Wormholes and the Information Paradox

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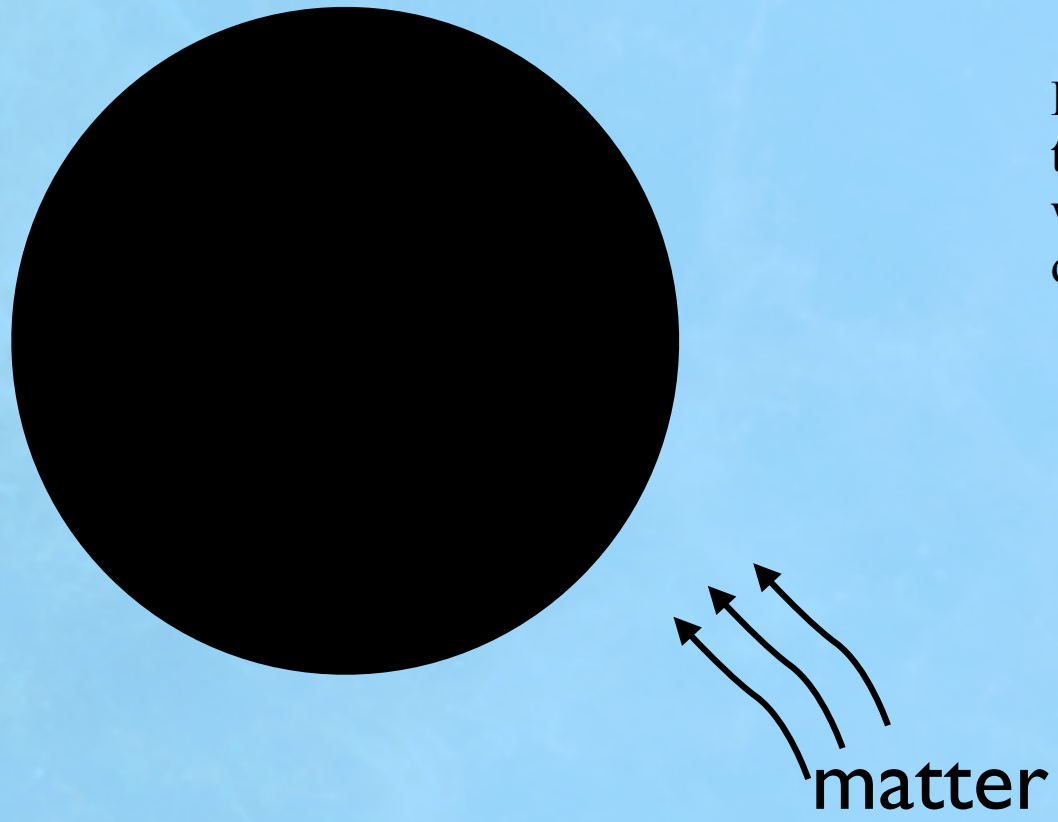
Black hole thermodynamics

[Bekenstein, Hawking, Christodoulou, etc. 1970's]



Black holes respond as if they are thermodynamic systems with a very large number of degrees of freedom.

If we assume the entropy of a black hole is

$$S = \text{entropy} = \frac{\text{Area}}{4} + \cdots$$

then the Einstein Equations can be rewritten as the "Laws of Black Hole Thermodynamics"

$$TdS = dE - \sum_i \mu_i Q_i$$

$$\frac{dS}{dt} \geq 0$$

This suggests black holes have a temperature *T* and an enormous number of microscopic d.o.f.

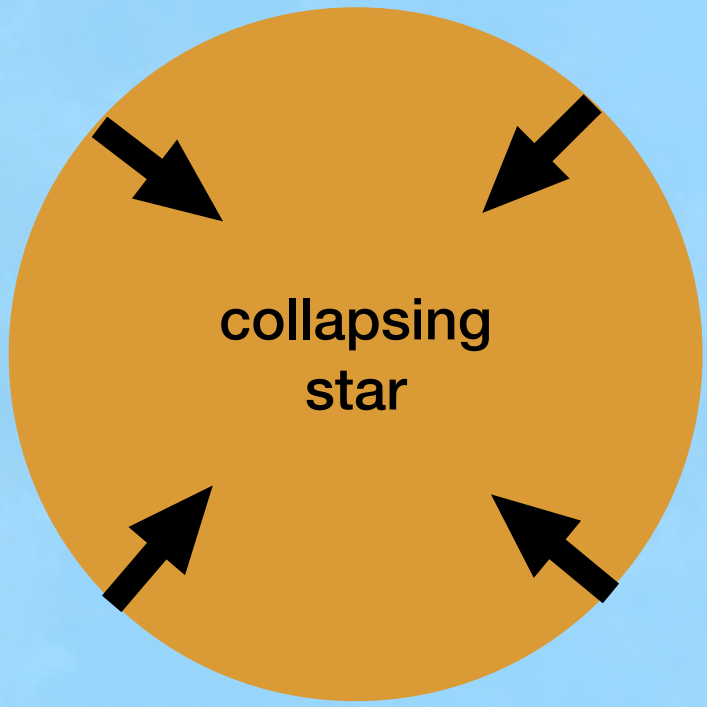
$$S = \log(\#states)$$

Sagittarius A*: $2^{10^{85}}$ states!

Hawking radiation

Hawking (1974) discovered that in quantum field theory, black holes do indeed radiate at the temperature

$$T_{\mathrm{Hawking}} = \frac{\hbar c}{4\pi R}$$



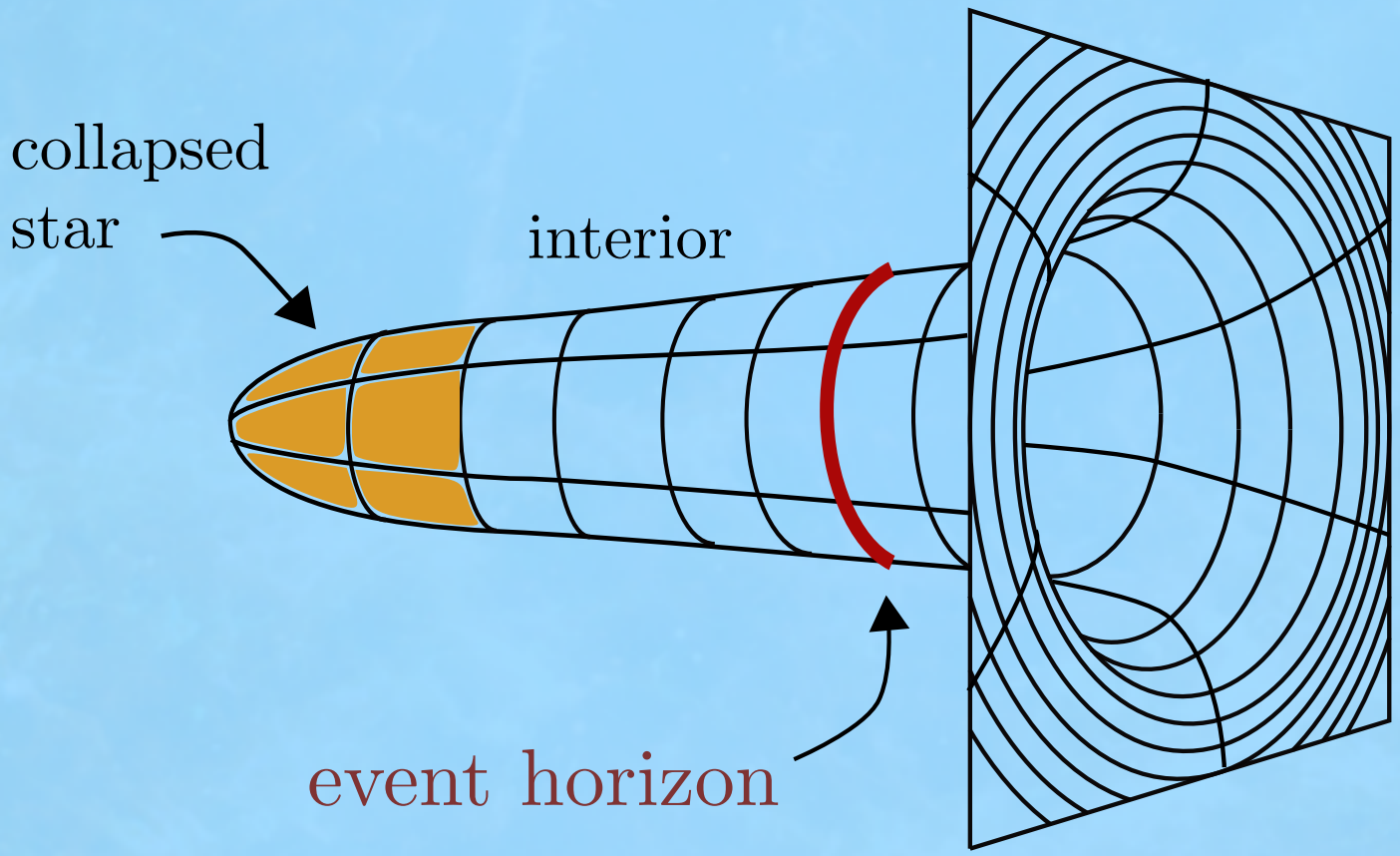
The information paradox

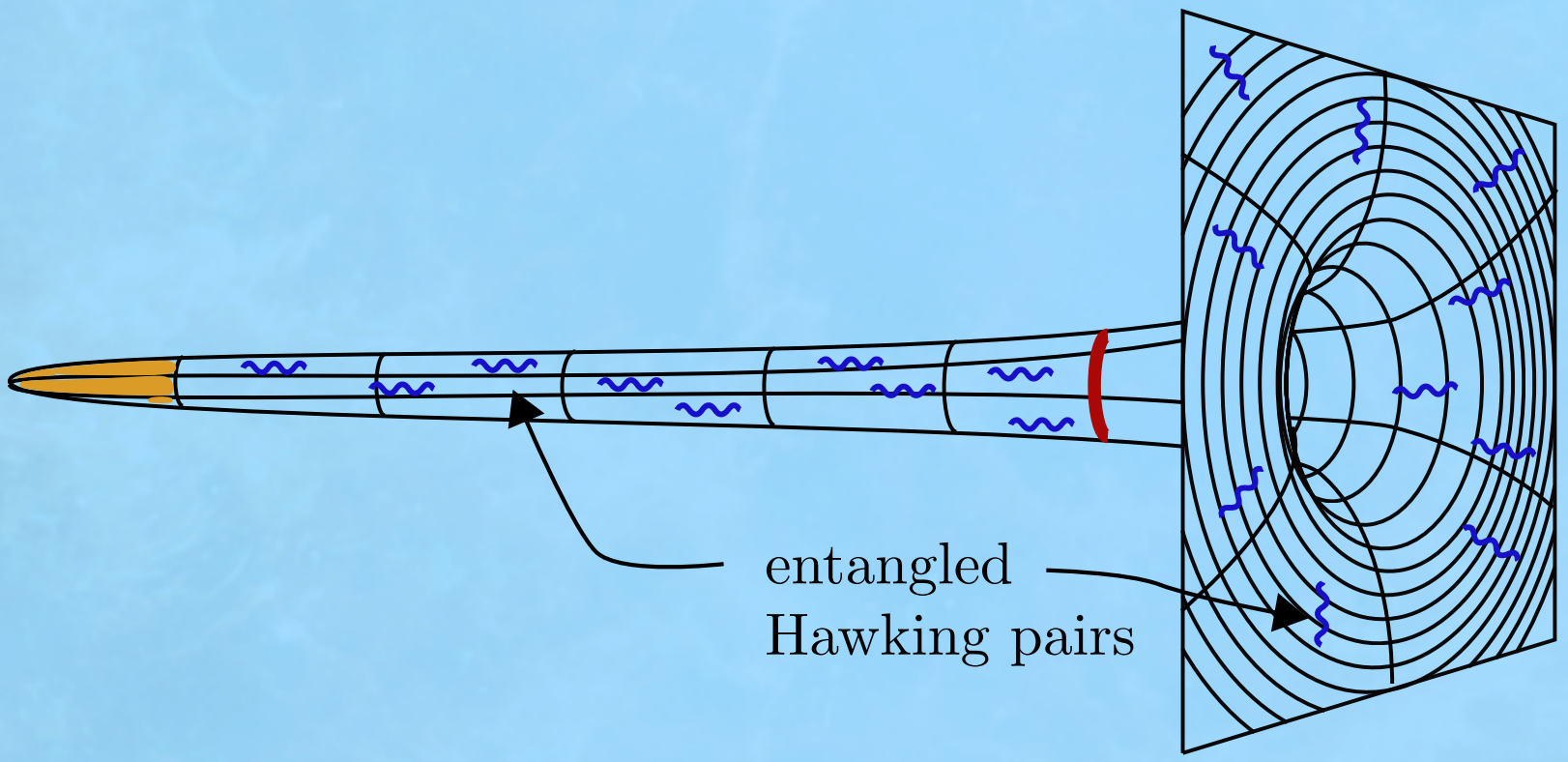
A *pure state* (the collapsing star) evolves into a *mixed state* (thermal Hawking radiation).

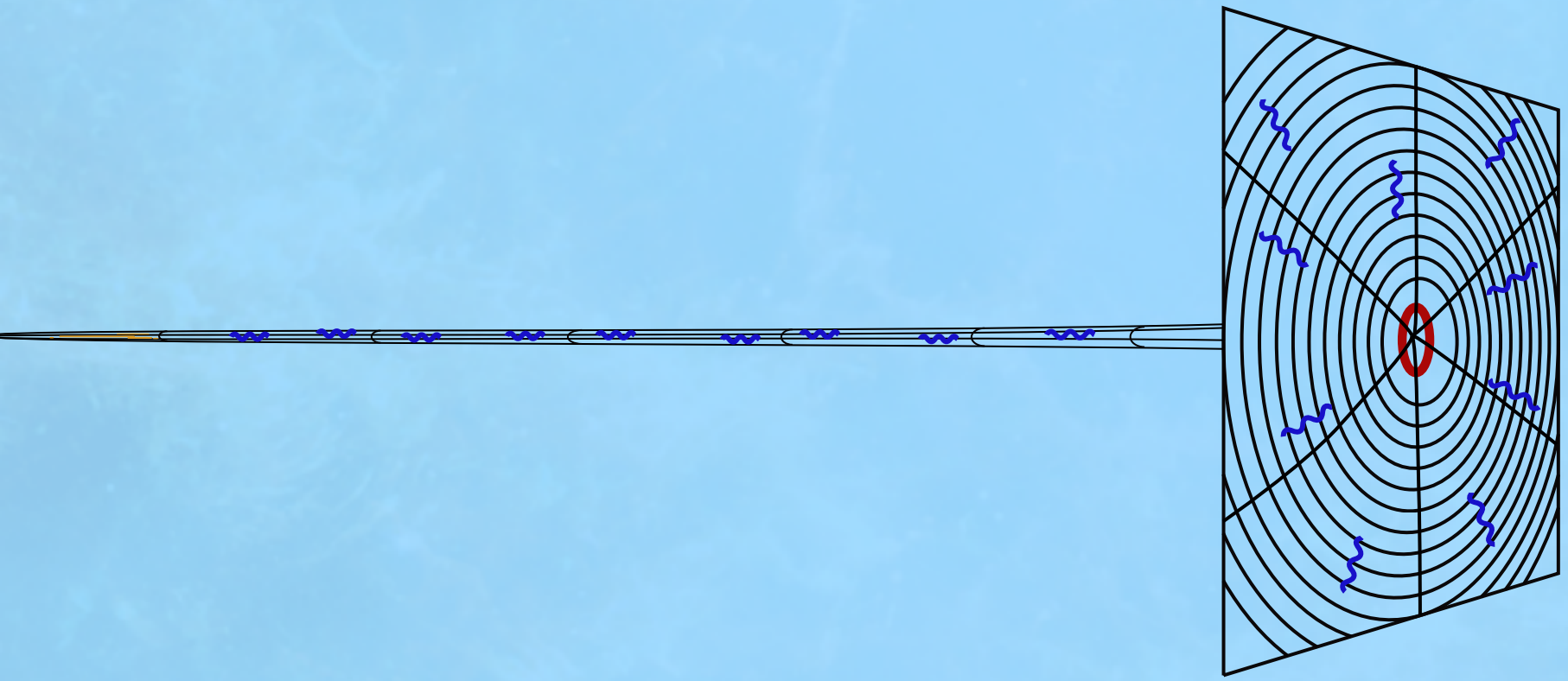
This is a violation of unitary quantum mechanics:

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$

This is the **black hole information paradox** [Hawking 1975]

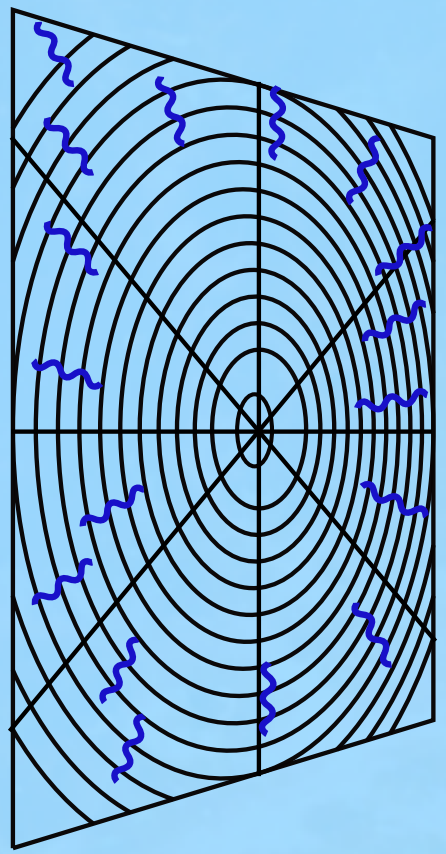

collapsed
star
interior
event horizon


entangled
Hawking pairs



1. Hawking radiation is a process of *entanglement production* between the black hole interior and the radiation.

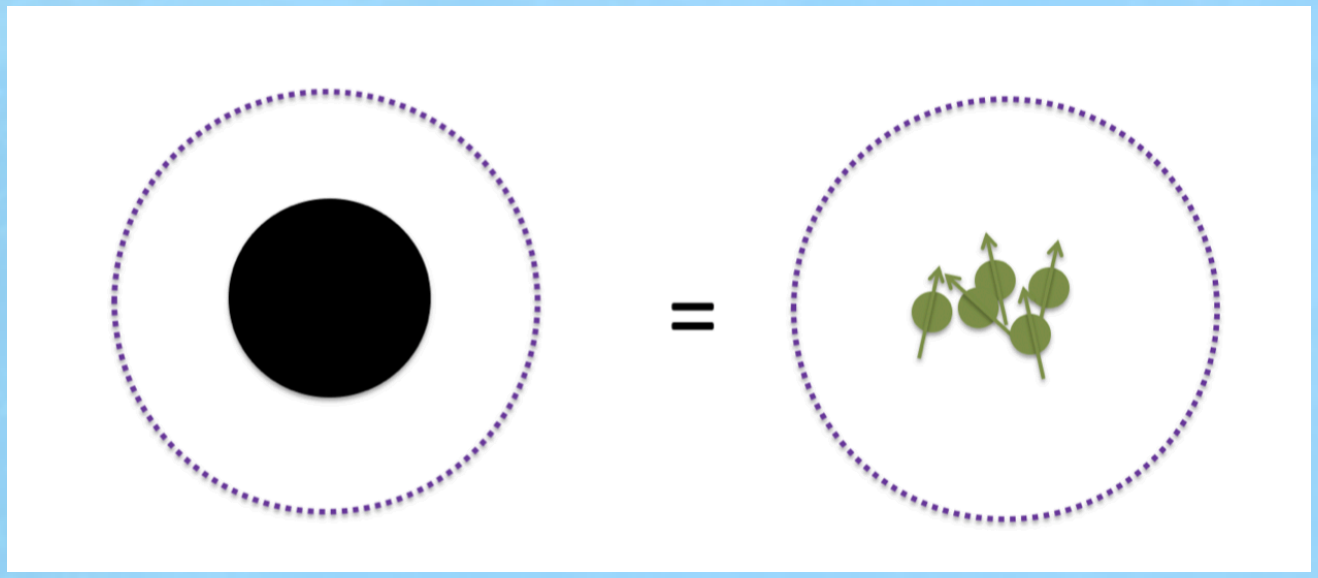
2. The paradox is that the final radiation has nothing to be entangled with.



The Unitary Black Hole Hypothesis

A black hole is an ordinary, unitary quantum system with $\dfrac{\text{Area}}{4}$ degrees of freedom.

These degrees of freedom are not visible in the classical theory, but must exist in quantum gravity.



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Consequences

1. Black hole thermodynamics is *ordinary* thermodynamics for these hidden degrees of freedom in the quantum theory.

2. Spacetime is an emergent phenomenon.

   Einstein's equations of general relativity govern collective long-distance modes (compare: hydrodynamics).

The consequences reach beyond black hole physics:

What is the statistical mechanics of spacetime?

Evidence for the unitary hypothesis

Black hole thermodynamics

String theory

Holographic duality (AdS/CFT correspondence)

However, the information paradox seems to be evidence *against* the unitary hypothesis

In the rest of this talk,

1. Pinpoint one aspect of the paradox: the large entropy of Hawking radiation

2. Describe a new effect leading to low entropy, consistent with unitarity

Nothing in the recent work requires string theory or duality.

The information paradox is (partly)
a problem with *entropy*.

Fine-grained vs. Coarse-grained entropy

The **fine-grained** entropy of a density matrix ρ is

$$S(\rho) = -\mathrm{tr}\rho\log\rho$$

Also called the “von Neumann entropy” or “entanglement entropy” or "quantum entropy”.

Zero in a pure state, $\rho = |\psi\rangle\langle\psi|$

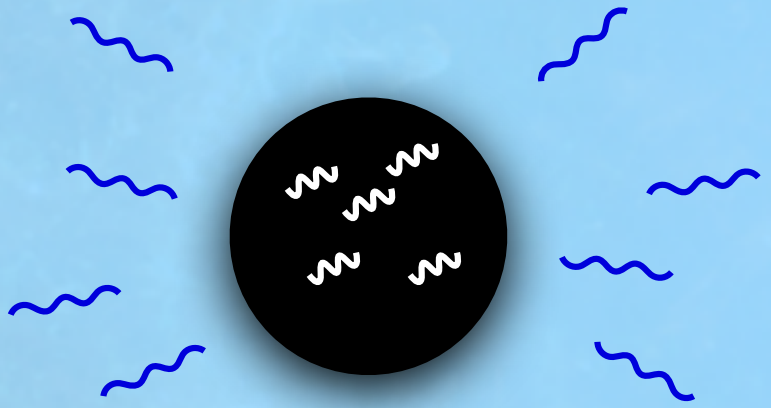
Invariant under unitary time evolution

Not to be confused with the **coarse-grained** or **thermodynamic** entropy,

$$S_{\mathrm{coarse}}(\rho) = \max_{\tilde{\rho}|E} S(\tilde{\rho})$$

which can increase under time evolution (2nd law of thermodynamics).

We are particularly interested in cases where the entropy is due to entanglement.



or $|\Psi\rangle = |00\rangle + |11\rangle$

In this case the fine-grained entropy is called ***entanglement entropy.***

For a system ***A*** entangled with a system ***B*,**

$$S(\rho_A) = S(\rho_B)$$

This gives an upper bound on how much entropy you can create with entanglement. For qubits,

$$S(\rho_A) \leq N_B \log 2$$

Entanglement entropy can be measured in ultracold atoms.

Measuring entanglement entropy through the interference of quantum many-body twins

Rajibul Islam, Ruichao Ma, Philipp M. Preiss, M. Eric Tai, Alexander Lukin, Matthew Rispoli, Markus Greiner
Department of Physics, Harvard University, Cambridge, MA 02138, USA
(Dated: September 4, 2015)

[Nature 2015]

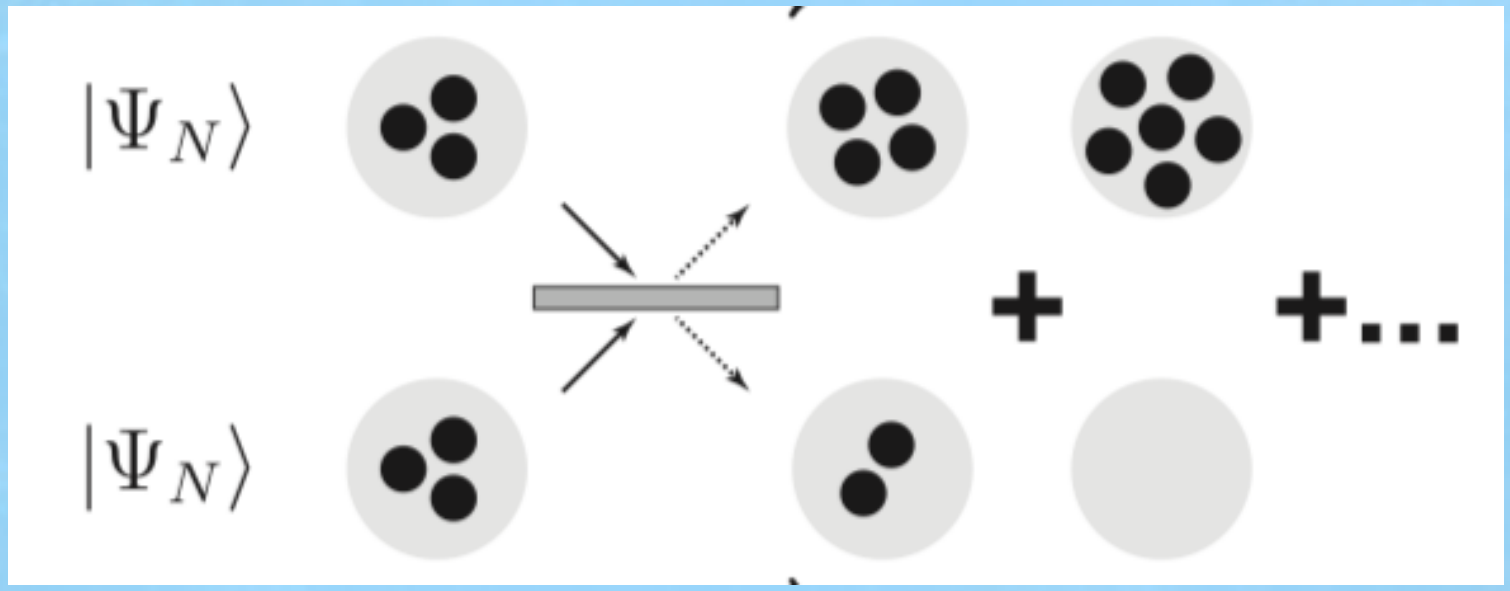
Actually, what they measure directly is a related quantity called the "purity" (or Renyi entropy)

$$Z_2 = \mathrm{tr}\,\rho^2$$

This is measured by "**replicating**":

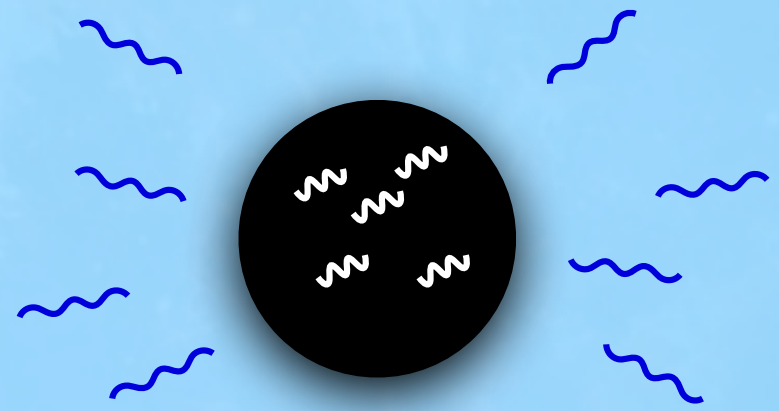
original

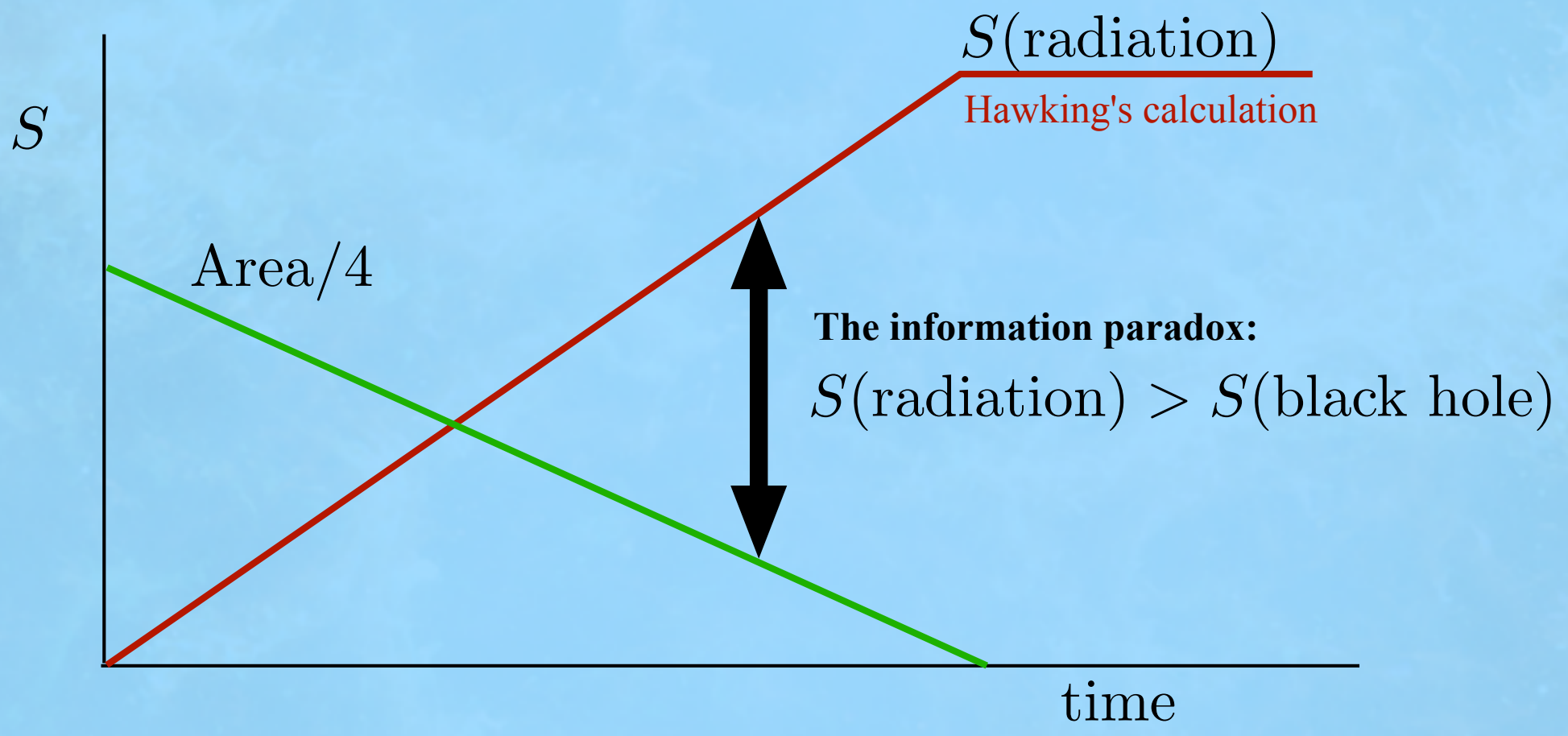
replica



The Page Curve

Compute the fine-grained entropy of the Hawking radiation as a function of time:





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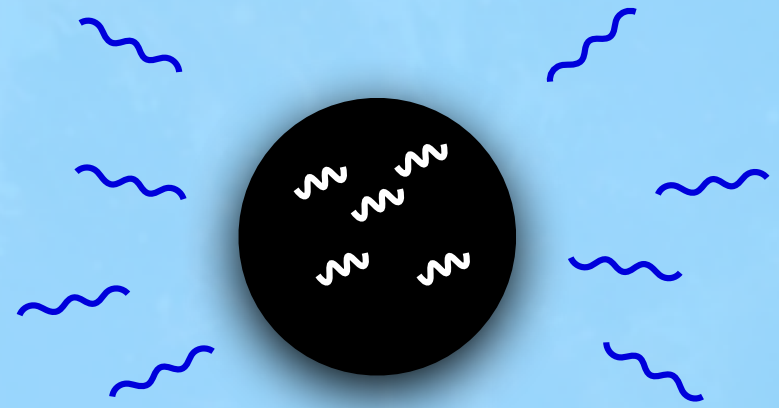
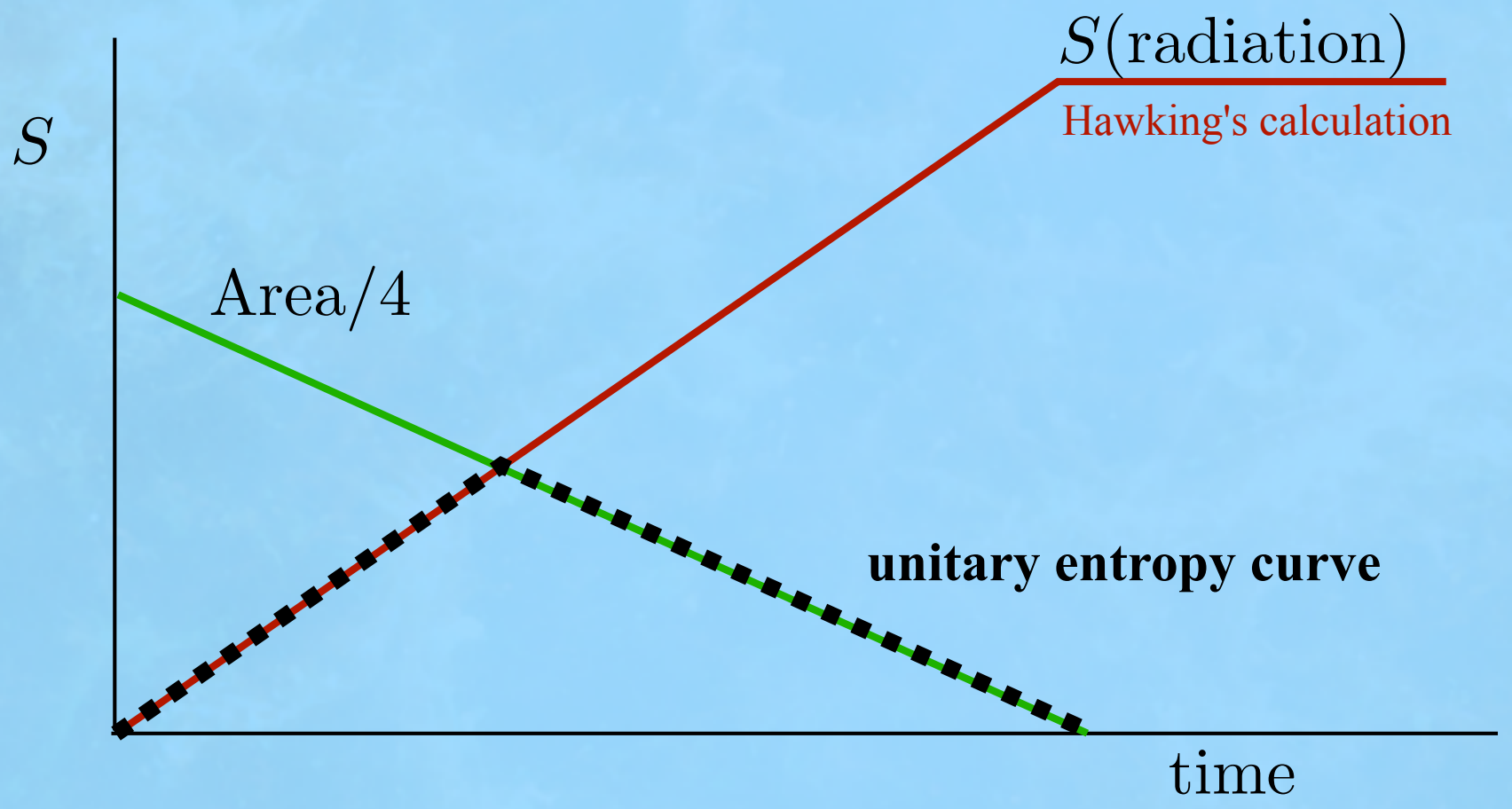




How can the entropy go back to zero?

It is impossible to solve the problem with *any* small, local corrections.

[Mathur '09; Almheiri, Marolf, Polchinski, Sully '12]

Any "fix" must come from non-perturbative quantum gravity,

$$e^{-M_{\rm Planck}}$$

Until recently we had no idea how to calculate these corrections.

I will now describe the corrections, discovered recently from the gravitational path integral.

Caveat: This is just one piece of the black hole information puzzle.

Recent progress

1. Holographic entanglement entropy

[Ryu and Takayanagi '06]
[Hubeny, Rangamani, Takayanagi '07]
[Lewkowycz, Maldacena '13]
[Barella, Dong, Hartnoll, Martin '13]
[Faulkner, Lewkowycz, Maldacena '13]
[Engelhardt, Wall '14]
[Dong, Lewkowycz '17]

2. The "Island formula" for the radiation entropy

[Penington '19]
[Almheiri, Engelhardt, Marolf, Maxfield '19]
[Almheiri, Mahajan, Maldacena, Zhao '19]

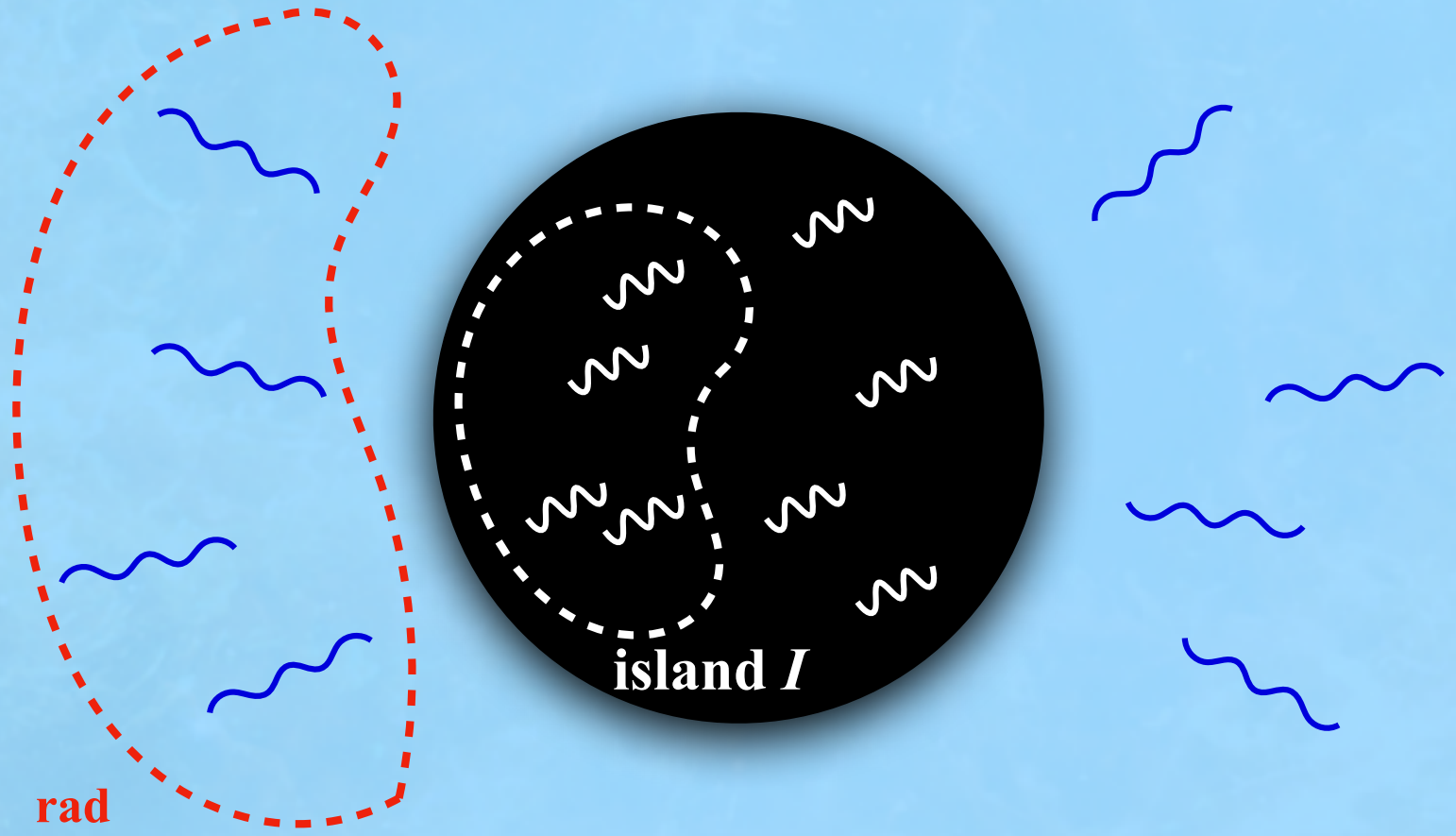
3. Replica wormholes

[Almheiri, TH, Maldacena, Shaghoulian, Tajdini '19]
[Penington, Shenker, Stanford, Yang '19]

Conceptual review article: arXiv 2006.06872 [Almheiri, TH, Maldacena, Shaghoulian, Tajdini]

Radiation entropy formula

[Penington '19]
[Almheiri, Engelhardt, Marolf, Maxfield '19]



$$S(\text{rad}) = \min_I \left[\frac{\text{Area}(I)}{4} + S(I \cup \text{rad}) \right] \approx \frac{1}{4}\text{Area}(I)$$

quantum gravity

QFT

at late times

Replica method

[Edwards and Anderson, 1975]

We are interested in the fine-grained entropy $S(\rho) = -\mathrm{tr}\rho\log\rho$

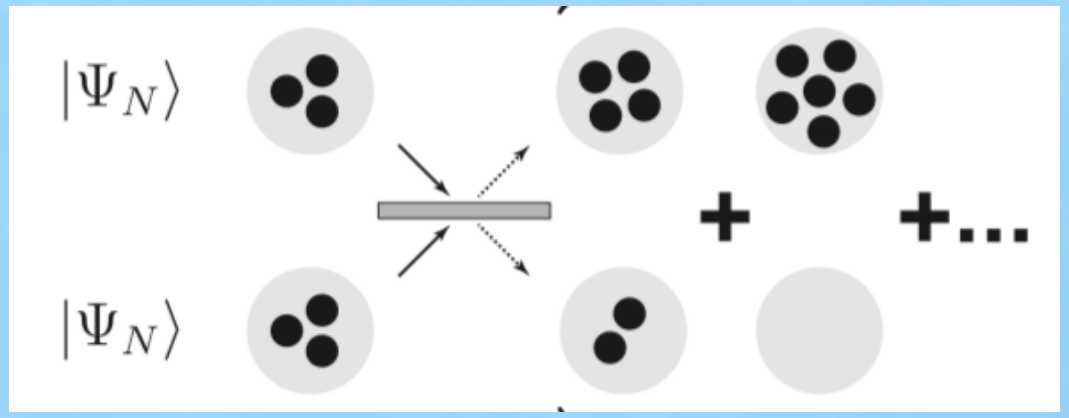
But instead, we introduce extra, “ficticious” copies of the system and calculate the "replica partition function” (a.k.a. Renyi entropy)

$$Z(n) = \mathrm{tr}(\rho^n)$$

This encodes the entropy by an analytic continuation in *n*

$$S(\rho) = -Z'(1)$$

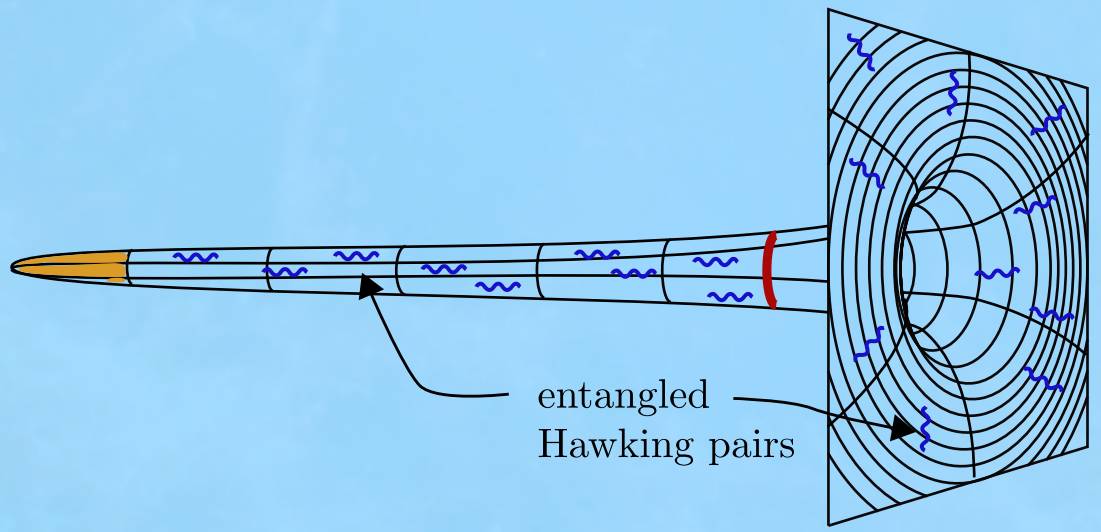
Compare: Ultracold atoms and the purity.



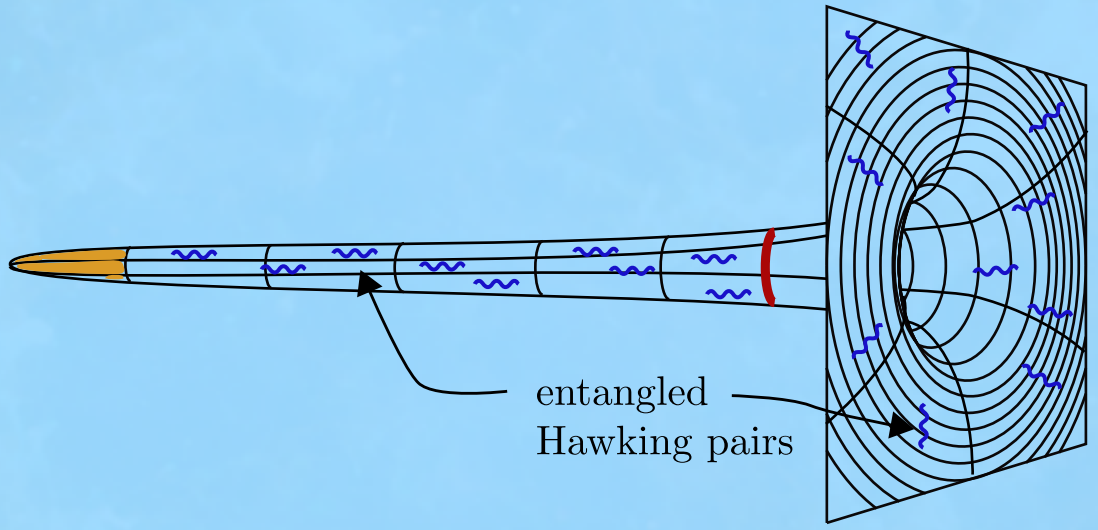
Replica method applied to Hawking radiation

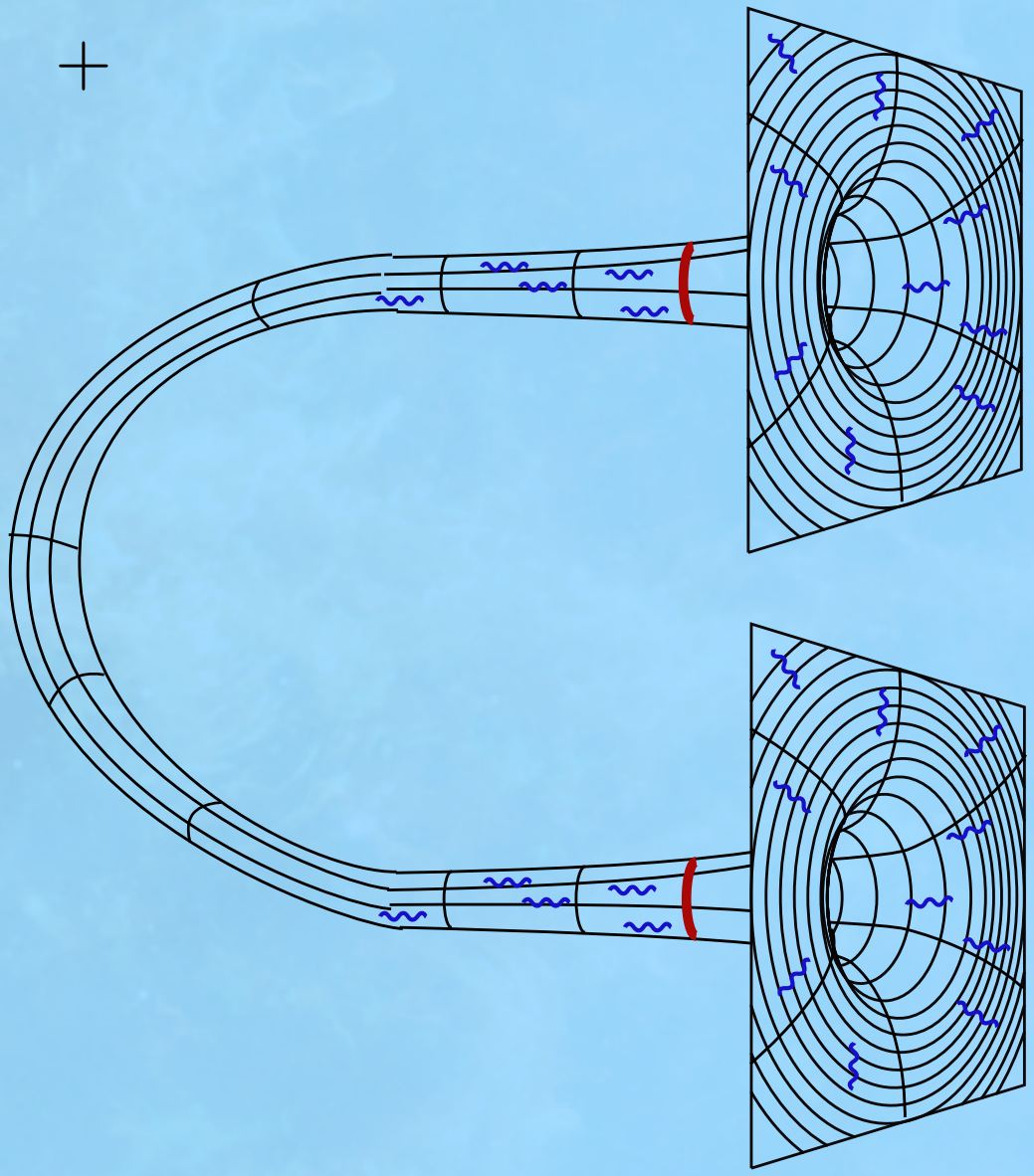
$$\mathrm{tr}\,\rho^2 \approx$$

original



replica





“Replica wormhole”

This is an exact solution of the nonlinear Einstein equations

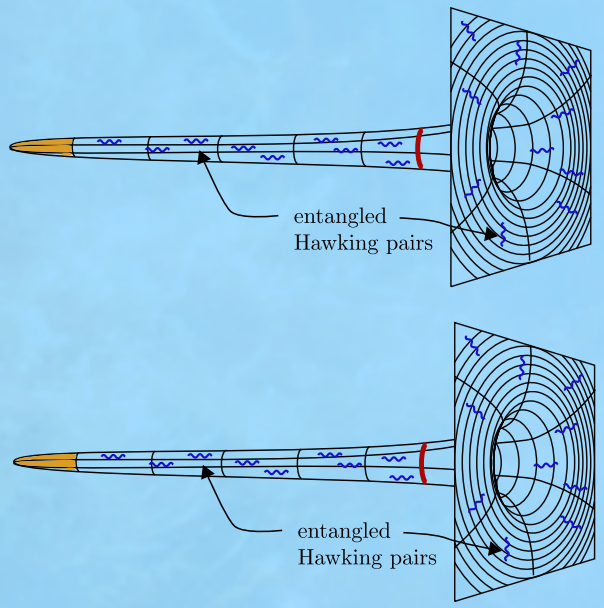
$$G_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle$$

This is actually an instanton, with a complex-valued spacetime metric.

The source on the right is a 1-loop, Casimir-like effect.

It is proportional to the entanglement of the Hawking radiation with the black hole interior.

$$\mathrm{Tr}\ \rho^2 =$$



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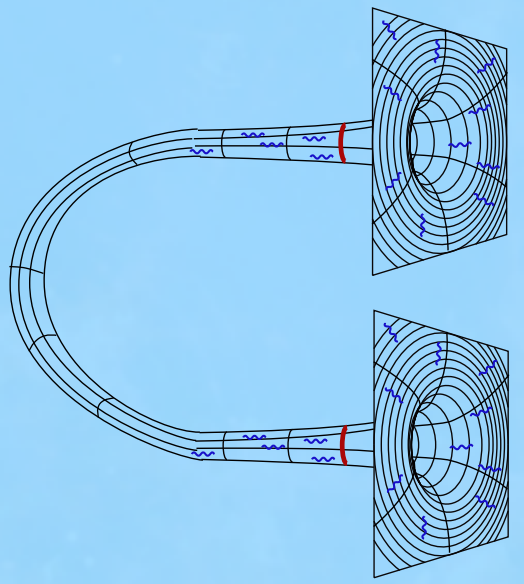
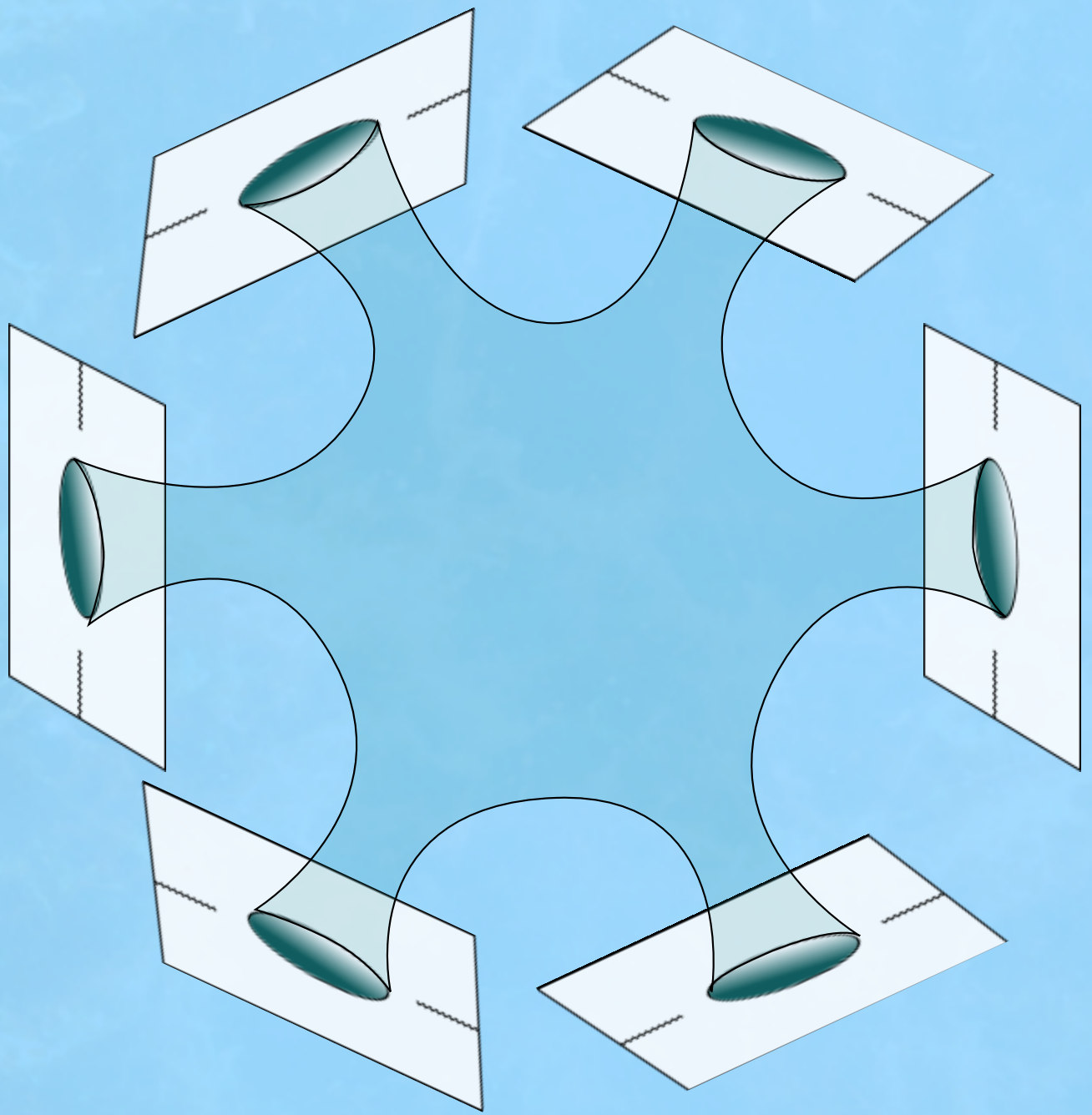


$+ \cdots$

$$= \quad e^{-S_2^{Hawking}} \quad + \quad e^{-S_2^{Wormhole}} \quad + \quad \cdots$$

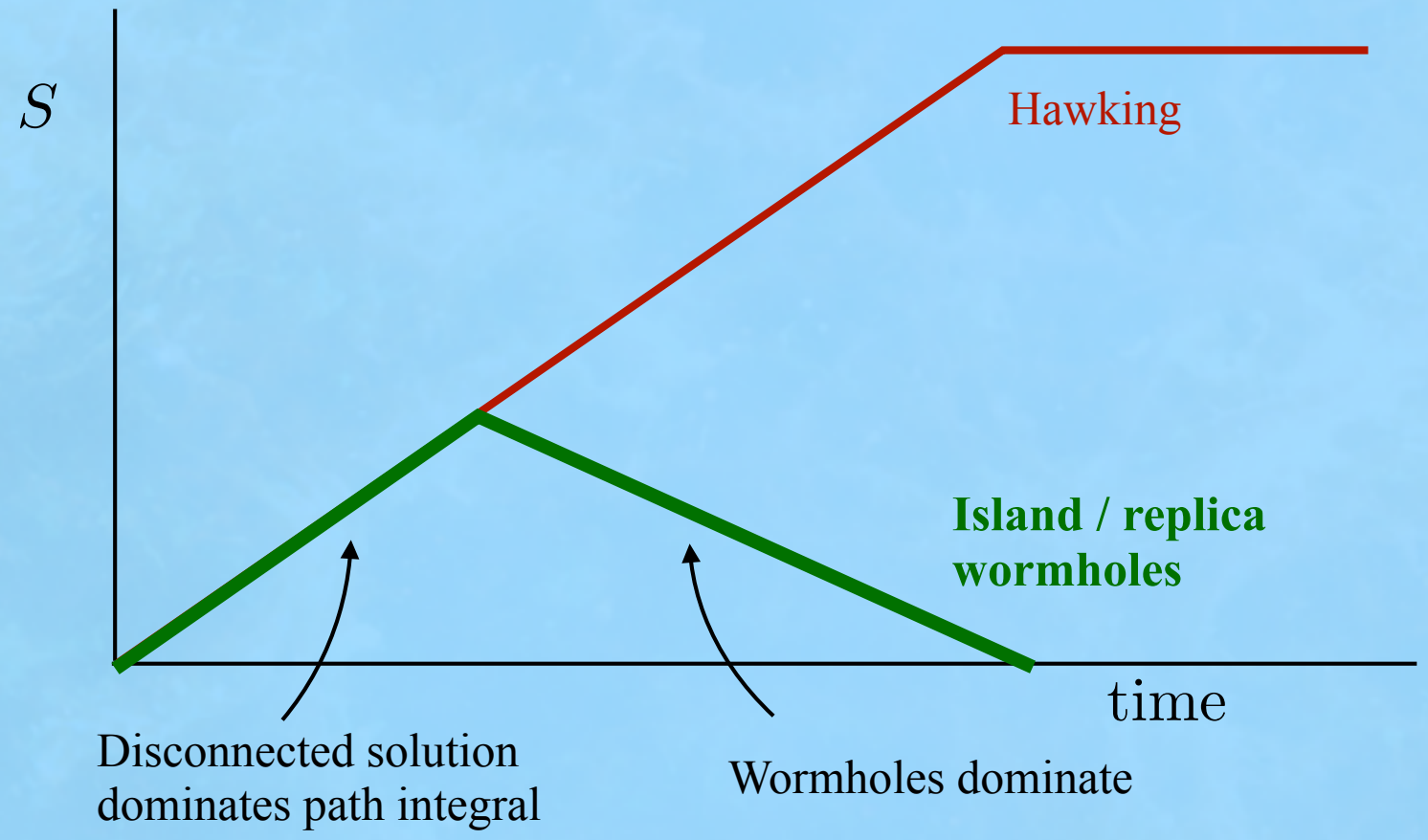


Entropy

$n \to 1$

The result is the "island formula."

And therefore the unitary Page curve.



The lesson:

This is strong evidence in favor of the unitary black hole hypothesis.

Higher topology spacetimes provide information about the microscopic theory.

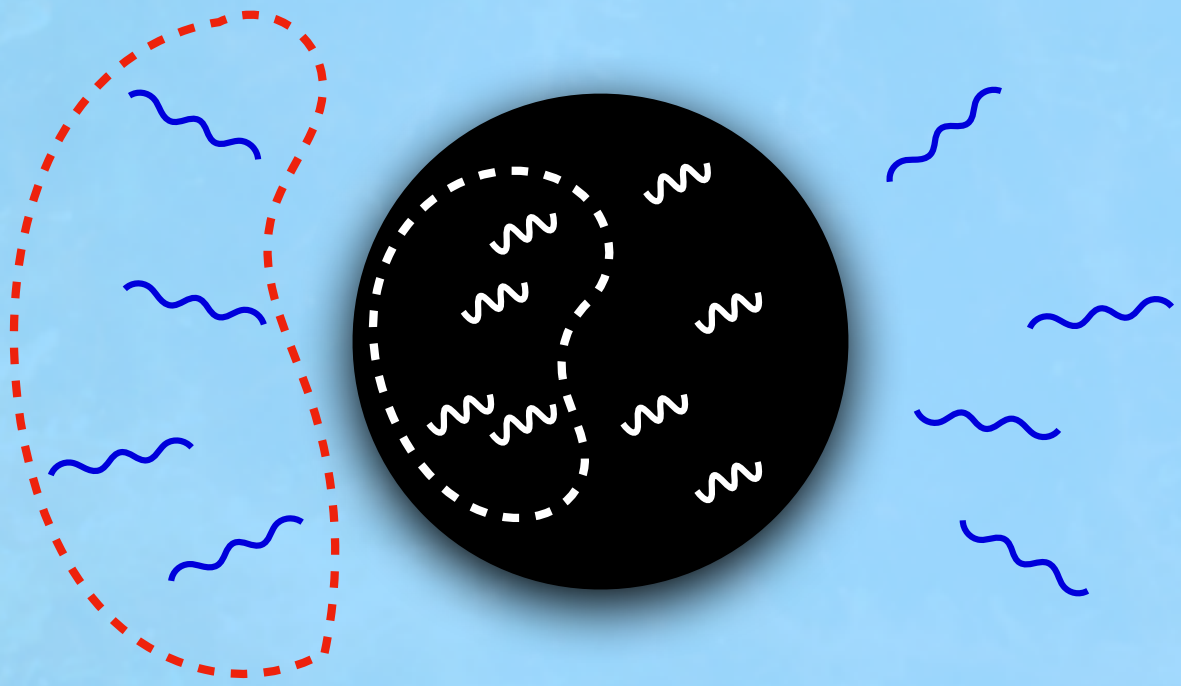
The surprise:

The low-energy theory (Einstein's theory coupled to quantum fields) is already "smart enough" to do this calculation.

This is remarkable, because the low-energy theory is *not* smart enough to calculate the wavefunction of the radiation.

Outlook

Interpretation of the island

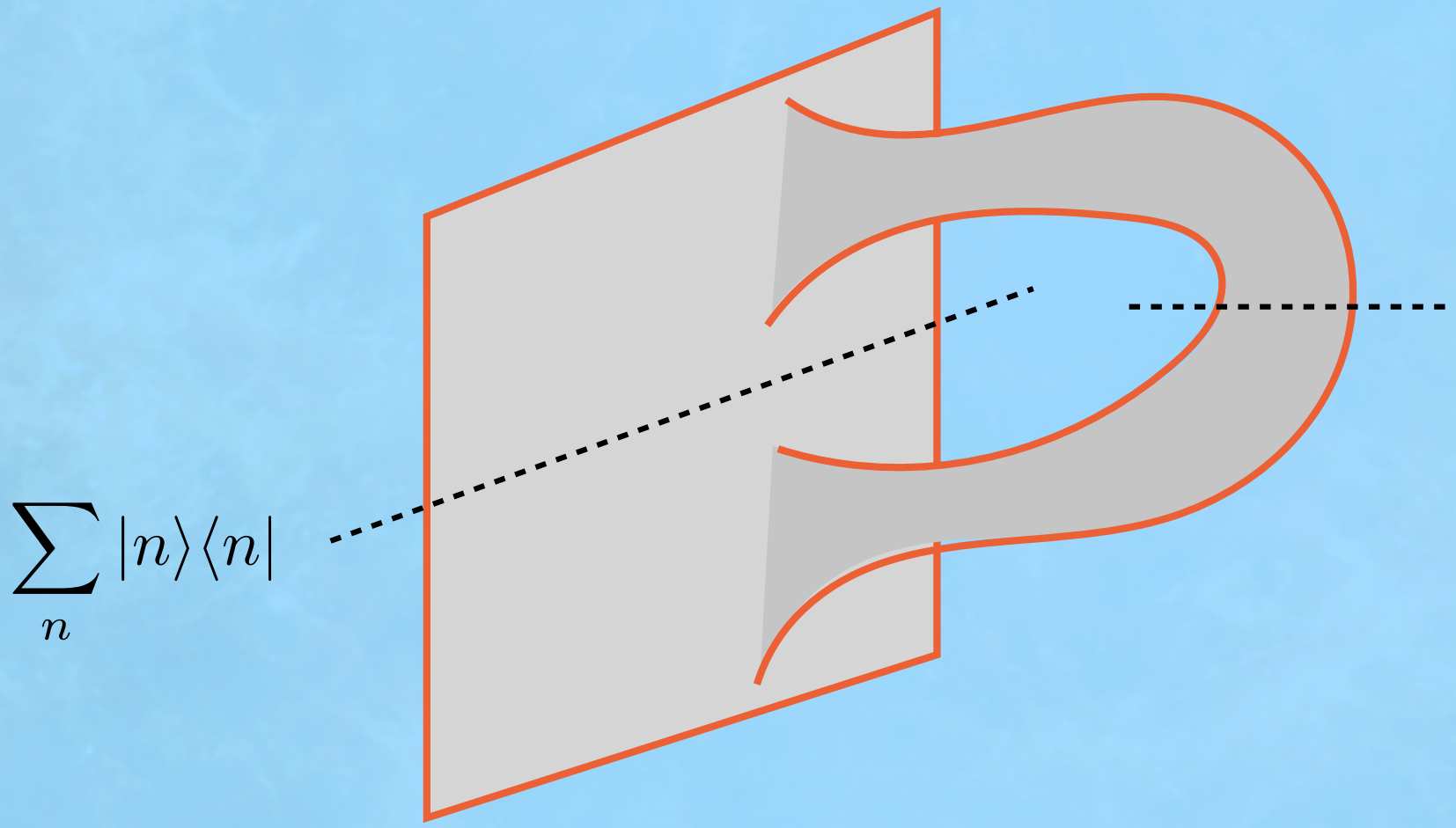


General arguments from quantum information indicate that the “island" is actually *fully encoded* in the radiation.

This is a subtle, non-local effect of quantum gravity.

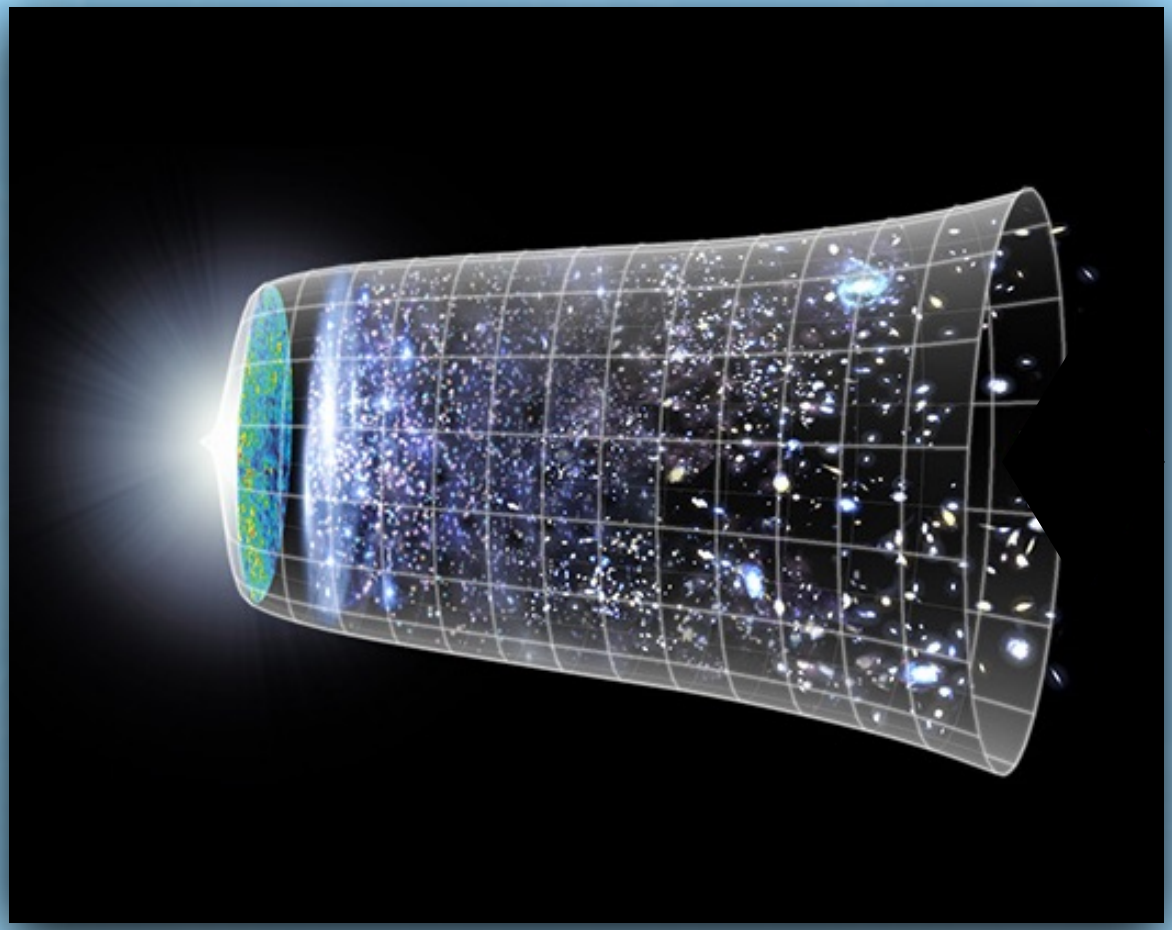
Work is underway to understand this encoding and its implications for quantum gravity.

Baby universes



[Old work by Coleman, Giddings, Strominger, etc.]
[Saad Shenker Stanford ’19]
[Marolf, Maxfield ’20]
etc.

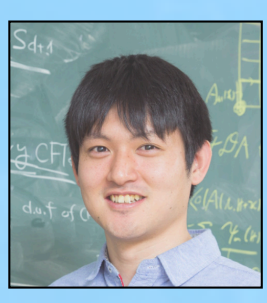
Quantum cosmology revisited?

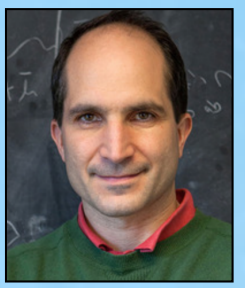


Thank you.

Collaborators on this and related work

Ahmed Almheiri
Kanato Goto
Yikun Jiang
Juan Maldacena
Edgar Shaghoulian
Andrew Strominger
Amir Tajdini

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