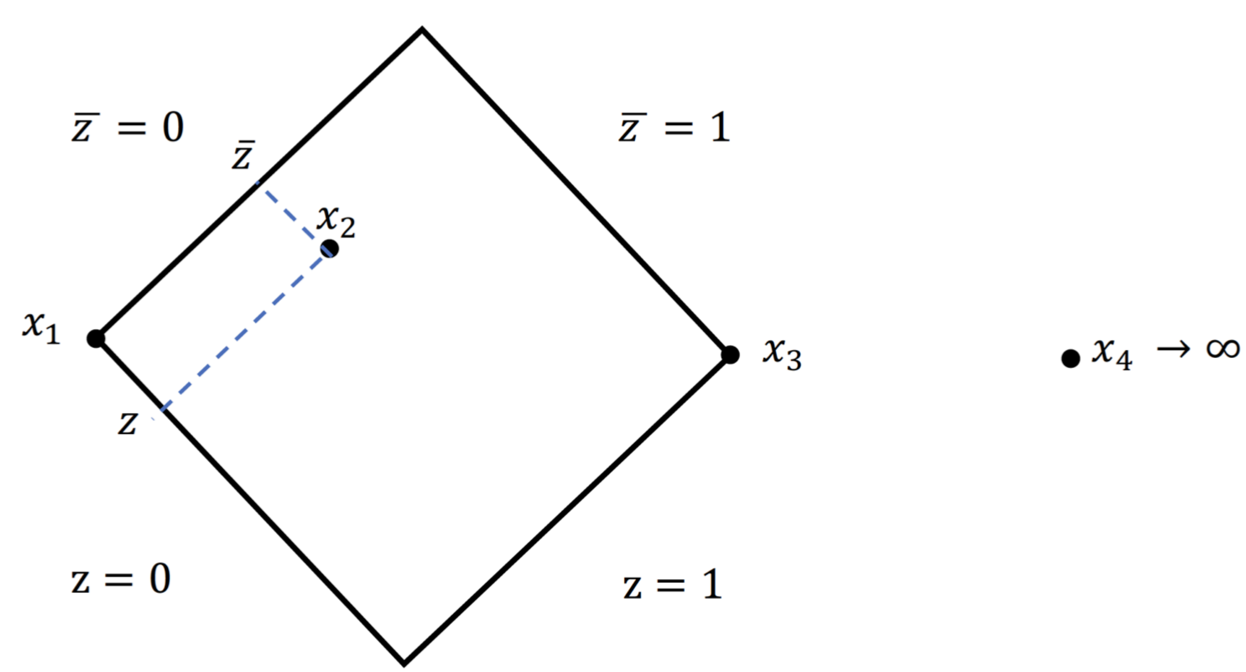

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3) If we also take the $v \to 0$ limit, in such a way that $z \ll 1 - \bar{z} \ll 1$, $\frac{v^{(\Delta-\ell)/2}}{v^{\Delta_O}} = 1$ and thus

$$\Delta = 2\Delta_O + \ell$$

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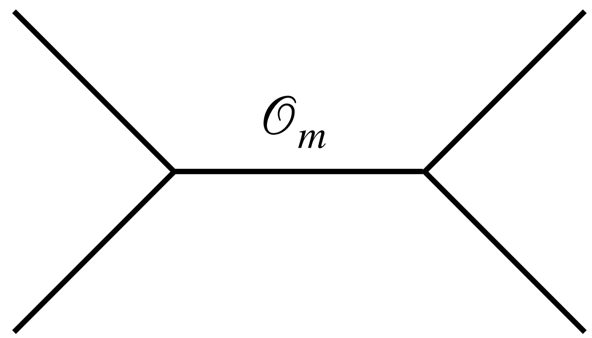
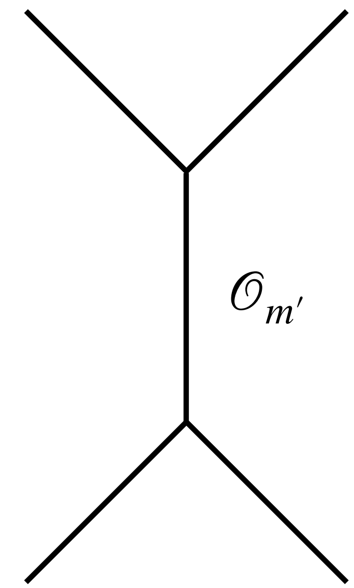
3) If we also take the $v \to 0$ limit, in such a way that $z \ll 1 - \bar{z} \ll 1$, $\dfrac{v^{(\Delta-\ell)/2}}{v^{\Delta_O}} = 1$ and thus

$$\boxed{\Delta = 2\Delta_O + \ell}$$ **“double” trace**

Large spin sector

$\sum_m$ 

$=$ $\sum_{m'}$ 

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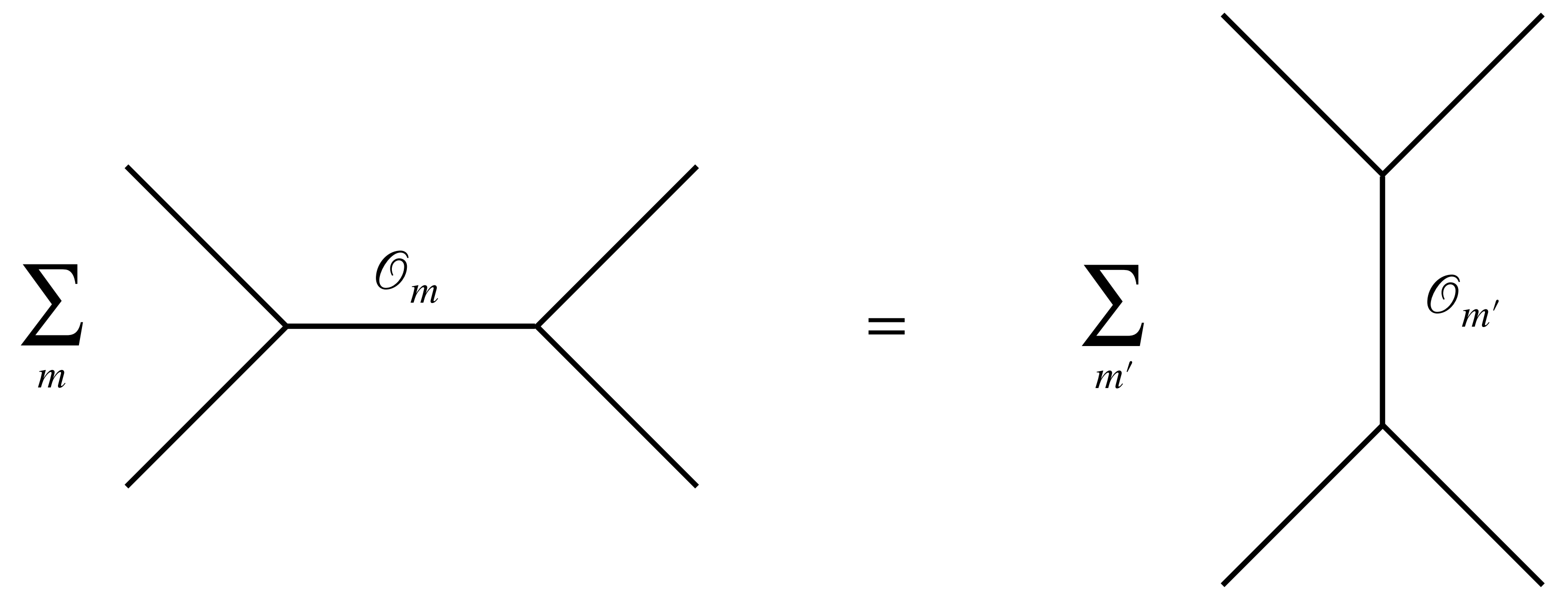

Identity Operator

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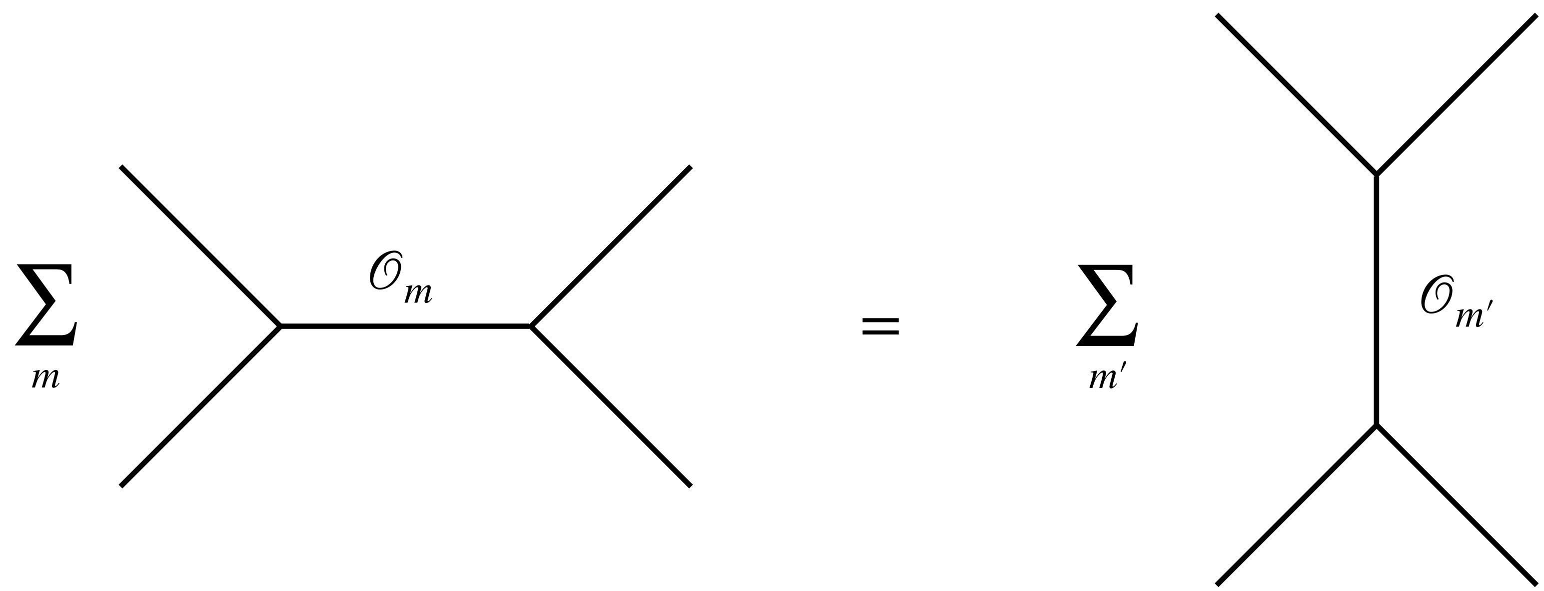

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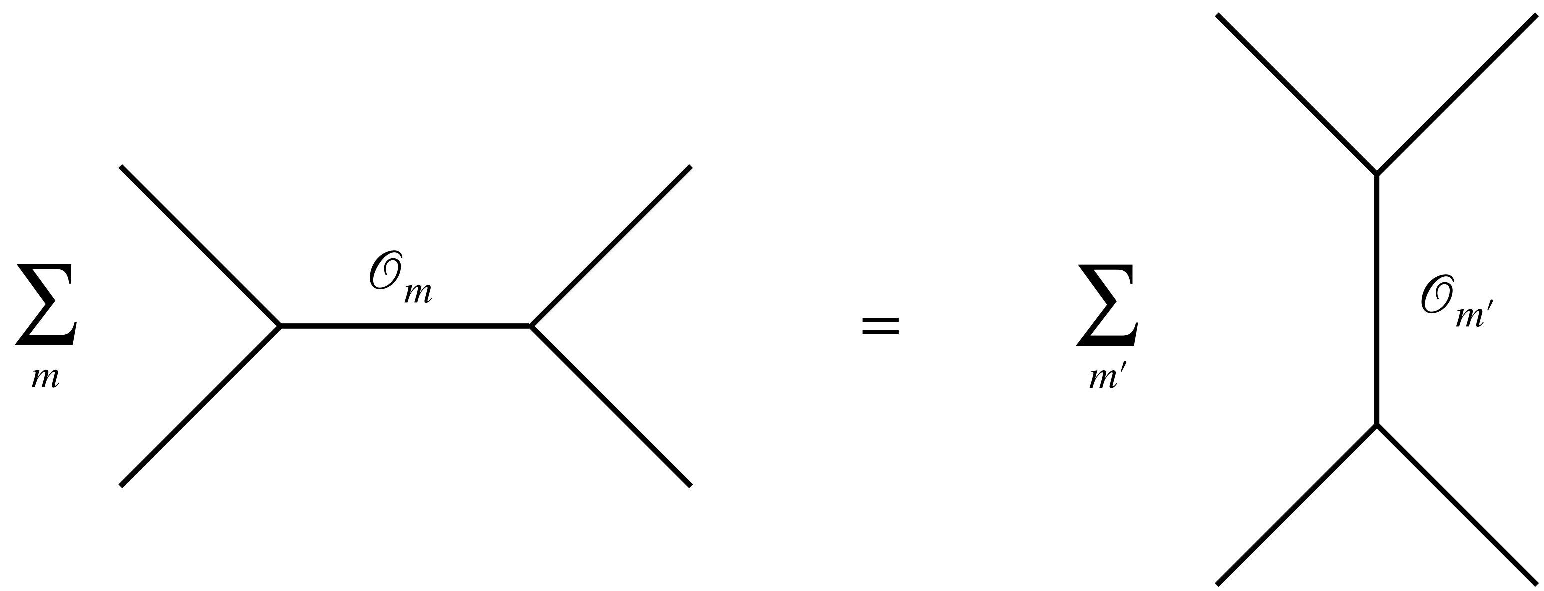

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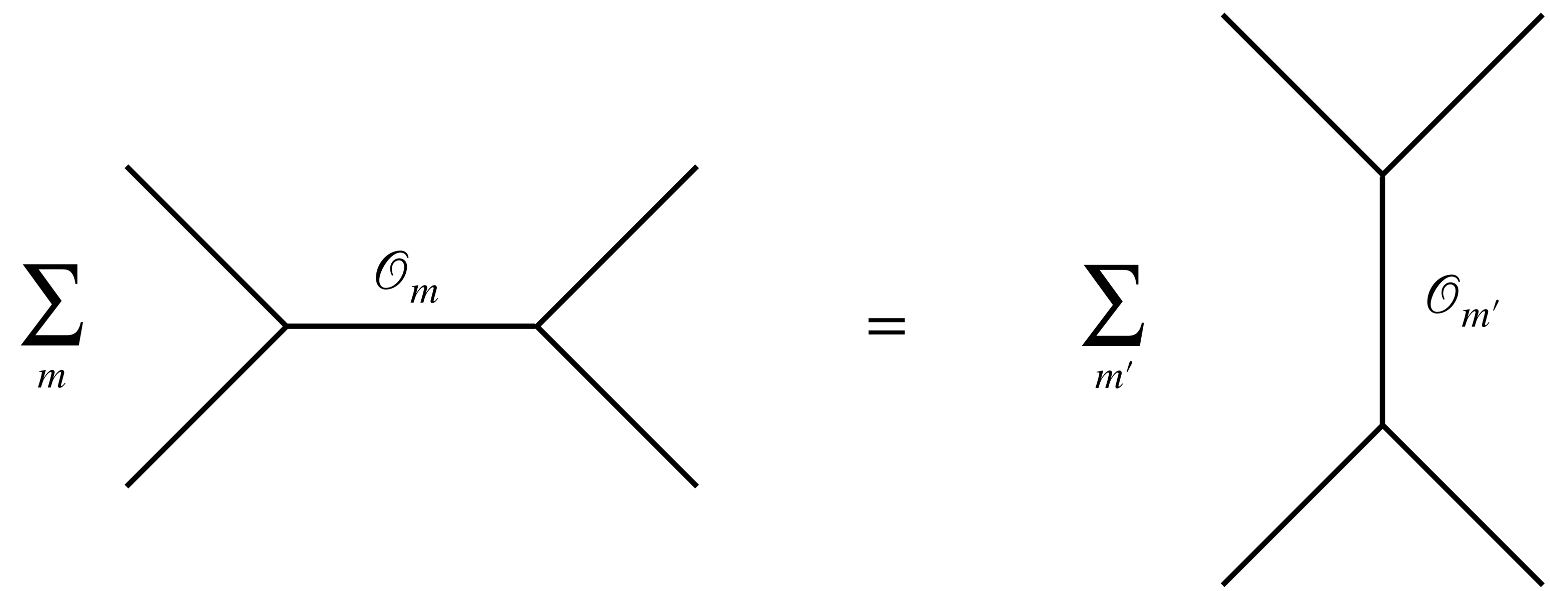

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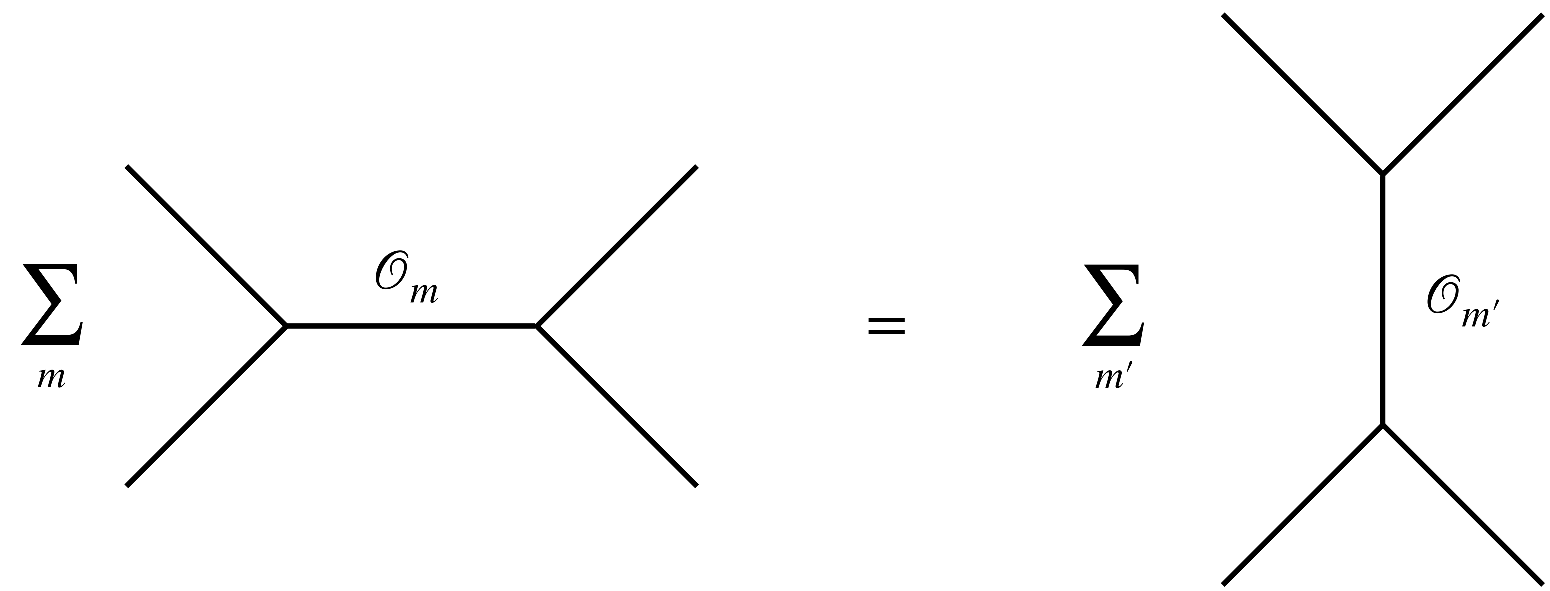

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It is possible to use the Casimir equation to iteratively find all the $1/\ell$ corrections and resum them to extrapolate for finite values of the spin.

Inversion formula

There is another approach to compute these quantities, less intuitive but computationally more powerful.

This approach allows inverting the OPE and prove why it is possible to resum the large spin series up to finite values of the spin.

Caron Huot 2017

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$$a_{\Delta,\ell} \xrightarrow[\Delta \to \Delta_k]{} \frac{c^2_{\Delta_k,\ell}}{\Delta_k - \Delta}$$

It has poles at the dimensions of the exchange operators with residues the square of the three point functions. The function is analytic in the spin for $\ell \geq 2$.

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Applicability

The applicability of these methods is pretty vast, and it mostly efficiently used when the theory has a small parameter (perturbation theory)

number of dimensions	$d = 4 - \epsilon$
rank of the gauge group	N
coupling constant	g or λ
...	

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Double trace operators $\mathcal{O}\,\Box^n\,\partial_{\mu_1}\ldots\partial_{\mu_\ell}\mathcal{O}$

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ignore the stress tensor

Large N

We expand all the quantities up to order N^{-4}:

$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \dots$$

$$\Delta = \Delta^{(0)} + \frac{1}{N^2}\gamma^{(1)} + \frac{1}{N^4}\gamma^{(2)} + \dots$$

$$c^2_{\Delta,\ell} = k^{(0)}_{\Delta,\ell} + \frac{1}{N^2}k^{(1)}_{\Delta,\ell} + \frac{1}{N^4}k^{(2)}_{\Delta,\ell} + \dots$$

The idea is to compute order by order in N, they CFT data. The main aim is to understand if we can predict the order N^{2k} using the $N^{2(k-1)}$ one.

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Let us start with the leading order

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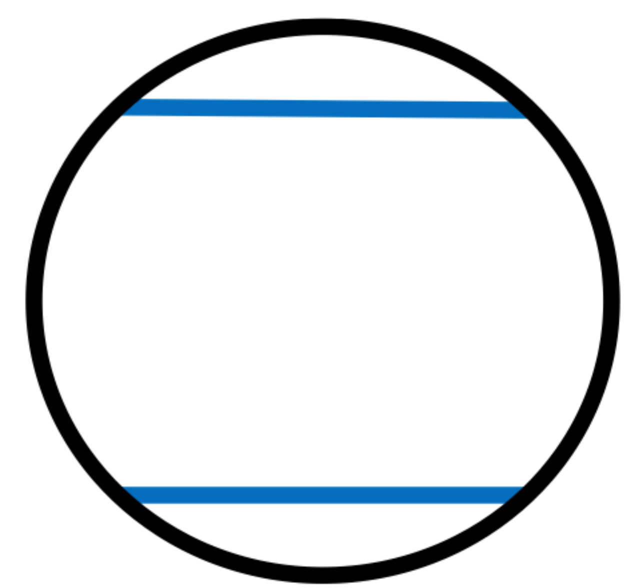

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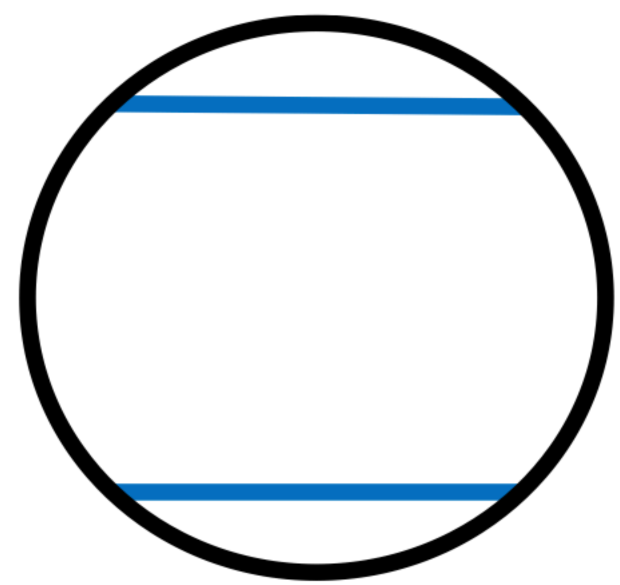

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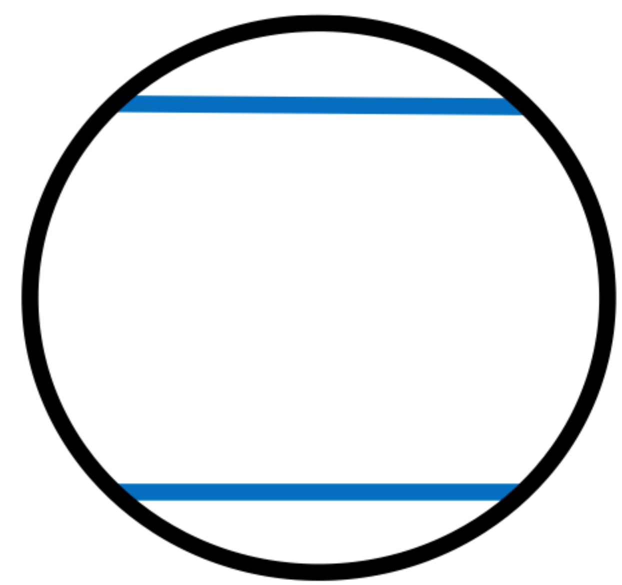

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Heemskerk Penedones Pochinski Sully 2009

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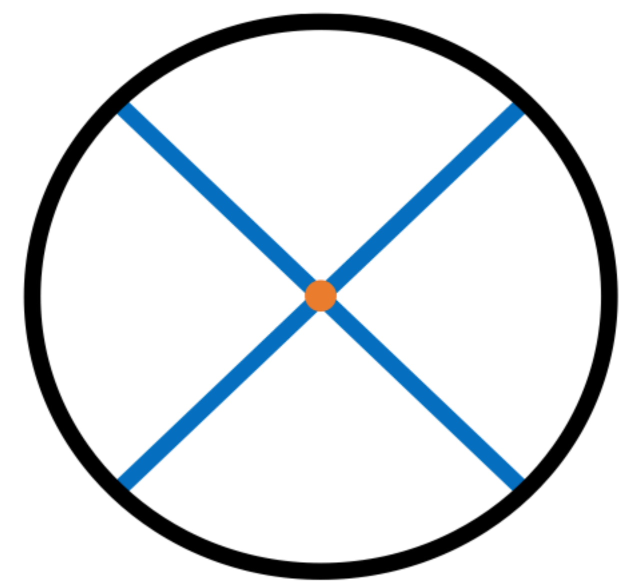

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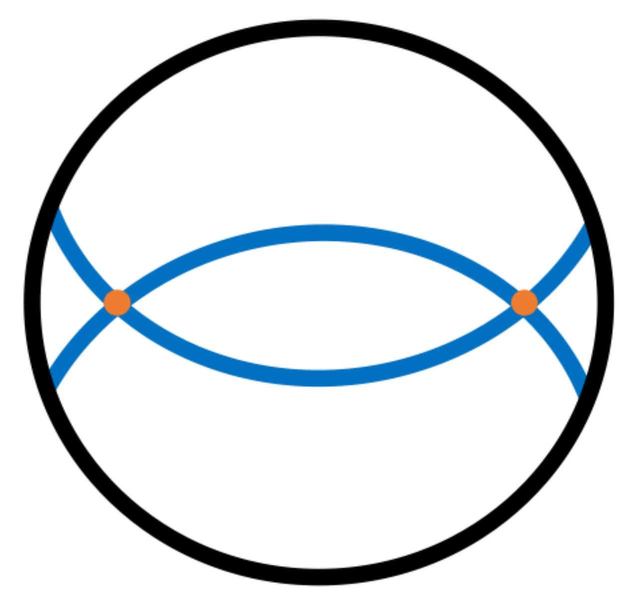

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In pictures

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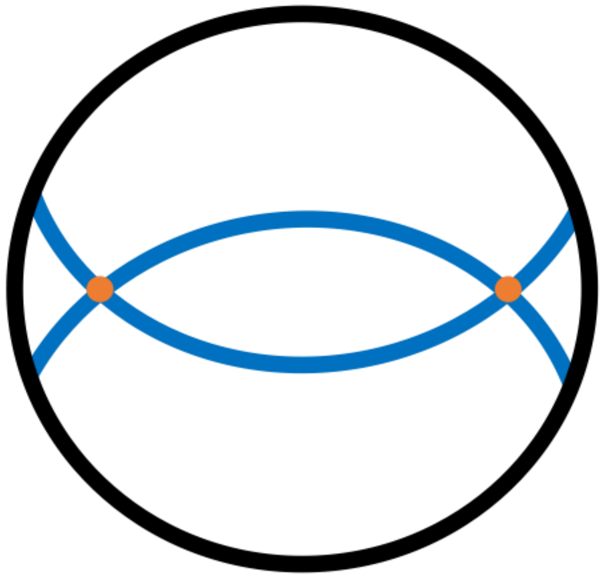

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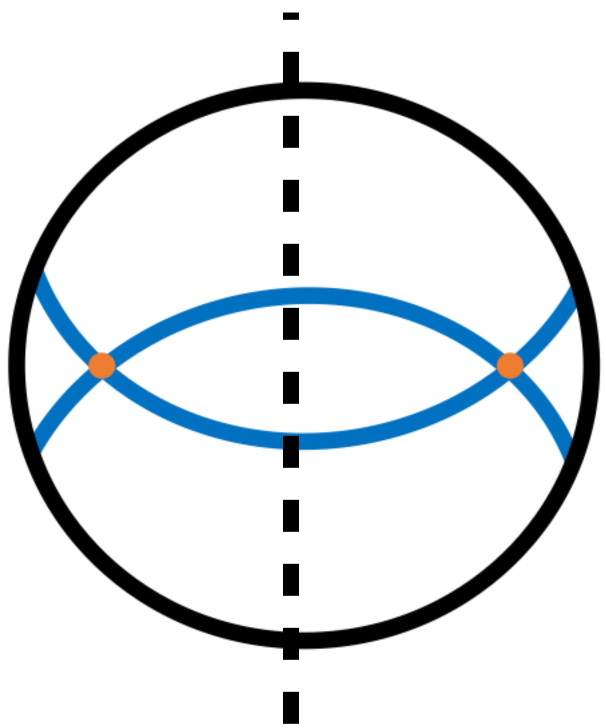

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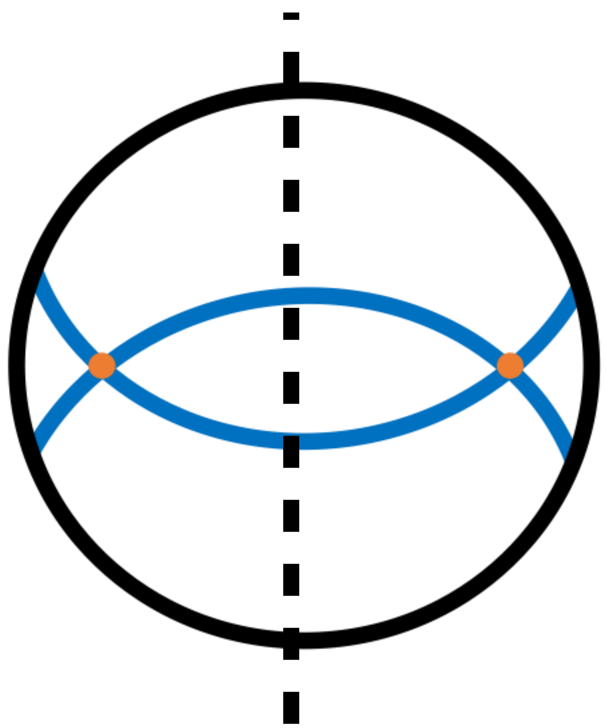

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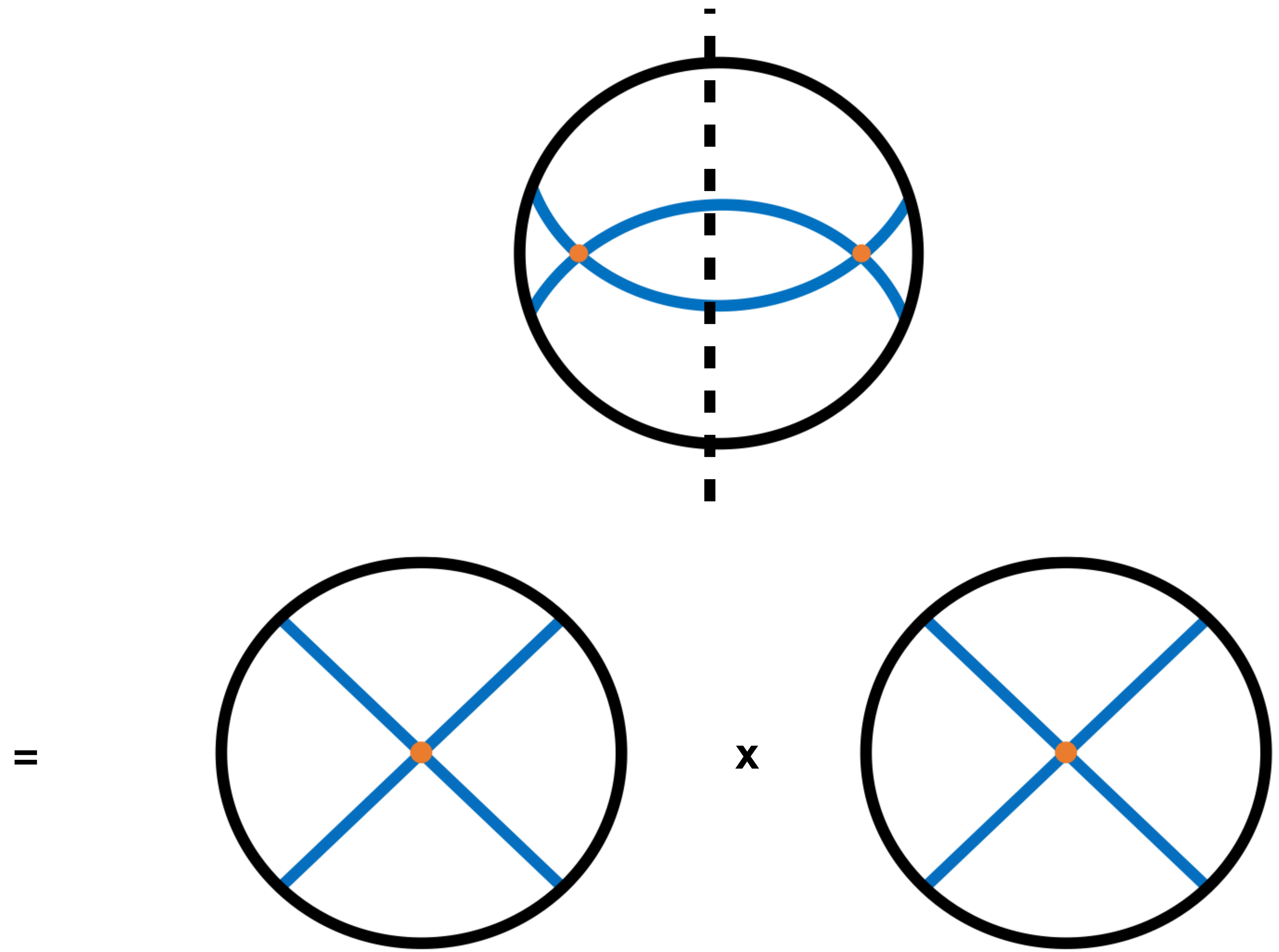
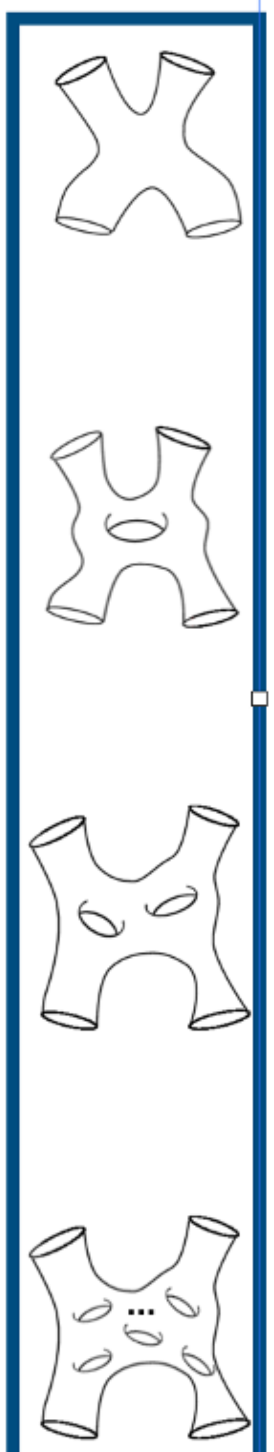
In pictures

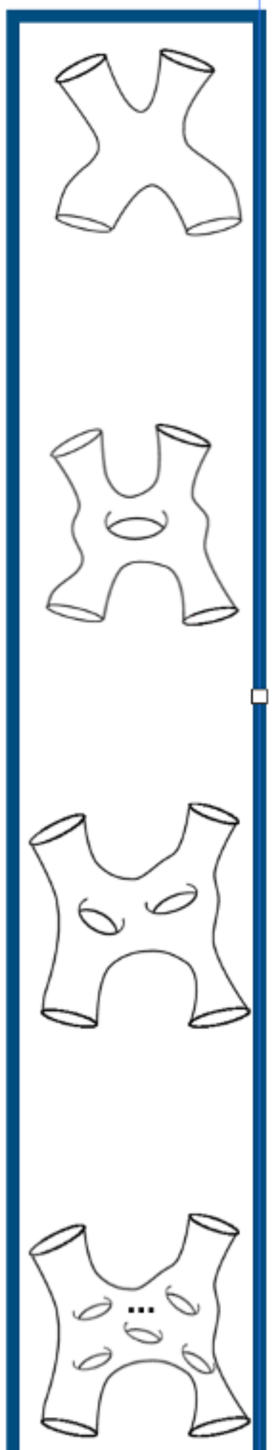


Other applications



This program has been carried over in several situations.

Other applications



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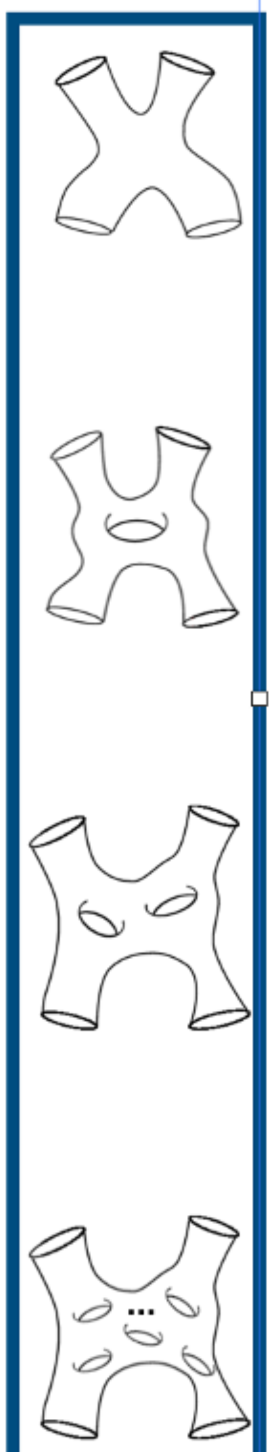
Using the AdS/CFT correspondence, it is possible to find amplitudes of supergravitons and supergluons on AdS spaces.

Other applications



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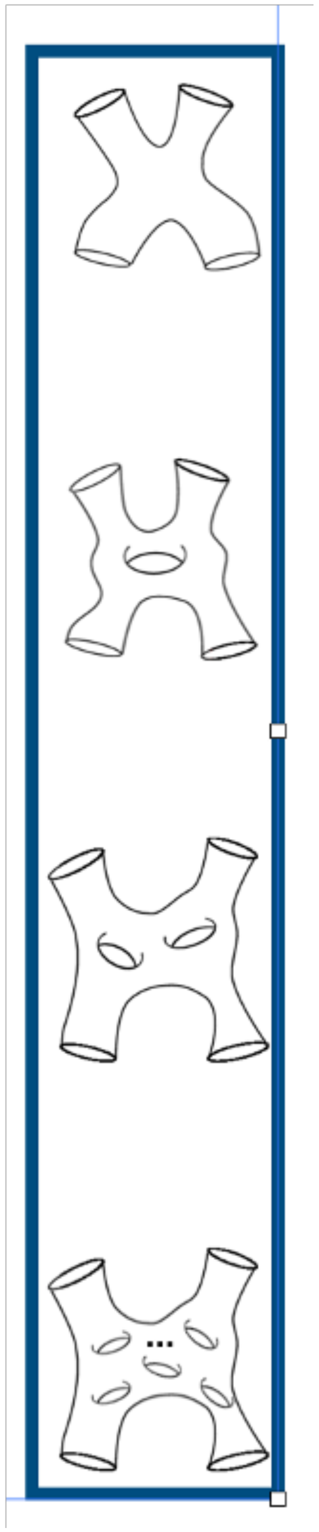

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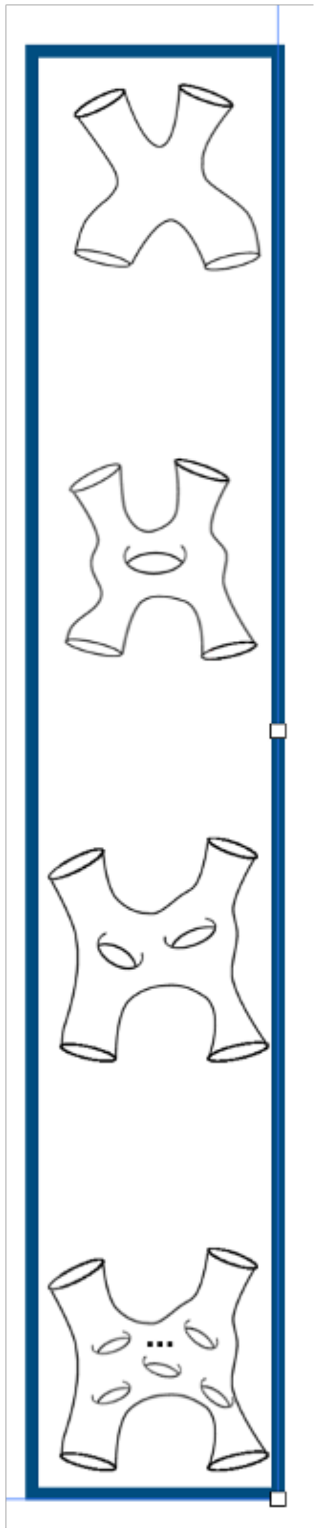

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Provides a unique framework to access scattering amplitudes in curved space-times, which are generically very hard/impossible to compute with other methods.

Conclusions

I presented a framework to analytically study CFT, using only the symmetries and the presence of an OPE expansion.

Mapping between singularities and OPE data.