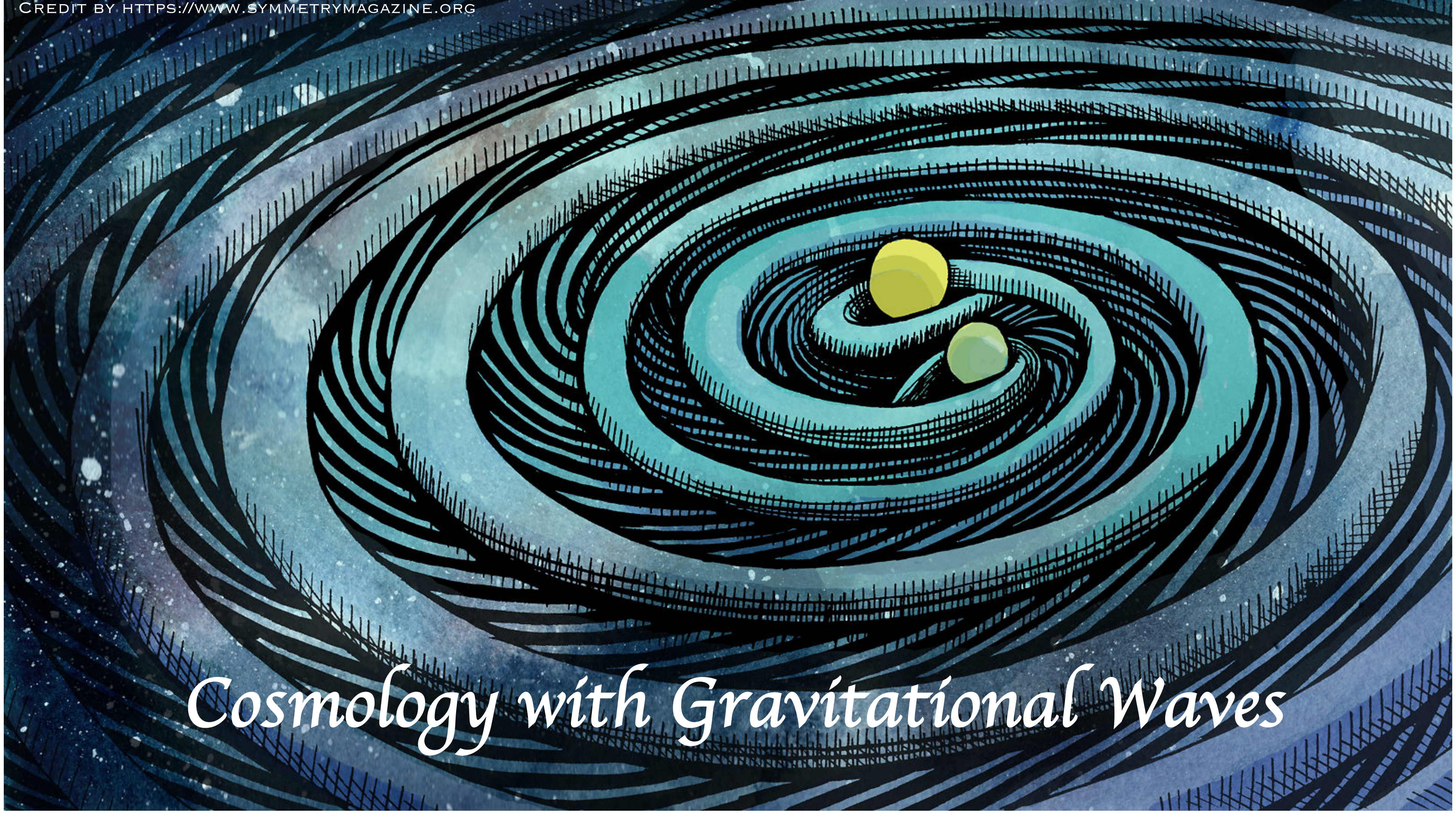

Cosmology with Gravitational Waves

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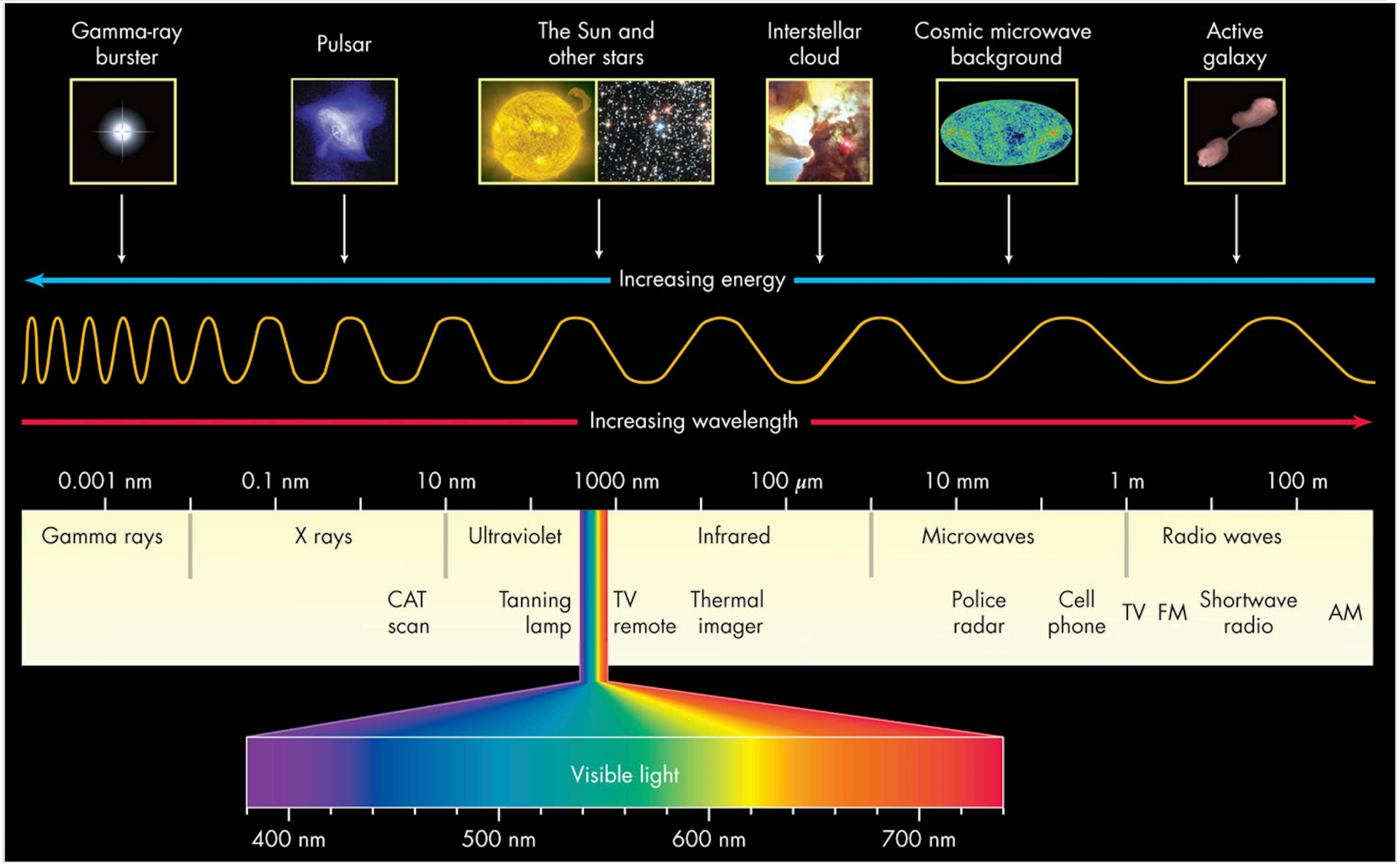
INFN

asi COSMOS

Tunder and lightning

Until a few years ago we have only seen the Universe (and ~ 95% of it is Dark: i.e. Dark Matter and Dark Energy)



(Pulsar): © Courtesy of NASA/CXC/ISAO; (The Sun): © SOHO (ESA & NASA); (other stars): NASA and The Hubble Heritage Team (STScI/AURA); (Interstellar cloud): A. Caulet (ST-ECF, ESA) and NASA; (Cosmic microwave background): Courtesy of NASA/WMAP Science Team; (Active galaxy): © NRAO/AUI/NSF

Tunder and lightning

Until a few years ago we have only seen the Universe (and ~ 95% of it is Dark: i.e. Dark Matter and Dark Energy)

Figure credit: LISA

Through the detection and observation of gravitational waves (GWs):

Trace

- Trace the formation, growth, and merger history of (massive) black holes (BHs), etc.

Explore

- Explore stellar populations and dynamics in galactic nuclei

Test

- Test General Relativity with observations

Probe

- Probe new physics and cosmology

Survey

- Survey compact stellar-mass binaries and study the structure of the Galaxy

What is a Gravitational Wave (GW)?

- In general one gives the name GW to a small ripples rolling across space-time.
- However it is not just a solution of Einstein's differential equations which contains “a lot of” wiggles for ripples in coordinates.
- It is important to distinguish the difference between the physical coordinate independent modes and modes that are purely coordinate artefacts.

Properties of GWs

- Propagate at the speed of light (in “standard” GR)
- Transverse to the direction of propagation
- Two polarizations (+ and x)

$$h_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{+} & h_{\times} & 0 \\ 0 & h_{\times} & -h_{+} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- Here we are assuming a weak gravitational field in GR where $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}\,, \quad |h_{\alpha\beta}| \ll 1$
- GWs are propagating in the *z*-direction;
- GWs are propagating waves of spacetime curvature, tidally stretching and squeezing as they radiate from their source into the Universe.
-The corresponding line element is

$$ds^2 = -dt^2 + (1 + h_{+})\,dx^2 + (1 - h_{+})\,dy^2 + 2h_{\times}\,dx\,dy + dz^2$$

Properties of GWs

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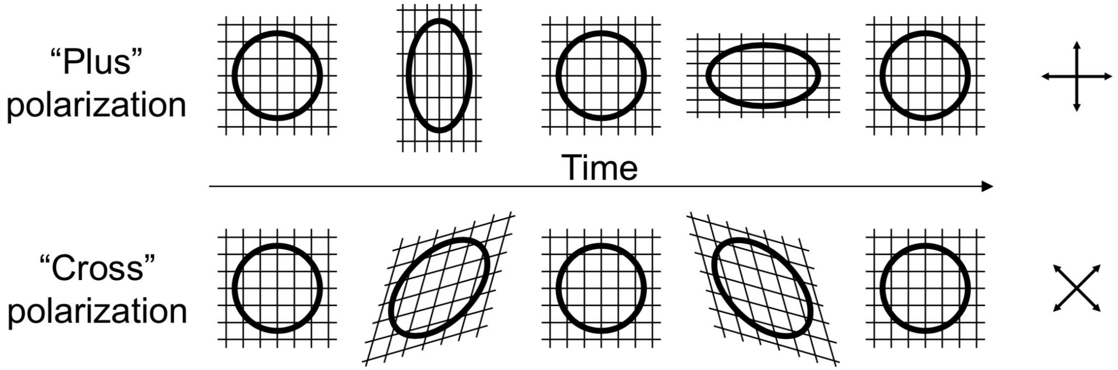
GWs are deformations of space itself, stretching it first in one direction, then in the perpendicular direction.



Properties of GWs

- Propagate at the speed of light (in "standard" GR)
- Transverse to the direction of propagation
- Two polarizations (+ and x)
- Carry energy

The GWs a source emits backreact upon it, which appears as a loss of energy and angular momentum.
The "quadrupole formula" predicts that a system with a time changing quadrupole moment will lose energy to GWs according to

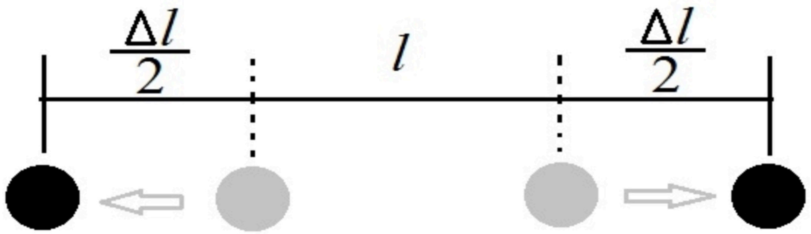
$$\frac{dE}{dt} = -\frac{1}{5}\frac{G}{c^5}\sum_{ij}\frac{d^3Q_{ij}}{dt^3}\frac{d^3Q_{ij}}{dt^3}$$

where

$$h_{ij} = \frac{2G}{c^4}\frac{1}{r}\frac{d^2Q_{ij}}{dt^2} \qquad Q_{ij} = \int \rho\left(x_i x_j - \frac{1}{3}\delta_{ij} r^2\right) dV$$

Properties of GWs

- Propagate at the speed of light (in “standard” GR)
- Transverse to the direction of propagation
- Two polarizations (+ and x)
- Carry energy
- Affect the relative separation of test particles



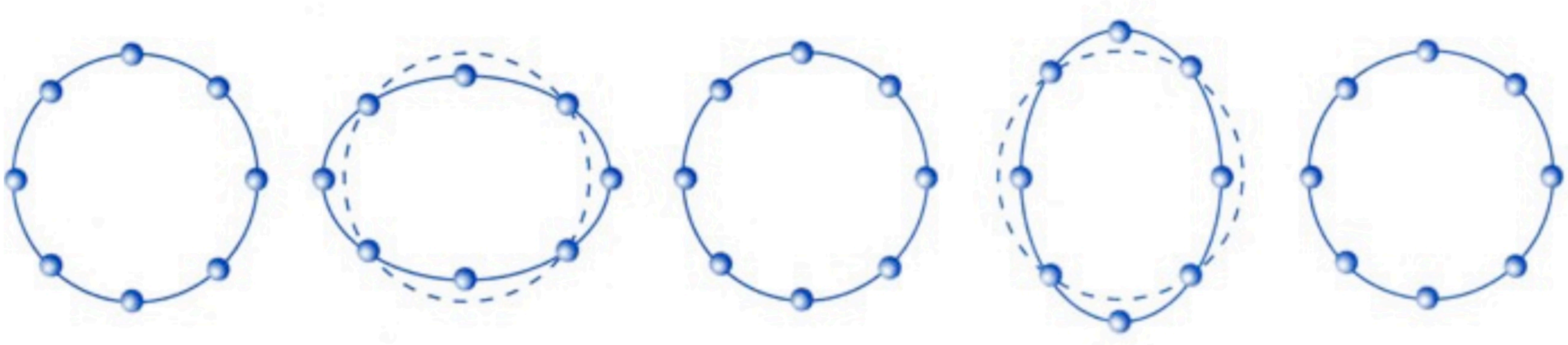
- Δl is the change in the spacing between particles due to gravitational wave,
- l is the initial distance between particles, and
- h is the fractional change in distance (strain) and given by

$$h = \Delta l / l$$

Properties of GWs

In general

- $h = \Delta l/l$ is more complex and depends upon the **geometry of the measurement device**, n^μ the (spatial and unit) separation vector orthogonal to the **arrival direction**, and the **frequency** and **polarization** of the GW
- h **stretch** and **shrink** the distance between two points



$$\frac{\Delta l}{l} = \frac{1}{2} h_{\mu\nu} n^\mu n^\nu$$

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- h **stretch** and **shrink** the distance between two points
- h is the magnitude of a typical component $h_{\alpha\beta}$

$$h \approx \frac{G}{c^4}\frac{mv^2}{r}$$

v is the typical speed associated with the source's quadrupolar dynamics, and *m* is proportional to the mass that participates in those dynamics.

$h = \Delta l / l \approx 10^{-22}$ **Gravitational waves are very very very weak!**

To produce strong GWs need large masses (e.g., at least the mass of the Sun) moving very fast (e.g., near the speed of light)

Properties of GWs

In general

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- h **stretch** and **shrink** the distance between two points
- h is the magnitude of a typical component $h_{\alpha\beta}$
- h depends on the kind of wave to be detected and this in turn depends on how the wave was produced and how far its source is from an observer!

$$
\begin{aligned}
h &\approx \frac{G}{c^4}\frac{mv^2}{r} \\
&\approx 10^{-22} \times \left(\frac{200\,\mathrm{Mpc}}{r}\right) \times \left(\frac{M}{3M_\odot}\right) \times \left(\frac{v}{0.3c}\right)^2 \\
&\approx 10^{-20} \times \left(\frac{6\,\mathrm{Gpc}}{r}\right) \times \left(\frac{M}{10^6\,M_\odot}\right) \times \left(\frac{v}{0.1c}\right)^2
\end{aligned}
$$

Numbers characterizes stellar mass sources that are targets for ground-based high- frequency detectors

Numbers characterizes massive black holes that are targets of space-based low-frequency detectors

Gravitational waves are classified into three types:

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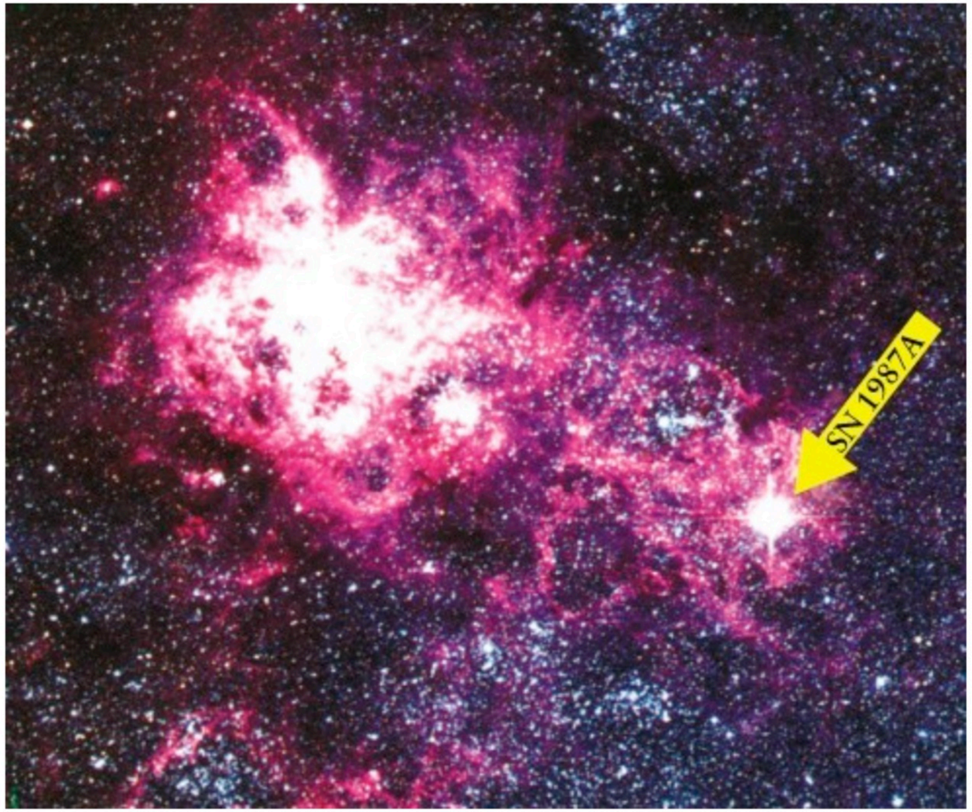
- Impulsive (bursts):

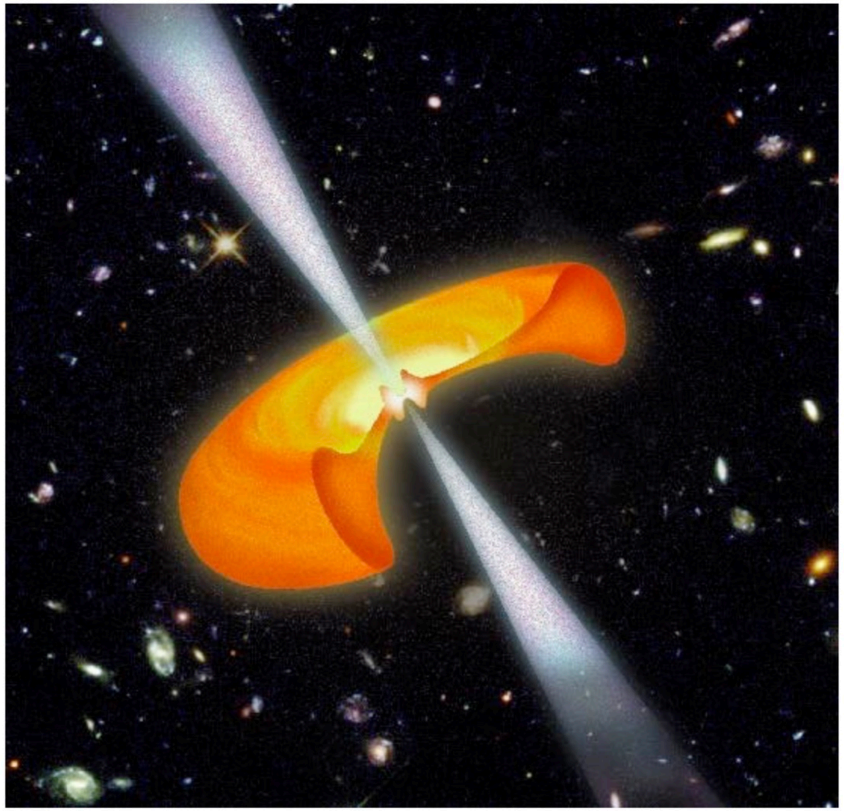
Intense gravitational radiation

→ produced by a supernova explosion

→ gamma ray bursters



SN 1987A



GRB / accreting BH

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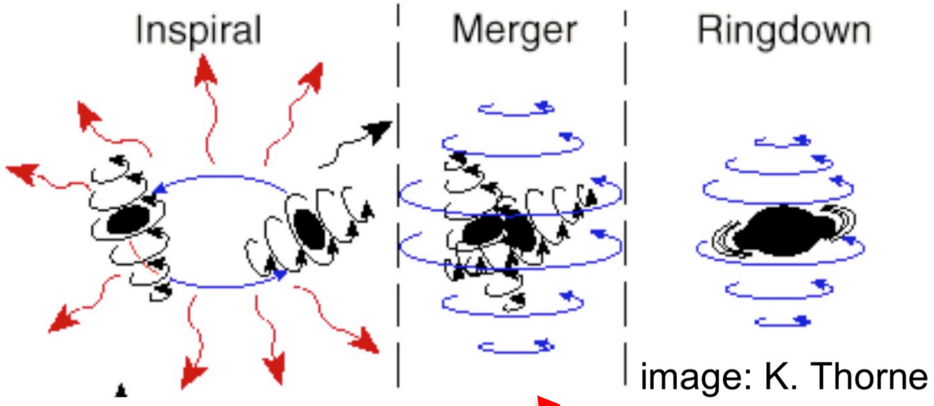
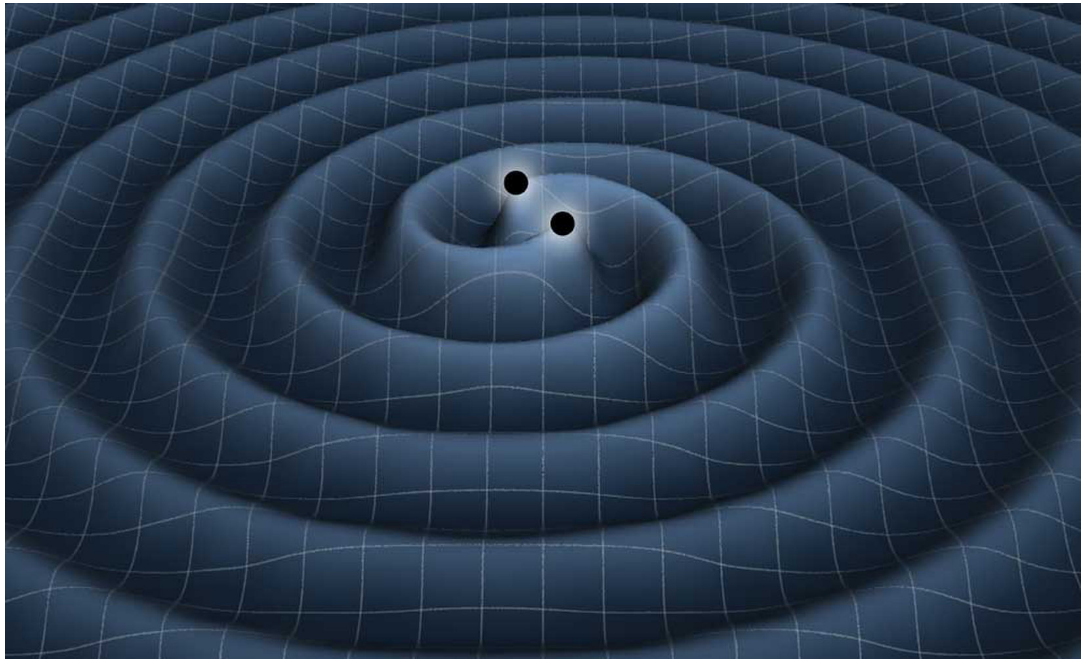
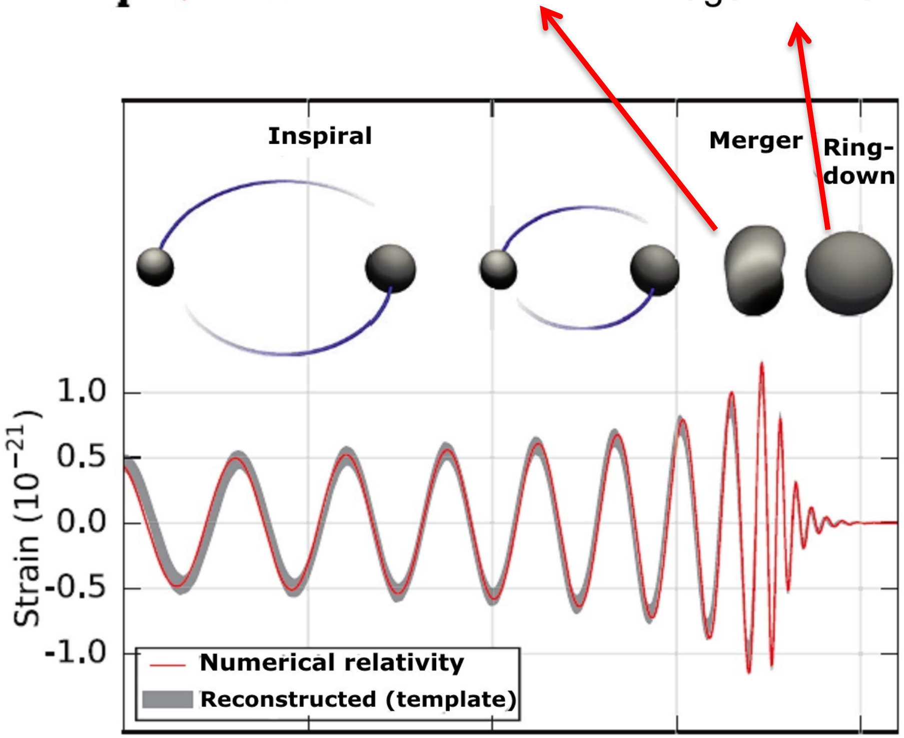


image: K. Thorne





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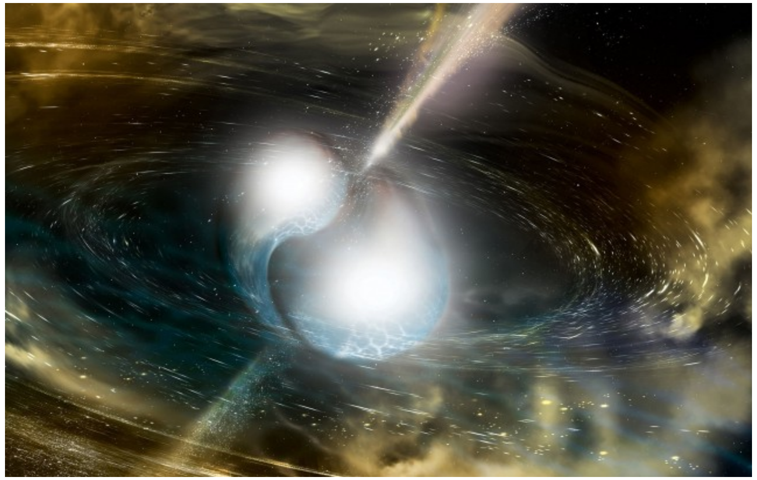

- Impulsive (bursts):

Intense gravitational radiation

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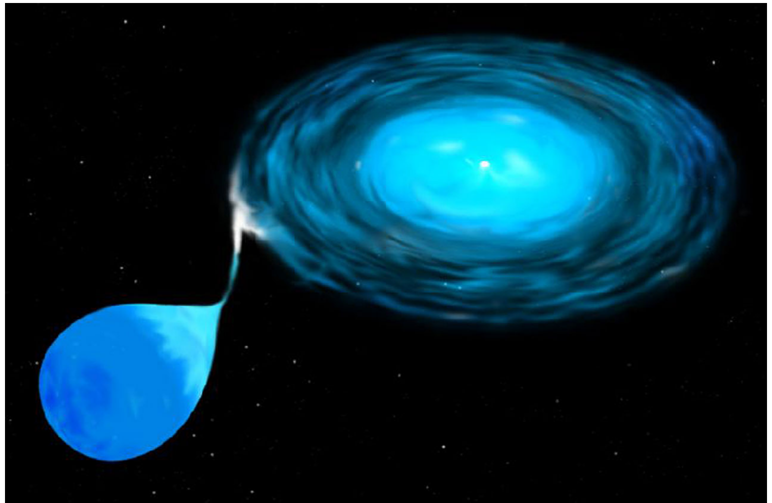
→ through the merging of two black holes.

- Periodic:

corresponds to those whose frequency is more or less constant for long periods of time

→ GWs may have their origin in binary neutron stars (BNS) [or BBH} rotating around their center of mass,

→ From a NS that is close to absorb material from another star (accreting neutron star)



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- Stochastic:

Stochastic waves contribute to the gravitational background noise

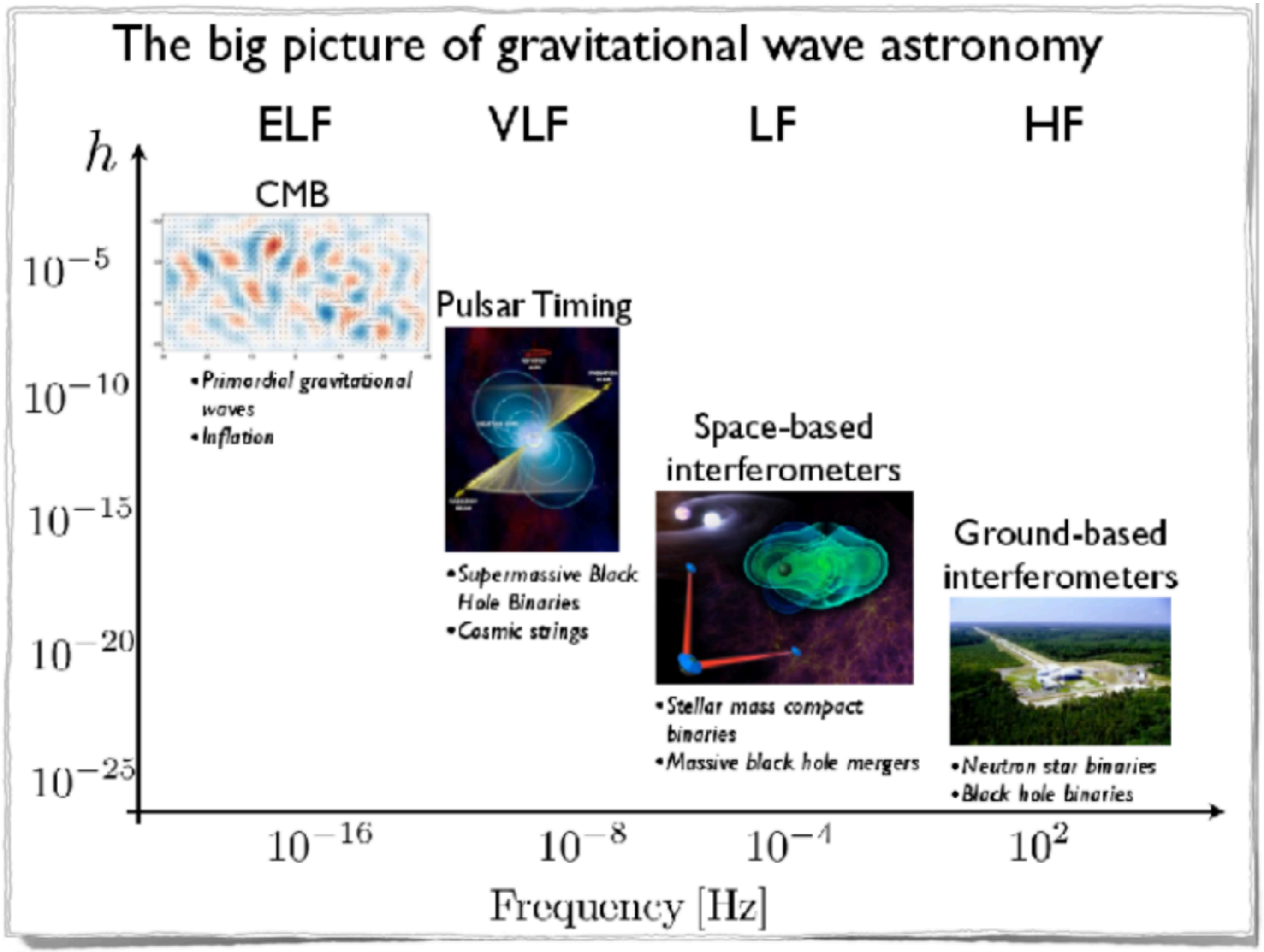
→ possibly have their origin in the Big Bang.

→ stochastic backgrounds due to BBHs or BNS coalescences.

Three different wave types appear in different parts of the spectrum

Gravitational wave spectrum

- GWs radiate at a frequency inversely proportional to their mass.
- Such sources are more intense and are expected to have higher amplitudes.



High frequency

- The high-frequency band of roughly 1-1000 Hz is targeted by ground-based laser interferometers.

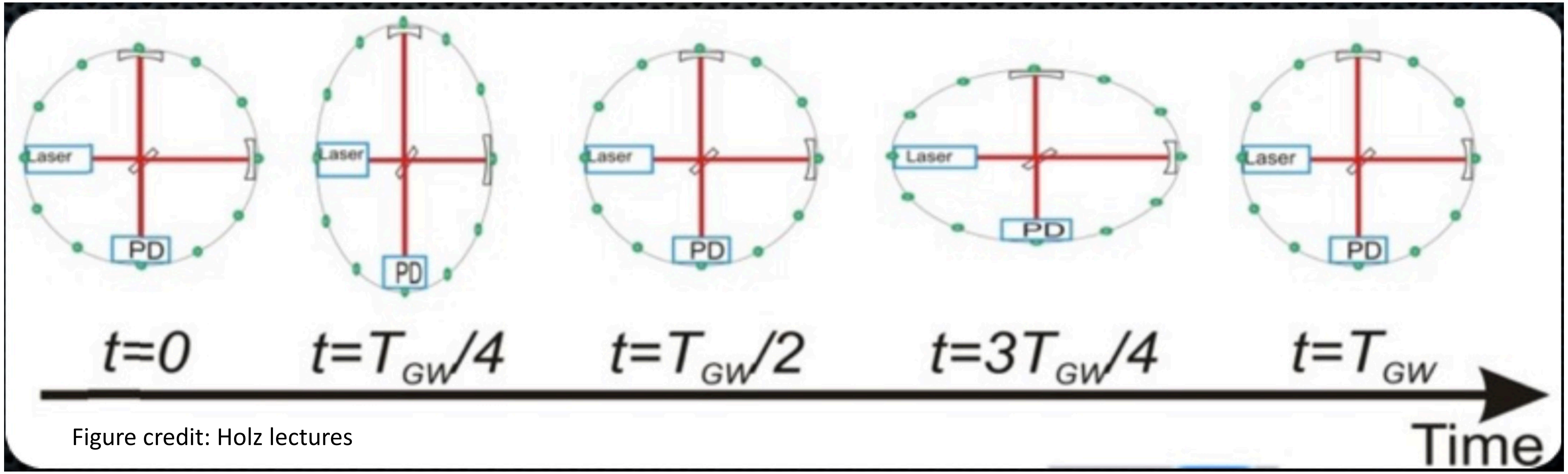
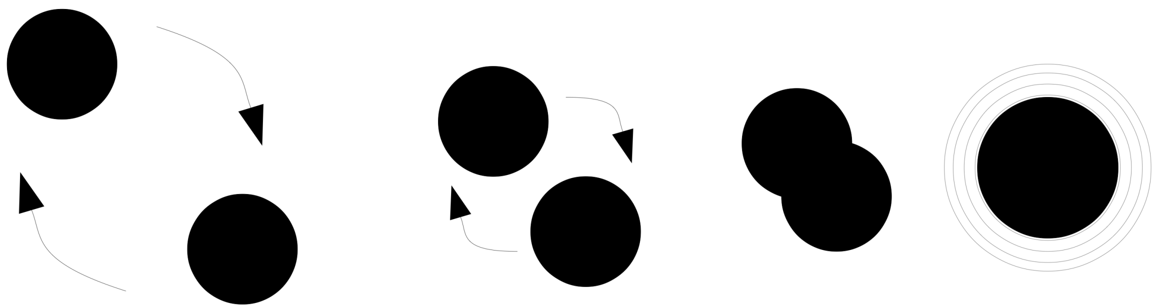


Figure credit: Holz lectures

If the distance between one of the mirrors to the light splitter varies by an amount Δl with respect to distance to the same splitter of the second mirror, then the recombined beam will change its intensity. From measuring the intensity change of the recombined light beam, it is possible to obtain Δl.

High frequency

- The high-frequency band of roughly 1-1000 Hz is targeted by ground-based laser interferometers.
- The expected sources need to be compact [e.g., neutron stars (NSs) or black holes (BHs)] and the inspiral needs to be in its final stages (last few minutes) in order for the GWs to be detectable by Earth-based interferometers.



- For such sources, the natural GW frequency is in the high-frequency band if $M \sim 1 - 100\ M_{\odot}$
- The most prominent facilities are those of LIGO in the USA, VIRGO in Italy, GEO600 in Germany, and KAGRA in Japan, which are all running.
- Finally with the third-generation ground-based interferometer Einstein Telescope (ET) and Cosmic Explorer, (CE) it will be possible to reach frequencies of the order of 1 Hz (since going underground helps to partially overcome the seismic noise).

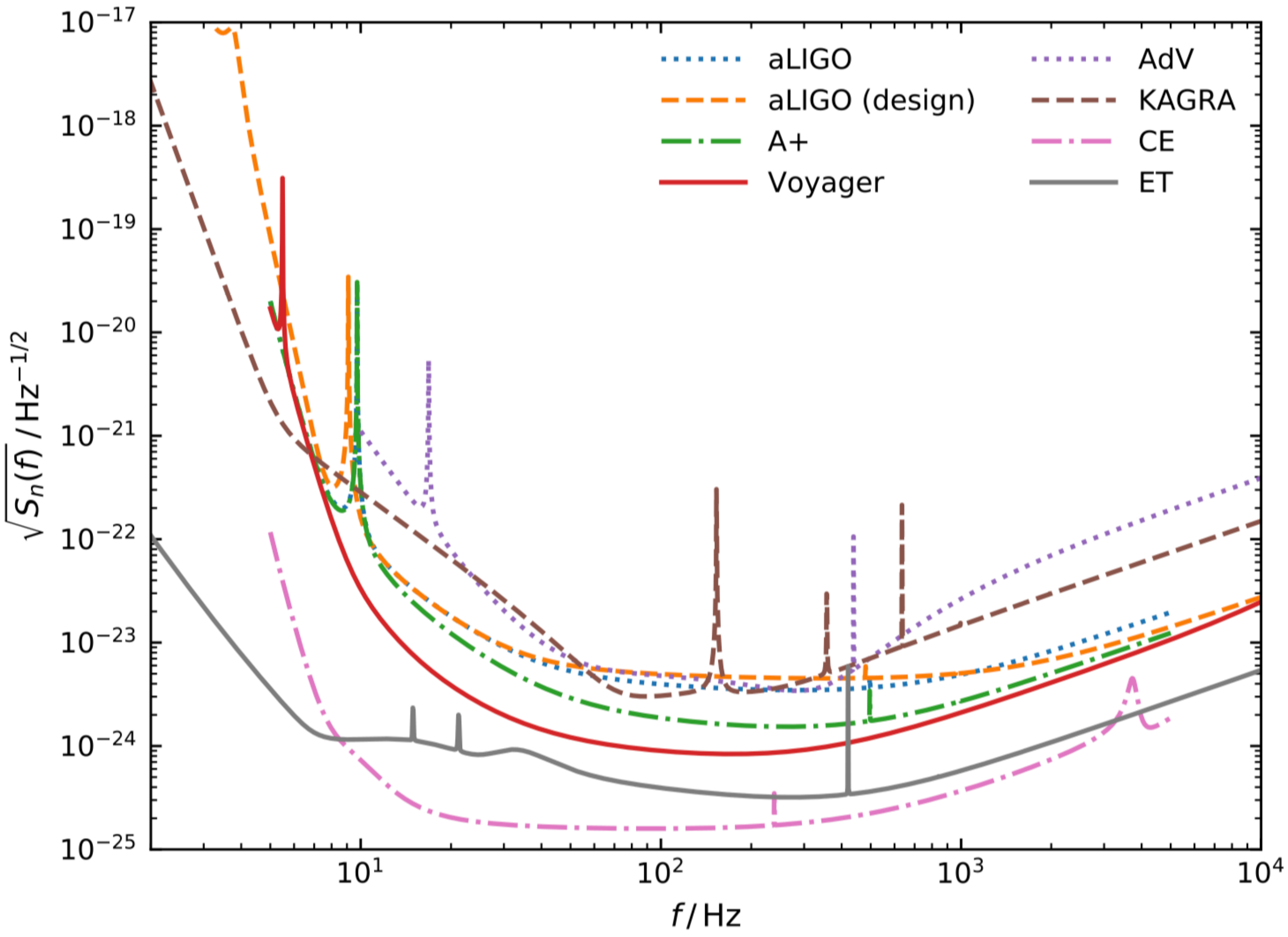
High frequency

Figure credit: Holz lectures

Terrestrial interferometers

High frequency

- Terrestrial interferometers
- Sensitivity evolution of current and proposed GW interferometric detectors.



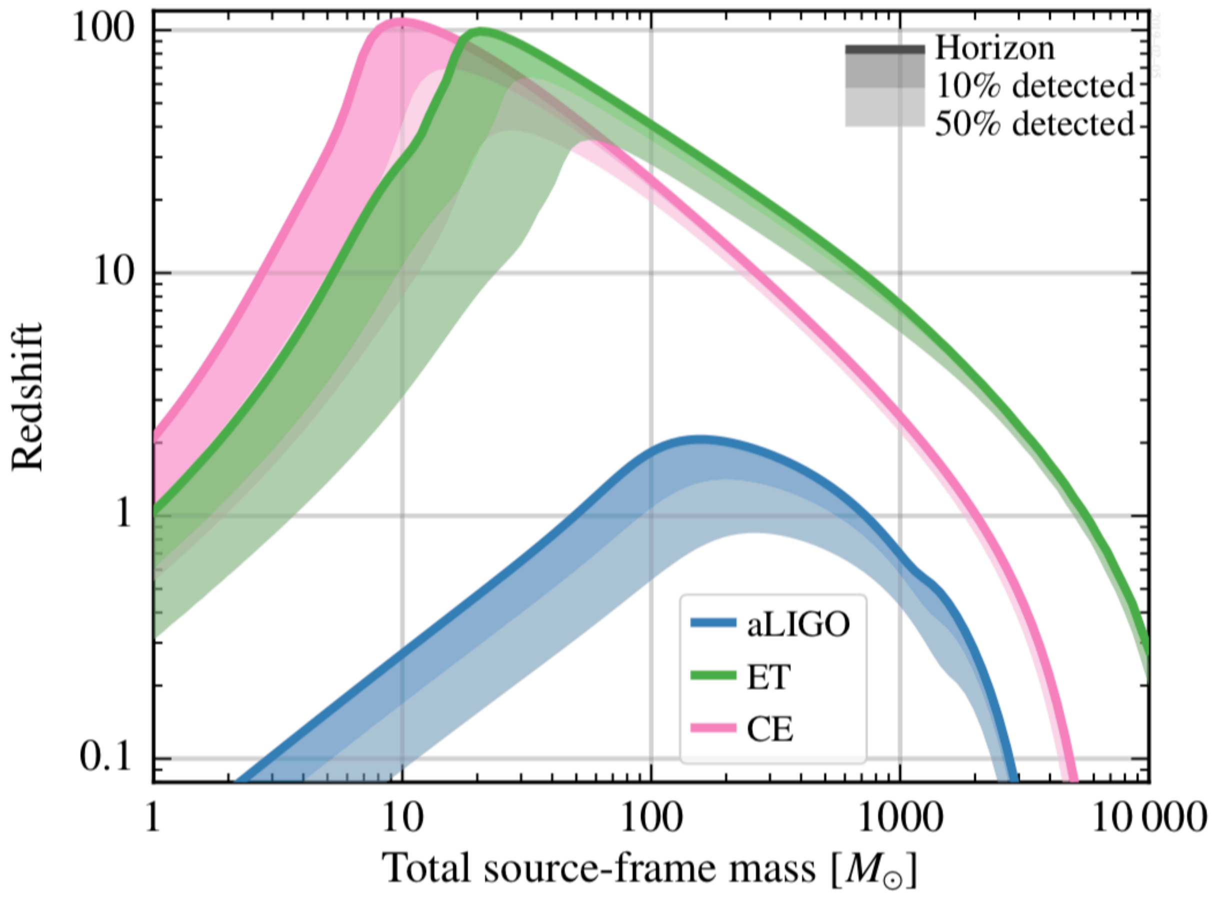
Third generation ground–based interferometric detector expected to be fully operational in early 2030s. It will be observing GWs emanated from BH-BH mergers up to $z \approx 20$, the coalescence of NS-NS systems up to $z \sim 2$, as well as from neutron star–black hole (NS-BH) inspirals up to $z \sim 8$.

Mitra et al. 2021

High frequency

An example of the extraordinary potential of 3G detectors

Maggiore et a. +DB 2020



Astrophysical reach for equal-mass, non-spinning binaries for Advanced LIGO, ET and CE

ET will uncover the full population of coalescing stellar BBH since the end the cosmological dark ages!

- We see that the coalescence of compact binaries with total mass (20 – 100) $M_\odot$, as typical of BH-BH or BH-NS binaries, will be visible by ET up to redshift $z \sim 20$ and higher, probing the dark era of the Universe preceding the birth of the first stars.
- By comparison, in the catalog of detections from the O1 and O2 Advanced LIGO/Virgo runs, the farthest BH-BH event is at $z \simeq 0.5$ and, at final target sensitivity, 2G detectors should reach $z \simeq 1$.
- The range of BH masses accessible will also greatly increase; ET will be able to detect BHs with masses up to several times 10^3 $M_\odot$, out to z~1–5.

ET and CE can contribute to uncover the star-formation history of the Universe!

Low frequency

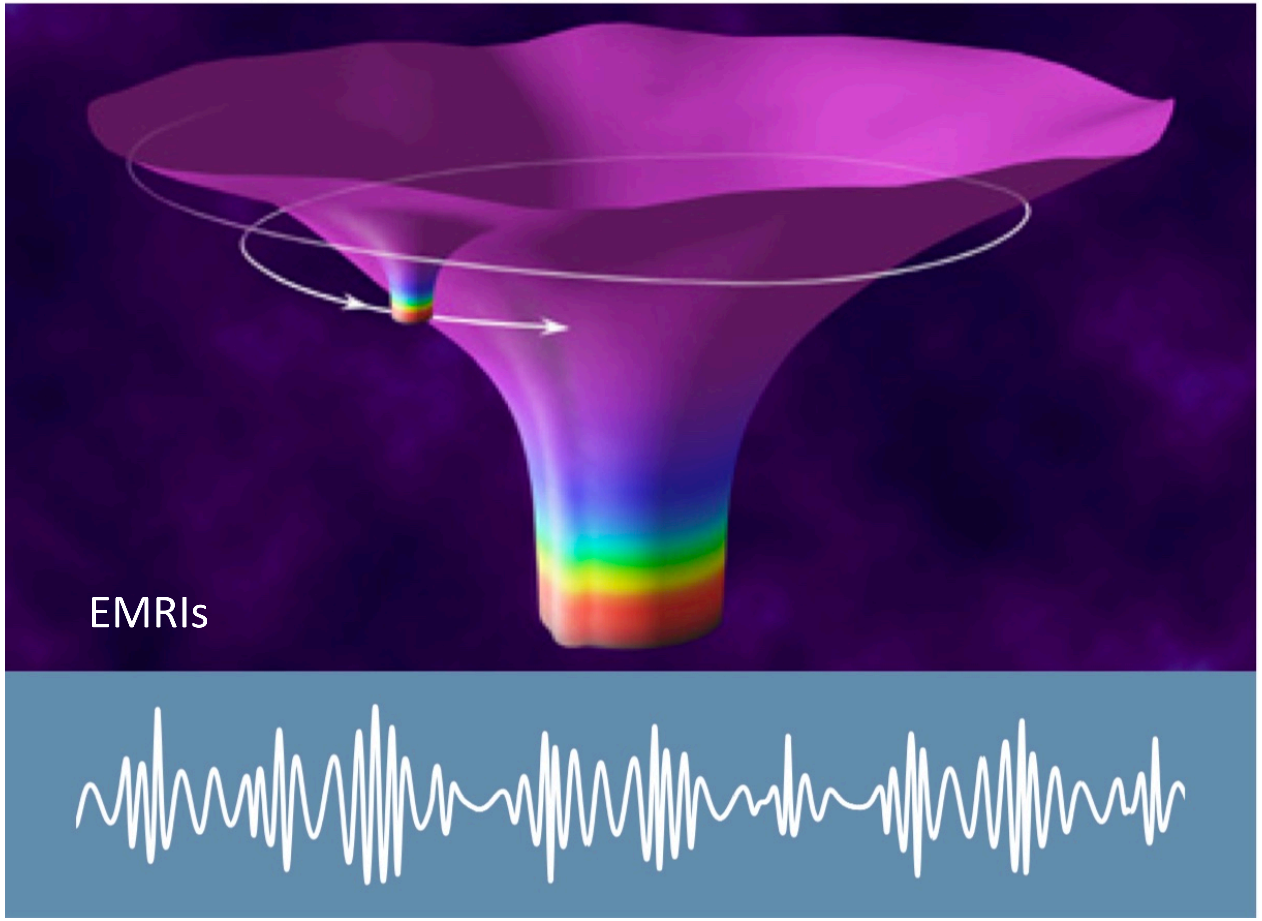
10^{-5} Hz <f< 0.1 Hz

- The expected sources are merging of very massive Black Holes at high redshifts

- It should also detect waves from tens of stellar-mass compact objects spiraling into onto strong-field orbits of central massive
  $\sim 10^6 M_\odot$ Black Holes [i.e. EMRIs Extreme Mass Ratio Inspirals (EMRIs)]

- Binary stars and binary white-dwarf systems formed in our Milky Way,

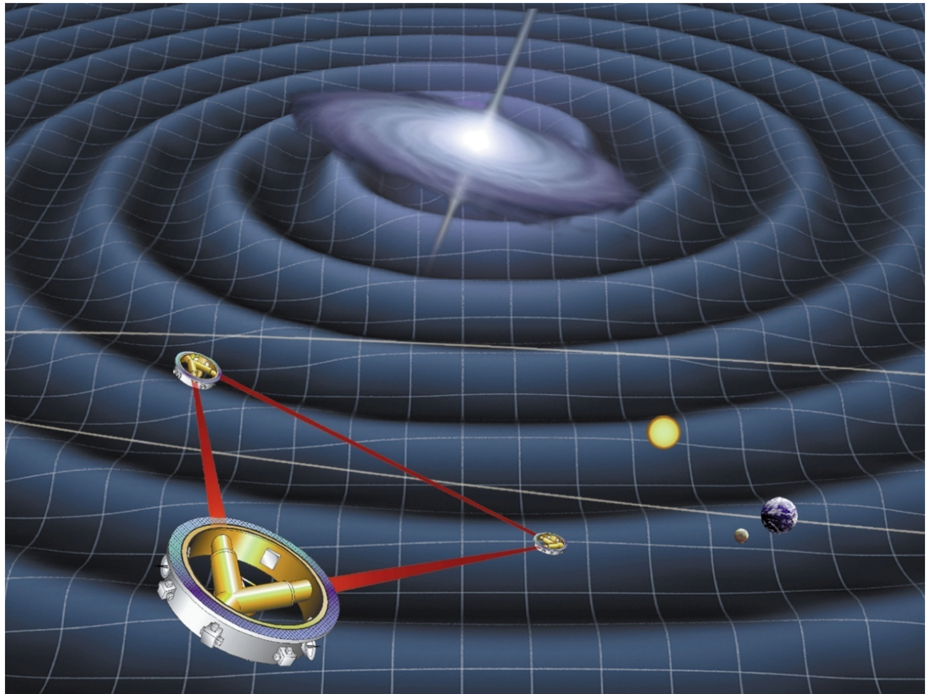
- It is targeted using laser interferometry between spacecraft.

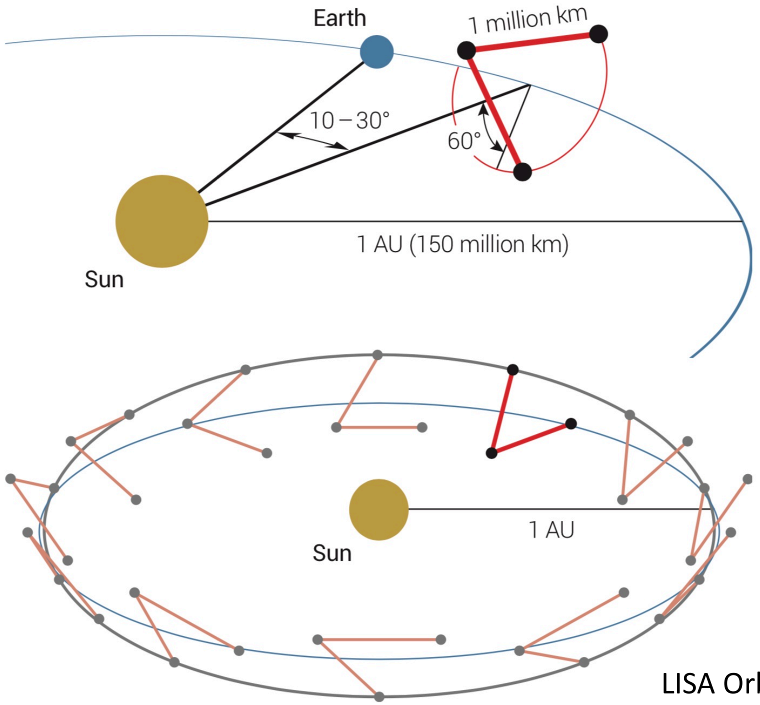


Low frequency 10^{-5} Hz <f< 0.1 Hz

The advantage of a space-based GW interferometer resides in its capability to reach high sensitivity in the intermediate frequency band below 1 Hz.

- In 2013, ESA has approved a GW observer in space as the L3 mission for launch in 2034, for which the LISA space-based interferometer is the main candidate.
- LISA plans to detect gravitational waves by measuring separation changes between fiducial masses in three spacecrafts that are supposed to be 5 million kilometers apart!

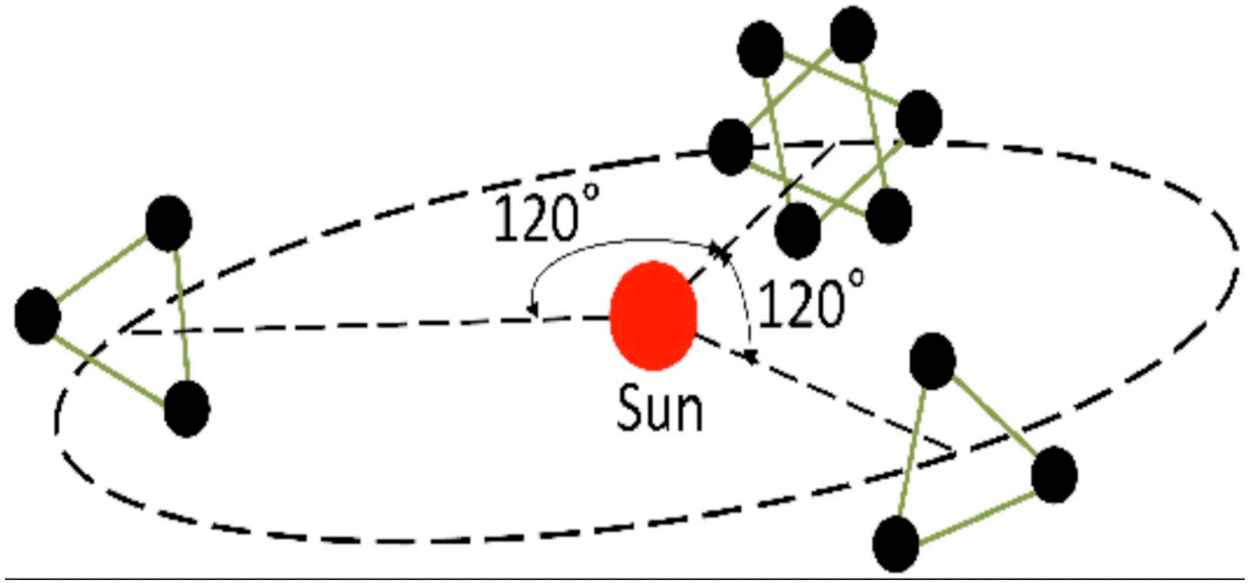




LISA Orbits

Low frequency

10^{-5} Hz <f< 0.1 Hz



The configuration of DECIGO and BBO. There are 8 effective interferometers in total.
BBO consists of four LISA-like triangular constellations orbiting the Sun at 1 AU
Decigo would be almost identical, except that the constellations are 50 times smaller than BBO

- Future space-based gravitational-wave (GW) detectors such as DECI-hertz Interferometer Gravitational-wave Observatory (DECIGO) and Big-Bang Observer (BBO) are the most sensitive to GWs in 0.1 − 1 Hz band
- DECIGO and BBO will aim at detecting the primordial GW background, the mergers of intermediate-mass black holes (BH), and a large number ($\sim 10^6$) of neutron-star (NS) binaries in an inspiraling phase.

DECIGO and BBO will provide a novel opportunity to measure the property of the Universe!

Very low frequency

$10^{-9} - 10^{-6}$ Hz

- In this frequency band, the two most plausible sources are

1) the coalescence of massive binary black holes

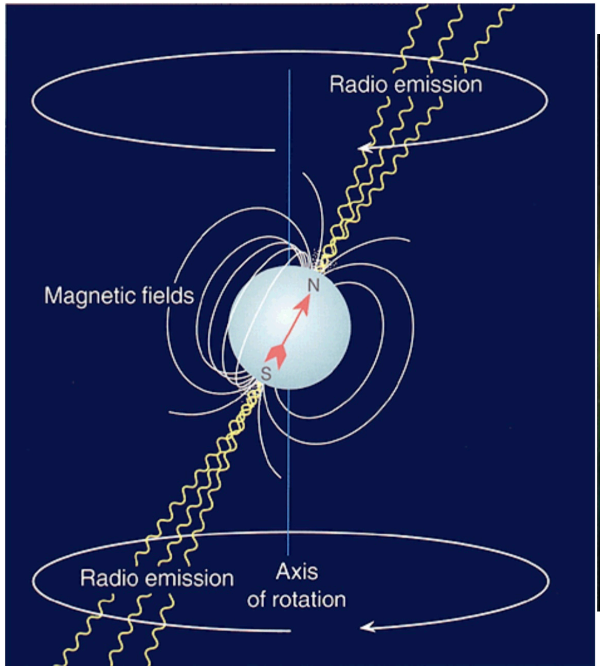
[Population synthesis estimates based on models of structure formation and galaxy growth suggest there should be a substantial population of such binaries whose members are black holes of $10^6 - 10^8\ M_\odot$]

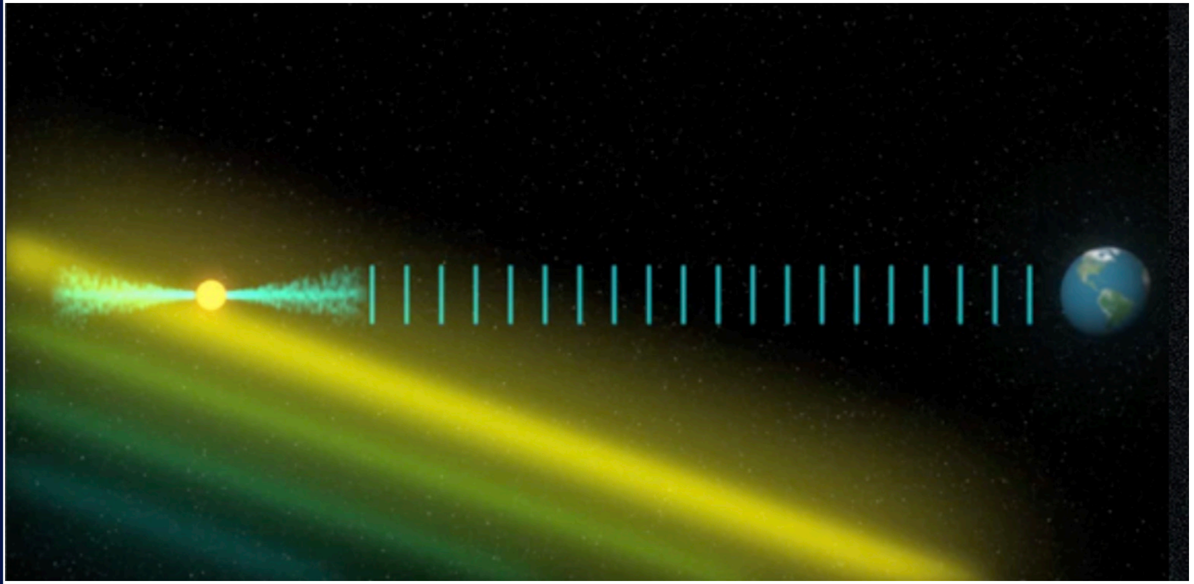
2) a high-frequency tail of the primordial GWs

The GWs produced by these binaries combine to form a stochastic background in the very low frequency band.

Very low frequency

$10^{-9} - 10^{-6}$ Hz





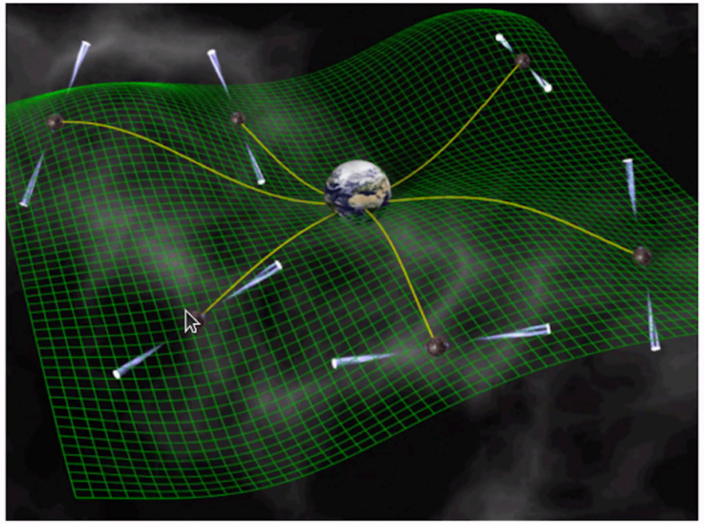
- This background is targeted by pulsar timing observations:

  This technique uses the fact that millisecond pulsars are very precise clocks; indeed, the stability of some pulsars rivals laboratory atomic clocks.

- Three Pulsar Timing Arrays are currently in operation – the EPTA in Europe, NANOGrav in the US and PPTA in Australia – sharing data under the aegis of the International Pulsar Timing Array (IPTA)

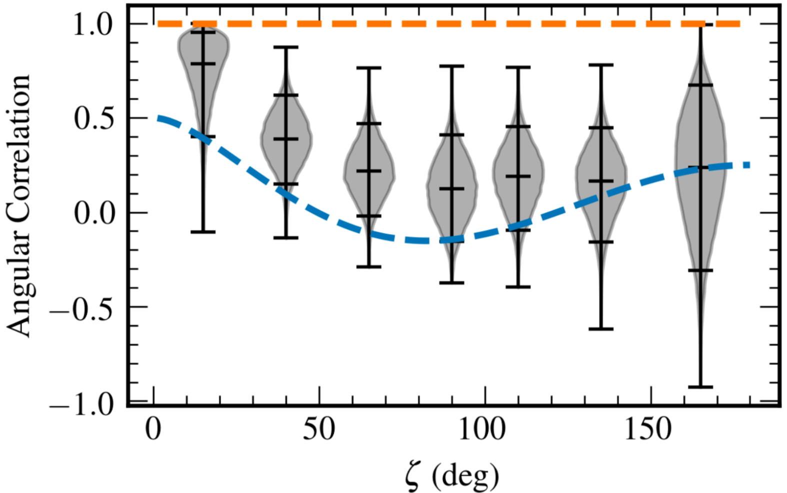
Very low frequency

The NANOGrav 12.5-year Data Set:
Search For An Isotropic Stochastic Gravitational-Wave Background (2009.04496.)



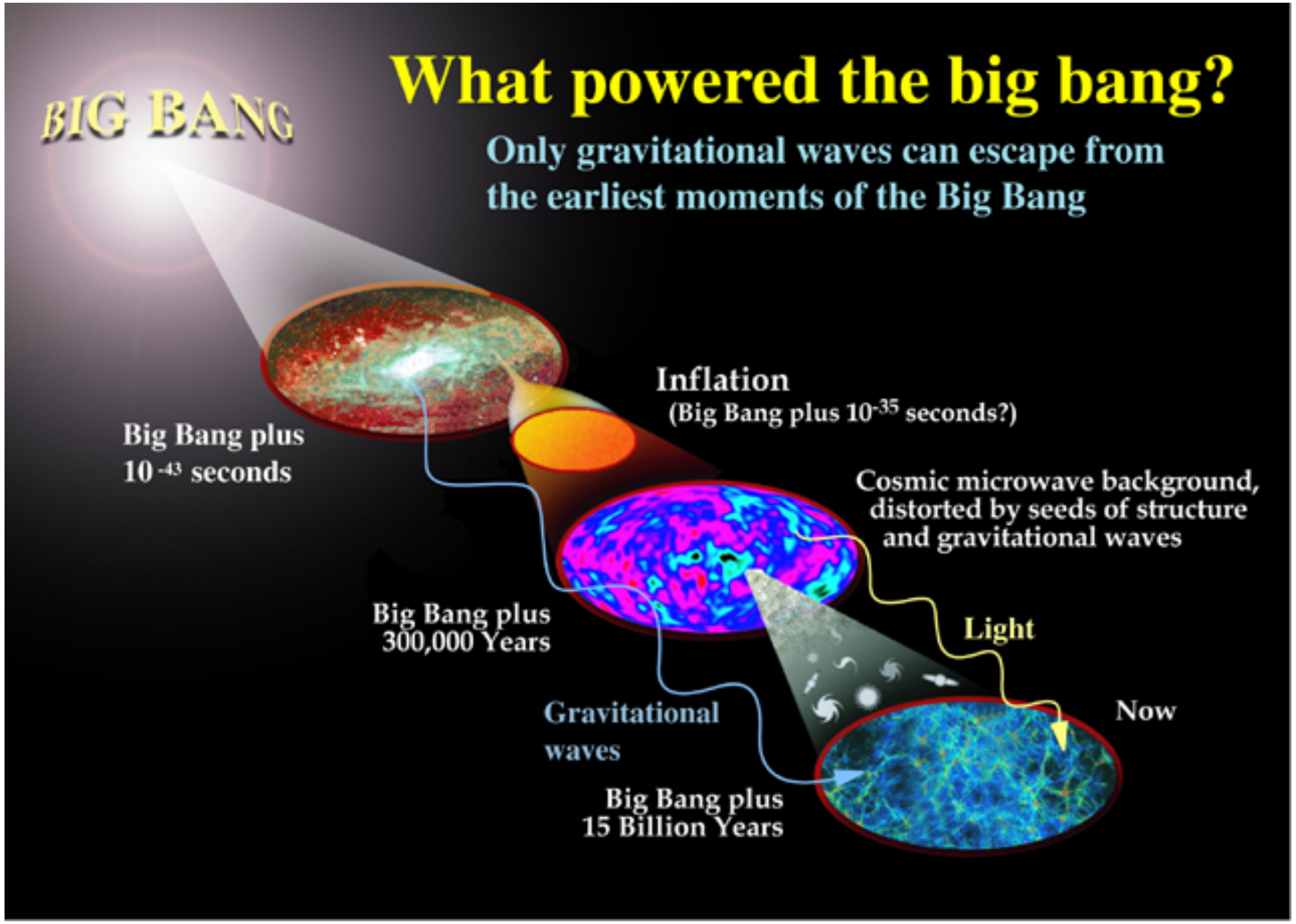
ABSTRACT

We search for an isotropic stochastic gravitational-wave background (GWB) in the 12.5-year pulsar-timing data set collected by the North American Nanohertz Observatory for Gravitational Waves. Our analysis finds strong evidence of a stochastic process, modeled as a power-law, with common amplitude and spectral slope across pulsars. Under our fiducial model, the Bayesian posterior of the amplitude for an $f^{-2/3}$ power-law spectrum, expressed as the characteristic GW strain, has median 1.92×10^{-15} and 5%–95% quantiles of $1.37\text{–}2.67 \times 10^{-15}$ at a reference frequency of $f_{\rm yr} = 1\ {\rm yr}^{-1}$; the Bayes factor in favor of the common-spectrum process versus independent red-noise processes in each pulsar exceeds 10,000. However, we find no statistically significant evidence that this process has quadrupolar spatial correlations, which we would consider necessary to claim a GWB detection consistent with general relativity. We find that the process has neither monopolar nor dipolar correlations, which may arise from, for example, reference clock or solar system ephemeris systematics, respectively. The amplitude posterior has significant support above previously reported upper limits; we explain this in terms of the Bayesian priors assumed for intrinsic pulsar red noise. We examine potential implications for the supermassive black hole binary population under the hypothesis that the signal is indeed astrophysical in nature.



Ultra low frequency 10^{-18} Hz< f< 10^{-13} Hz

- This low frequency GW band is best described using wavelength: it consists of GWs with $c/H_0 > \lambda > 10^{-5} c/H_0$.
- These are waves that vary on length-scales comparable to the size of our Universe!
- Following inflation, the GWs that are produced by this process propagate through the Universe.
- GW barely interact with matter as they propagate, just stretching and squeezing the primordial plasma in the young expanding Universe.



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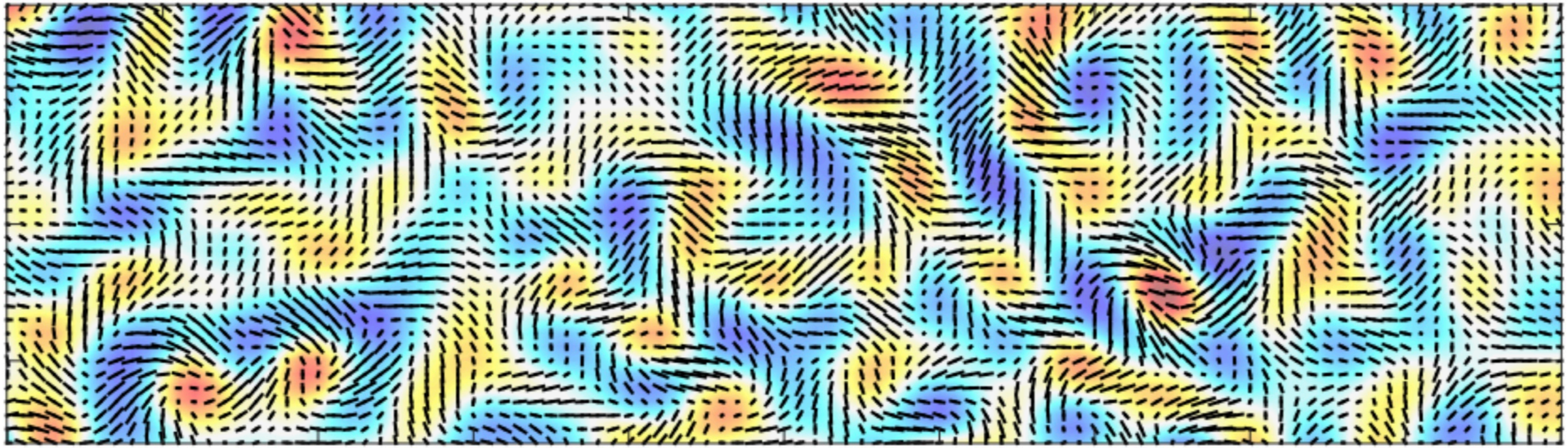


- This stretching and squeezing creates a quadrupolar temperature anisotropy in the plasma at recombination, which causes the CMB to be linearly polarized. The GWs thus leave an imprint on the CMB.

- Then we expect primordial gravitational waves stemming from the inflationary era of the very early Universe. Primordial quantum fields fluctuate and yield space–time ripples at a wide range of frequencies. These could in principle be detected as B-mode polarization patterns in the CMB radiation, at large angles in the sky.

Gravitational Universe

The beauty of General Relativity (GR) is that it is a falsifiable theory

Thus even a single experiment incompatible with a prediction of the theory would lead to its invalidation

(For example) To test GR:

- **in strong gravity regime**
- **in cosmology**

Tests of strong gravity and non-linear regime

To answer these questions and learn about the fundamental nature of gravity is by observing the vibrations of the fabric of spacetime itself.

Leaving open several questions:

- *Does gravity travel at the speed of light ?* **YES! From GW170817**
- *Does the graviton have mass?*
- *How does gravitational information propagate: Are there more than two transverse modes of propagation?*
- *Does gravity couple to other dynamical fields, such as, massless or massive scalars?*
- *What is the structure of spacetime just outside astrophysical black holes? Do their spacetimes have horizons?*
- *Are astrophysical black holes fully described by the Kerr metric, as predicted by General Relativity?*
- …

in cosmology:

Gravitational wave maps of resolved events: GW standard sirens

Stochastic backgrounds (SGWB)

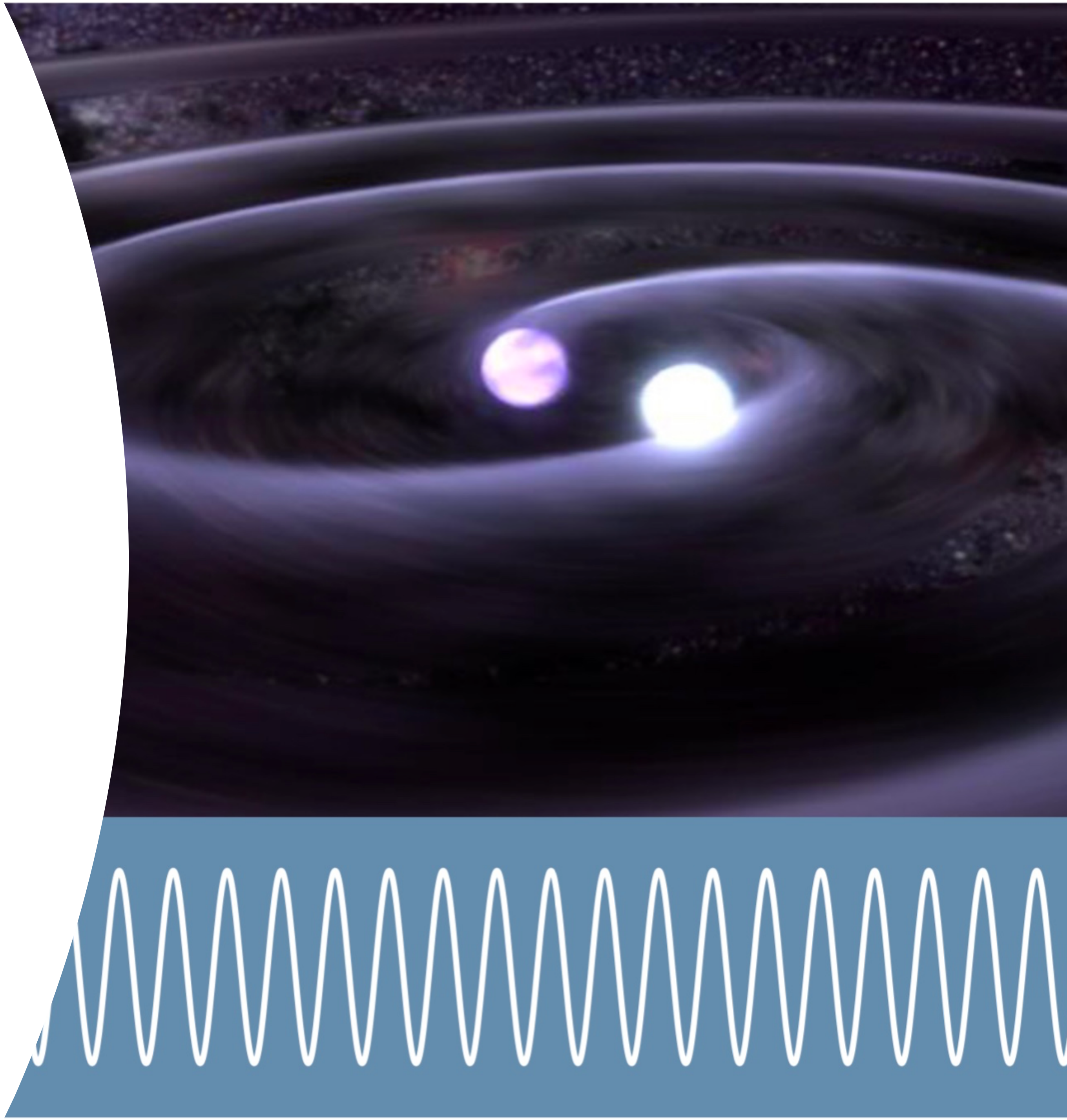
GW standard sirens

- GWs provide a unique, purely gravitational way to measure distance (Schutz 1986)
- As GW detections can be thought of as aural rather than optical (Hughes 2003), a more appropriate term for a GW standard candle is a “standard siren”
- BH binary mergers as "standard sirens" to extract information on the expansion of the Universe, by measuring the expansion history with completely different techniques to electromagnetic probes.

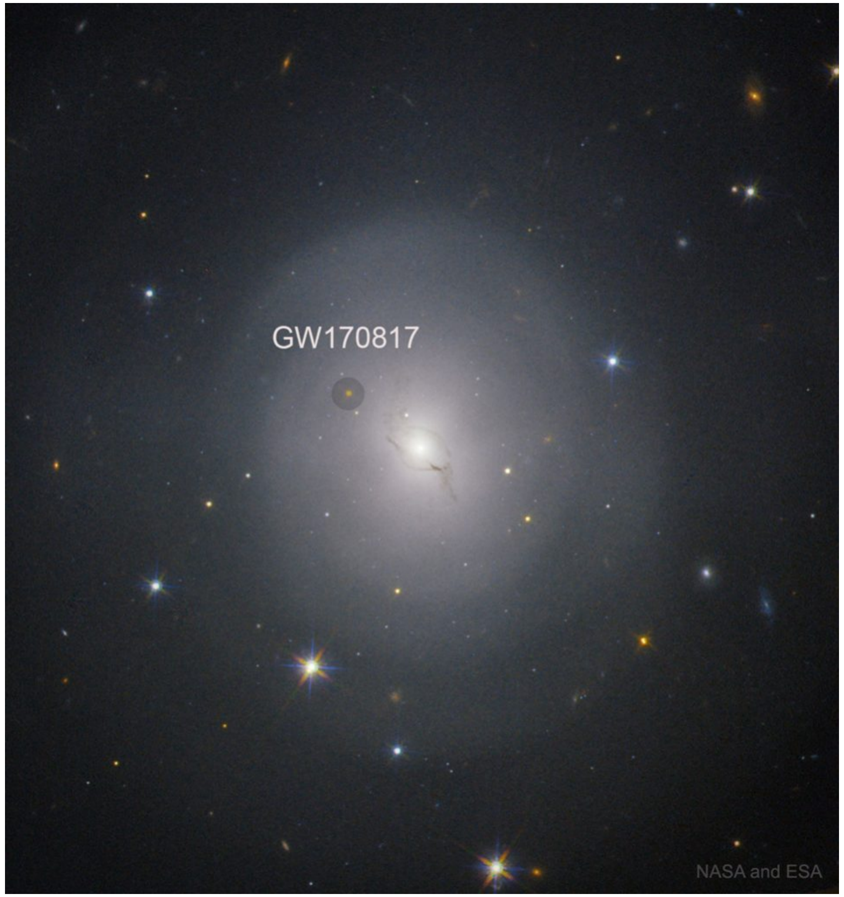
GW standard sirens

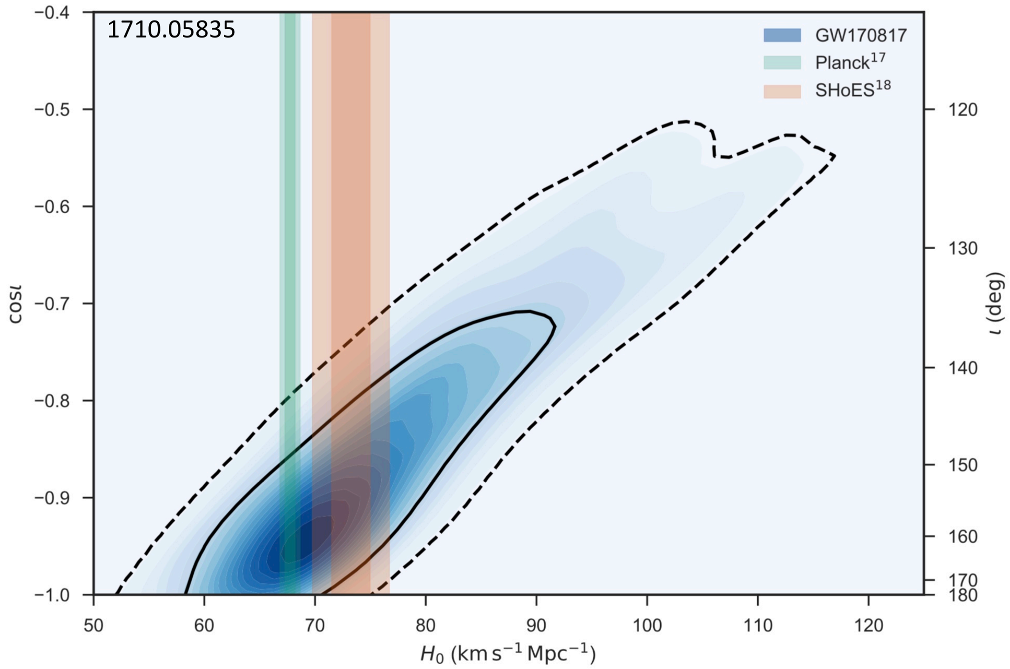
- Black hole coalescences could serve as standard sirens for cosmography by providing absolute and direct measurements of the luminosity distance D_L

- One advantage of standard sirens over SNIa is that they allow for a direct measurement of D_L up to large redshift, unlike optical measurements, which require cross-calibrations of successive distance indicators at different scales;



GW standard sirens

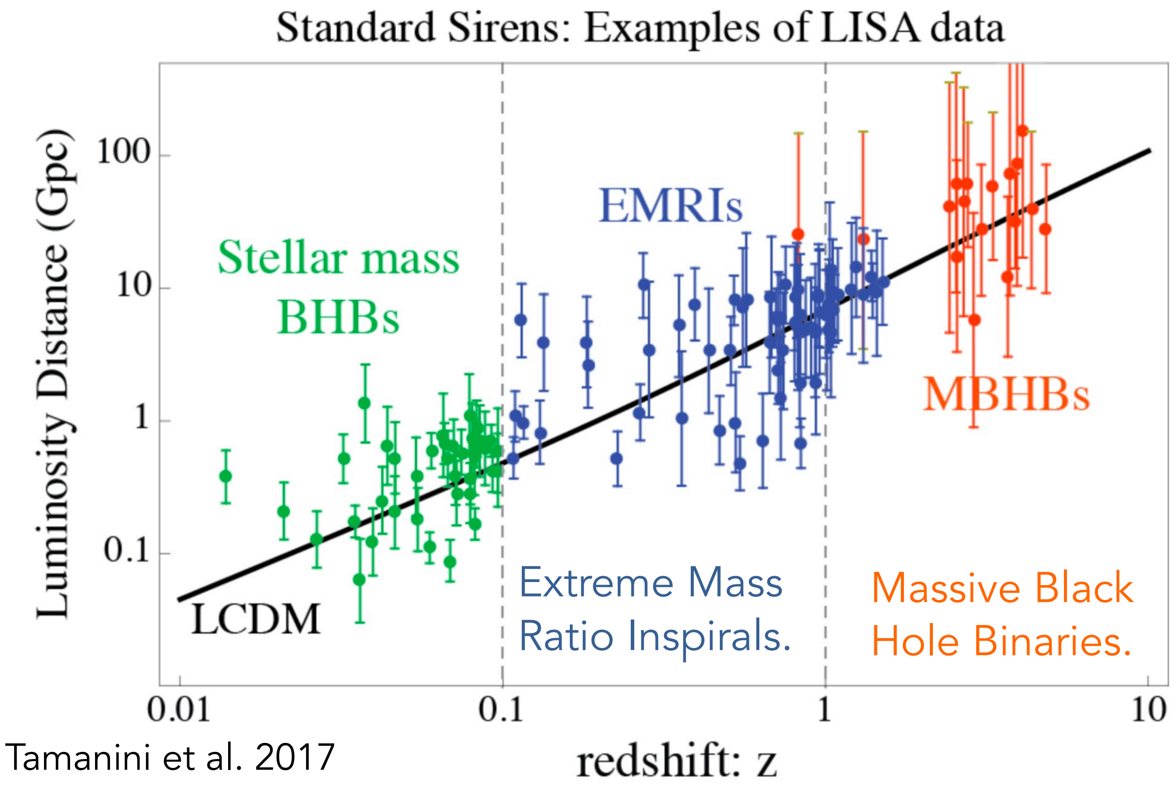




Posterior density of H_0 and cos ι from the joint GW-EM analysis (blue contours). Shading levels are drawn at every 5% credible level, with the 68.3% (1σ, solid) and 95.4% (2σ, dashed) contours in black. Values of H_0 and 1- and 2σ error bands are also displayed from Planck (Planck Collaboration et al. 2016) and SHoES (Riess et al. 2016).

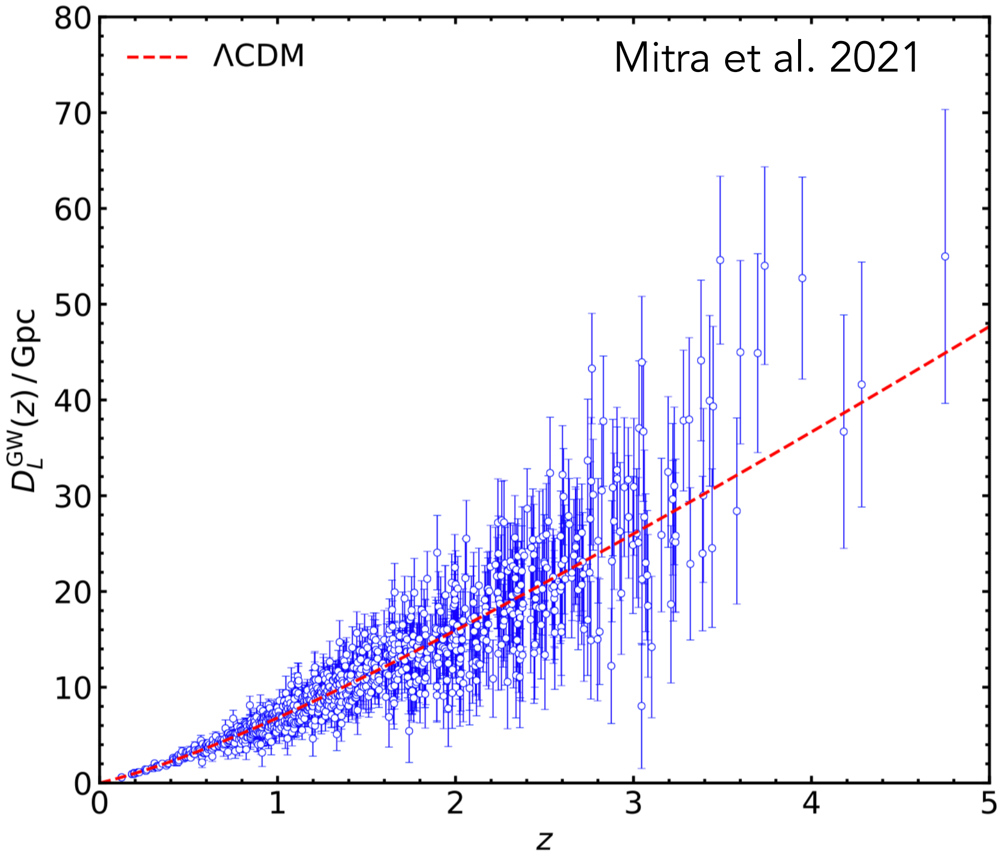
- This method has recently been used for the first (albeit still low-precision) gravitational measurement of the Hubble constant, using LIGO/VIRGO binary neutron star event, GW170817.

Approaches to standard siren cosmology



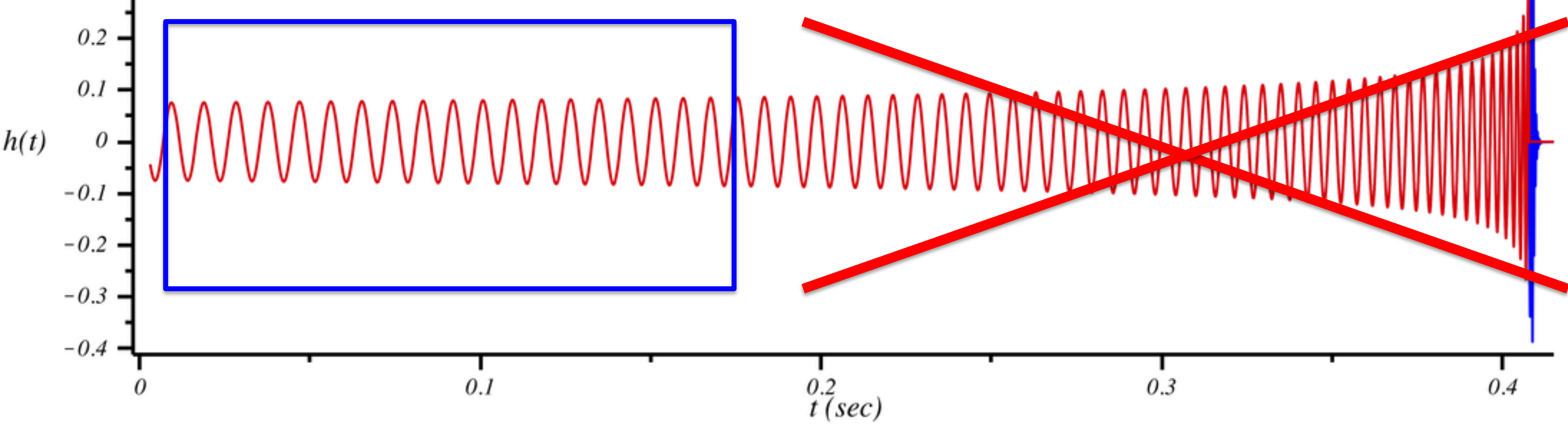
Tamanini et al. 2017

Observation of electromagnetic counterparts, from which the source redshift can be extracted, is required.



Cosmography with the Einstein Telescope

GW standard sirens offer a completely different way to measure distances than standard candles (type SNe1a) and standard rulers (baryon acoustic oscillations).



Cosmology with GW measurements

- Binary black hole inspirals are well understood, theoretically clear and well-modelled (see Schutz 1986; Holz & Hughes 2005; Dalal, Holz, Hughes & Jain 2006; Cutler and Holz 2009; Nissanke et al. 2010, 2013)
- For distance determination the most interesting epoch of BH-BH coalescence is the inspiral, when the binary's members are widely separated and slowly spiral together due to back-reaction from GW emission.

[In general, the merger (in which the holes come into contact, forming a single body) and the ring- down (the final, simple stage of the ringdown, in which the merged binary is well-modelled as a single, distorted black hole) are irrelevant.]

GWs from binary system

Schematically, a measured binary waveform takes the form:

$$h_+ = \frac{2\mathcal{M}_z^{5/3}[\pi f(t)]^{2/3}}{D_L}\left[1 + (\hat{L}\cdot\hat{n})^2\right]\cos[\Phi(t)]\ ,$$

$$h_\times = \frac{4\mathcal{M}_z^{5/3}[\pi f(t)]^{2/3}(\hat{L}\cdot\hat{n})}{D_L}\sin[\Phi(t)]\ .$$

- luminosity distance D_L
- accumulated GW phase $\Phi(m_1, m_2, \mathbf{S}_1, \mathbf{S}_2, t)$
- GW frequency $f(t) = (1/2\pi)d\Phi/dt$
- $\hat{n}$ points from the center of the barycenter frame to the system, and hence defines its position on the sky;
- $\hat{L}$ points along the binary's orbital angular momentum, and hence defines its orientation.
- Redshifted chirp mass $\mathcal{M}_z = (1+z)(m_1 m_2)^{3/5}/(m_1+m_2)^{1/5}$

Distance, but not redshift!

- Gravitational waves provide a direct measure of luminosity distance, but they give no independent information about redshift

- GWs from a local binary with masses (m_1, m_2)

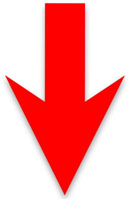
- at redshift z is indistinguishable from a local binary with masses

$$(m_1, m_2) \rightarrow \left(\frac{m_1}{(1+z)}, \frac{m_2}{(1+z)}\right)$$

- In general to measure cosmology, need independent measurement of redshift

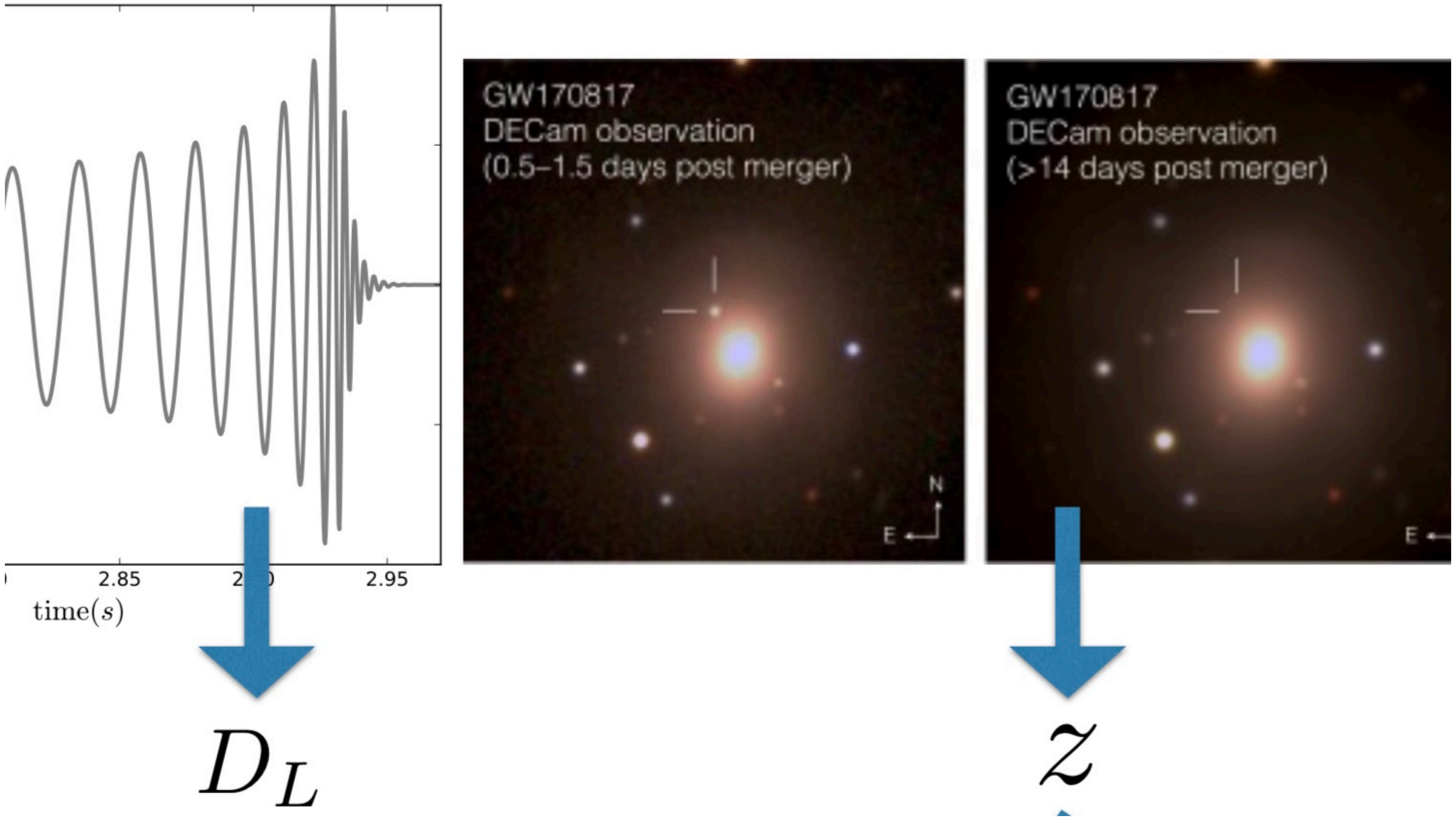
Approaches to standard siren cosmology

- **To determine cosmological parameters (e. g. H_0)**
- **To establish luminosity distance-redshift relation**



Using the electromagnetic (EM) counterpart

Using a statistical solution-method (absence of EM counterparts)



Approaches to standard siren cosmology (1)

- EM counterpart case, electromagnetic observations identify a counterpart to the GW source.
- This can be done by directly observing a transient electromagnetic source, such as a Gamma-Ray Burst (GRB)/afterglow or a kilonova
- Short GRBs are known to occur at low redshift ($z < 0.2$) and are thought to be the result of binary mergers (NS or BH)
- The redshift can then be determined, either directly from the **EM counterpart**, or by identifying the **host galaxy** associated with the counterpart and measuring its redshift instead.

Approaches to standard siren cosmology (1)
Possible issues…

- Using GRB counterparts for example, host galaxy identification can sometimes be unreliable.
- We also require that the emission cone from the GRB is coincident with our line of sight.
- Current estimates [e.g. see Nakar (2007), where they coupled only some short-hard GRBs to measure redshifts] show that **only a small fraction (~10^{-3}) of BNS events will be useful as standard sirens**.
- Then, in general, we have the **incompleteness of the catalog**! [E.g. when the *real host is too faint to be detected by the survey* and thus to be considered for the analysis]. Then, each single event posterior distribution for sky position, redshift etc.., will be displaced compared to the case in which the true host is included.
- Other systematic error: coherent peculiar velocity of the merging at low redshift could make a misidentification of the host redshift.

2) Standard siren cosmology without EM counterpart case: Statistical method

- **Assuming a source redshift distribution based on galaxy catalogs**. Since each GW event typically has a large error volume on the sky, then by combining a large number of such sources, a value of the cosmic expansion rate can be obtained maximizing the likelihood fit. Such a method, however, could be limited to redshifts $z < 1\text{-}2$ where galaxy catalogs are complete, see MacLeod and C. J. Hogan 2008; Petiteau et al 2011.

- Namikawa, Nishizawa & Taruya, (2016a, b); Oguri (2016) build up a sufficient catalog of GW events that the anisotropies in the distribution of events on the sky can be matched to the known clustering of large-scale structure. Essentially, **one cross-correlates galaxies with gravitational-wave standard sirens in order to maximize the likelihood of cosmological parameters, and thereby determine the distance-redshift relationship.**

- **Using a Bayesian formalism** to analyse catalogs of NS-NS inspiral detections Taylor, Grair & Mandel 2012 investigated a novel approach to measuring the Hubble constant using GW signals from compact binaries by exploiting the narrowness of the distribution of masses of the underlying neutron-star population.

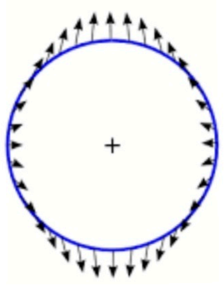
2) Standard siren cosmology without EM counterpart case: Cosmological inference using only GWs

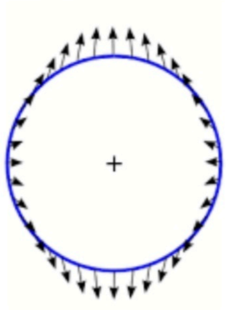
- **The Bayesian inference** developed by Del Pozzo 2012 is useful **to facilitate the inclusion of all assumptions and prior information about a GW source within a single data analysis framework.**

- **This approach guarantees the minimisation of information loss and the possibility of including naturally event-specific knowledge** (such as the sky position for a Gamma Ray Burst-GW coincident observation) in the analysis.

See also Ghosh, Del Pozzo & Ajith 2015

3) Standard siren cosmology without EM counterpart case: using tidal effects





The tidal effects provide additional contributions to the phase evolution of the gravitational wave signal that **break a degeneracy between the system's mass parameters and redshift** and thereby allow **the simultaneous measurement of both the effective distance and the redshift for individual sources**, see Messenger & Read 2012 and Del Pozzo Li & Messenger 2017

Appearing at 5th PN order, these effects will be measurable using 3rd generation gravitational wave detectors, e.g. ET

Credit Del Pozzo

Cosmological perturbation effects on GW luminosity distance estimates

- In DB, Raccanelli, Bartolo & Matarrese (1702.01750), we consider the observer frame as reference system (the usual calculation is performed in a homogeneous and isotropic FLRW Universe, see Laguna et al 2010);

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[Note that we are assuming that the observer “at the emitted position” is within a region with a comoving distance to the source sufficiently large so that the gravitational field is “weak enough” but still “local”, i.e. the gravitational wave wavelength is small w.r.t. the comoving distance from the observer]

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- We consider only the regime called of "quasi- circular" motion (i.e. the approximation in which a slowly varying orbital radius is applicable)
- Geometric optics approximation: the propagation of high frequency GWs travelling over a smooth background, which varies over scales much larger than the GW wavelengths;

Geometric optics approximation

Geometric optics approximation, the space-time metric can be written as the sum of two parts:

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} + \epsilon h_{\mu\nu}$$

where $\tilde{g}_{\mu\nu}$ is usually named "background metric" and describes both the FRW metric and first-order perturbations, and $h_{\mu\nu}$ is the gravitational wave metric perturbation

In the shortwave approximation and neglecting the response of matter background effect to the presence of $h_{\mu\nu}$, a gravitational wave can be described as

$$\bar{h}_{\mu\nu} = A_{\mu\nu}\, e^{i\varphi/\epsilon} = e_{\mu\nu}\, \mathcal{A}\, e^{i\varphi/\epsilon} = e_{\mu\nu} h\,,$$

where $\bar{h}_{\mu\nu} = h_{\mu\nu} - \tilde{g}_{\mu\nu} h/2$ and h is the trace of $h_{\mu\nu}$ w.r.t. the background metric $\tilde{g}_{\mu\nu}$. Here $e_{\mu\nu}$ is a polarization tensor and $\mathcal{A}$ and φ are real functions of retarded time and describe respectively the amplitude and the phase of the GW.

Isaacson (1967,1968)

Geometric optics approximation

It is convenient to use the comoving metric $\hat{g}_{\mu\nu} = \tilde{g}_{\mu\nu}/a^2$, where a is the scale factor. Defining the GW wave-vector

$$k_\mu = -\nabla_\mu \varphi,$$

we have $k^\mu k_\mu = 0$, $k^\mu \nabla_\mu k^\nu = 0$, and

$$\frac{d}{d\chi} \ln(\mathcal{A}a) = -\frac{1}{2} \nabla_\mu k^\mu ,$$

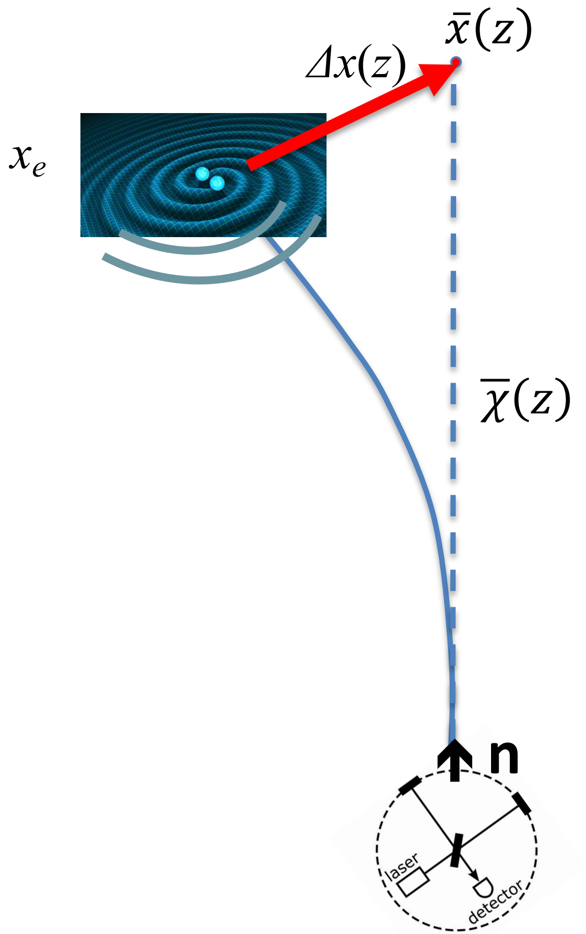
where we have used $d/d\chi \equiv k^\mu \nabla_\mu$.

Isaacson (1967, 1968), Bertacca et al. (2017)

COSMIC LABORATORY

(cosmic rulers)

What we observe GW is the *apparent* position at which it appears in a given direction **n** and redshift *z* **(if we know the EM of the host galaxy)**.



In observers frame the GW geodesics are given by (in conformal coordinates)

$$\bar{x}(z) = [\eta_0 - \bar{\chi}(z), \bar{\chi}(z)\mathbf{n}]$$

$\bar{\chi}$ *:* comoving distance in the observed frame

In a generic perturbed Universe we have:

$$a_e = \bar{a}\,[1+\Delta \ln a] \qquad \Delta \ln a\,(z) = \bar{a}\,\mathcal{H}\Delta x^0(z) = \bar{a}\,\mathcal{H}[\delta\chi - \delta x^0](z)$$

$$x_e = \bar{x} + \Delta x \qquad \Delta x^i(z) = n^i\,\delta\chi(z) + \delta x^i(z)$$

Bertacca et al. 2017

- The perturbed gravitational waves can then be fully described as

$$h(\eta_e, \mathbf{x}_e) = \mathcal{A}(\eta_e, \mathbf{x}_e)\, e^{i\varphi(\eta_e, \mathbf{x}_e)} = \frac{\mathcal{Q}(1+z)}{\bar{\chi}}(1 + \Delta \ln \mathcal{A}) e^{i(\bar{\varphi}+\Delta\varphi)} ,$$

where the perturbations of amplitude and phase are

$$\Delta \ln \mathcal{A} = \delta \ln \mathcal{A} + \Delta x^0 \bar{\partial}_0 \ln \bar{\mathcal{A}} + \Delta x_\parallel \bar{\partial}_\parallel \ln \bar{\mathcal{A}} = \delta \ln \mathcal{A} - \left(1 - \frac{1}{\mathcal{H}\bar{\chi}}\right) \Delta \ln a + \frac{T}{\bar{\chi}} ,$$

$$\Delta\varphi = \delta\varphi + T ,$$

with $-T = \Delta x^0 + \Delta x_\parallel = \delta x^0 + \delta x_\parallel$, and

$$\frac{\mathrm{d}}{\mathrm{d}\bar{\chi}} \delta \ln \mathcal{A} = -\frac{1}{2}\left[\frac{\partial}{\partial \bar{x}^0}\left(\delta k^0 + \delta k_\parallel\right) + \frac{\mathrm{d}}{\mathrm{d}\bar{\chi}} \delta k_\parallel - 2\frac{\mathrm{d}}{\mathrm{d}\bar{\chi}}\kappa + \delta\Gamma^\mu_{\mu\nu}\bar{k}^\nu\right] ,$$

where κ is the weak lensing convergence term

$$\kappa = -\frac{1}{2}\bar{\partial}_{\perp i}\Delta x^i_\perp .$$

$\mathcal{Q}$ is constant along the null geodesic.
Here $\mathcal{Q}$ is determined by the local wave-zone source solution and contains all the physical information on the spiralling binary. $\mathcal{Q}$ is defined directly in the observed frame $\bar{x}^\mu = (\bar{\eta}, \bar{\mathbf{x}})$

Bertacca et al. 2017

- Therefore, we can express perturbations in amplitude and phase, in the Poisson gauge, as

$$\begin{aligned}\Delta \ln \mathcal{A} &= \Psi - \kappa - \left(1 - \frac{1}{\mathcal{H}\bar{\chi}}\right)\Delta \ln a + \frac{T}{\bar{\chi}}\,,\\ \Delta\varphi &= 0\,.\end{aligned}$$

- The correction in $\Delta \ln \mathcal{A}$ can be related to the luminosity distance $\mathcal{D}_L$, in the following way

$$\frac{\Delta \mathcal{D}_L}{\bar{\mathcal{D}}_L} = -\Psi - \kappa + \left(1 - \frac{1}{\mathcal{H}\bar{\chi}}\right)\Delta \ln a - \frac{T}{\bar{\chi}}\,,$$

taking into account that

$$h_e = \frac{\mathcal{Q}(1+z)^2}{\mathcal{D}_L} e^{i\bar{\varphi}}$$

and

$$\Delta \ln \mathcal{A} = -\frac{\Delta \mathcal{D}_L}{\bar{\mathcal{D}}_L}\,,$$

defining $\bar{\mathcal{D}}_L = (1+z)\bar{\chi}$ the observed average luminosity distance taken over all the sources with the same observed redshift z, with $\mathcal{D}_L = \bar{\mathcal{D}}_L + \Delta\mathcal{D}_L$.

- The gravitational wave observed at the detector is red-shifted, hence we find

$$h_r \equiv \frac{h_e}{(1+z)} = \frac{\mathcal{Q}(1+z)}{\mathcal{D}_L} e^{i\bar{\varphi}}\,.$$

Cosmological perturbation effects on GW luminosity distance estimates

Computing the modifications of the value of the luminosity density D_L inferred from gravitational waves, due to perturbations, we can write the correction to the luminosity distance in Poisson gauge as

$$\frac{\Delta \mathcal{D}_L}{\bar{\mathcal{D}}_L} = \left(1 - \frac{1}{\mathcal{H}\bar{\chi}}\right) v_\parallel - \frac{1}{2}\int_0^{\bar{\chi}} d\tilde{\chi} \frac{(\bar{\chi} - \tilde{\chi})}{\tilde{\chi}\bar{\chi}} \triangle_\Omega (\Phi + \Psi) + \\ + \frac{1}{\mathcal{H}\bar{\chi}}\Phi - \left(1 - \frac{1}{\mathcal{H}\bar{\chi}}\right) \int_0^{\bar{\chi}} d\tilde{\chi} (\Psi' + \Phi') - (\Phi + \Psi) + \frac{1}{\bar{\chi}} \int_0^{\bar{\chi}} d\tilde{\chi} (\Phi + \Psi)$$

where $\triangle_\Omega \equiv \bar{\chi}^2 \bar{\nabla}_\perp^2 = \bar{\chi}^2 (\bar{\nabla}^2 - \bar{\partial}_\parallel^2 - 2\bar{\chi}^{-1}\bar{\partial}_\parallel) = (\cot\partial_\theta + \partial_\theta^2 + \partial_\varphi / \sin^2\theta)$

We can recognize the presence of a velocity term (the first r.h.s. term), followed by a lensing contribution, and the final four terms account for SW, ISW, volume and Shapiro time-delay effects.

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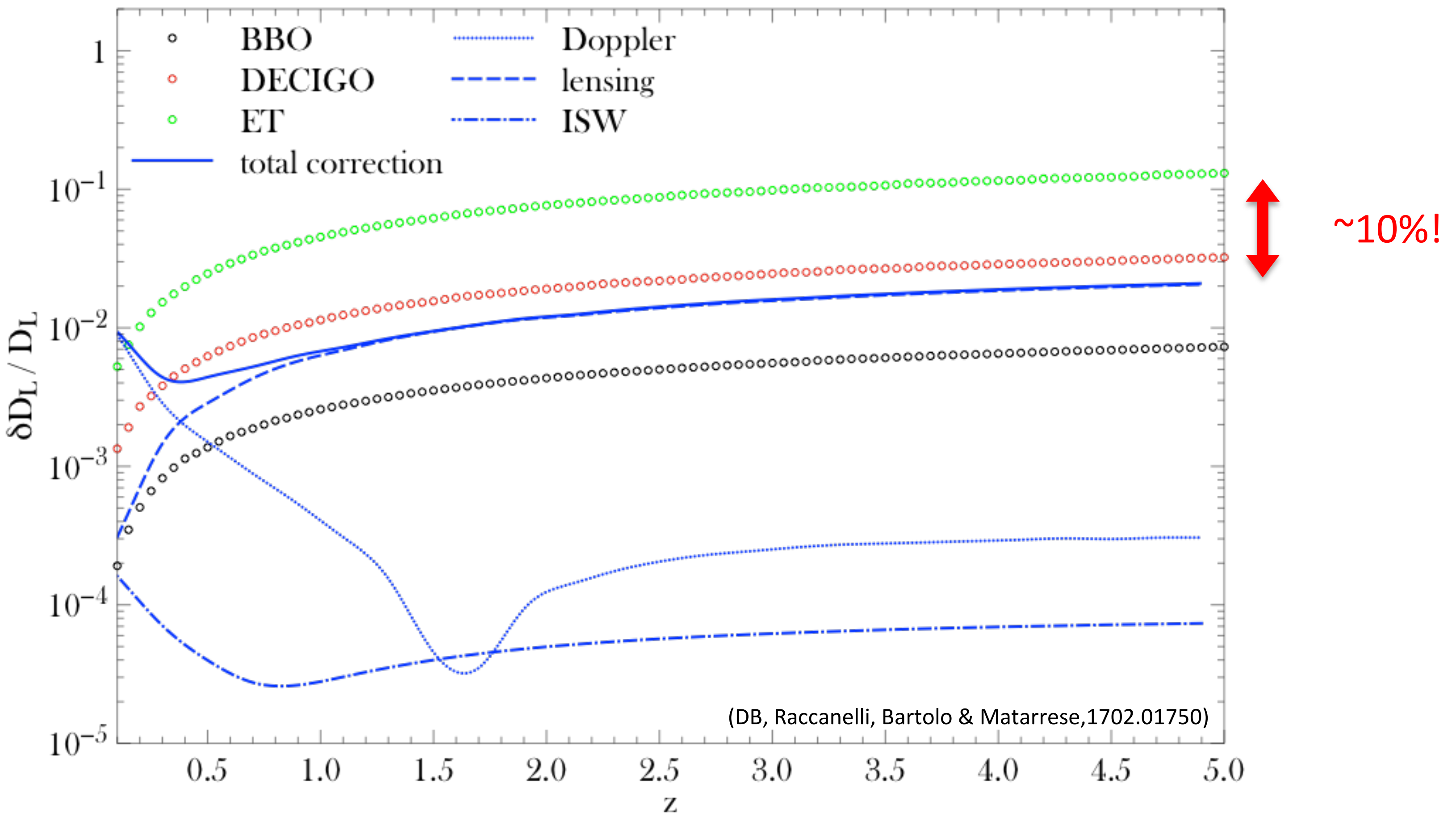
To numerically compute the magnitude of this effect, we calculate the mean fluctuation of the effect, at any given redshift, as

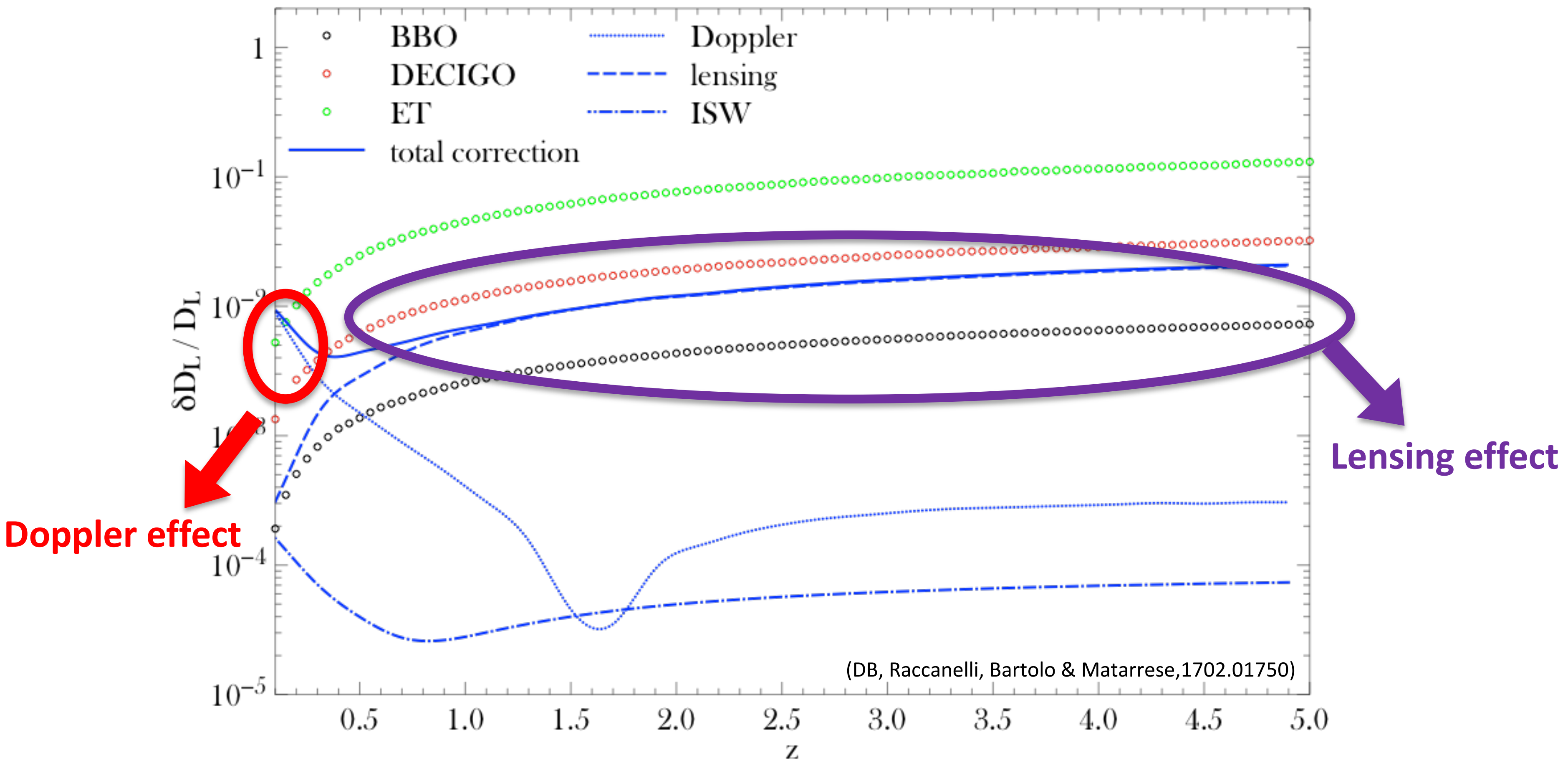
$$C_\ell^{\mathcal{D}_\mathrm{L}} = \left\langle \frac{\Delta\mathcal{D}_L}{\bar{\mathcal{D}}_L}\ \frac{\Delta\mathcal{D}_L}{\bar{\mathcal{D}}_L}^{*}\right\rangle$$

Total correction to luminosity distance estimates due to perturbations



Points show the predicted precision in measurements of the luminosity distance, at any redshift, for the Einstein Telescope (green points) [Taylor et al 2012], DECIGO (red points) and the Big Bang Observer (black points) [Camera & Nishizawa 2013]

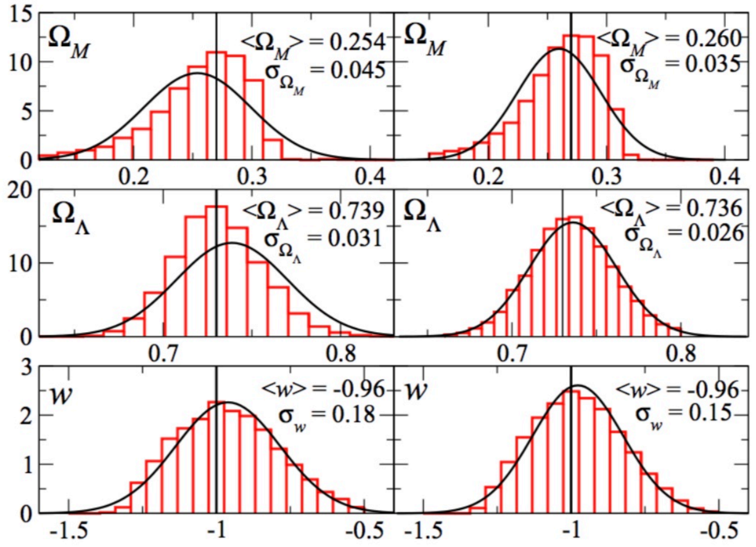
Total correction to luminosity distance estimates due to perturbations



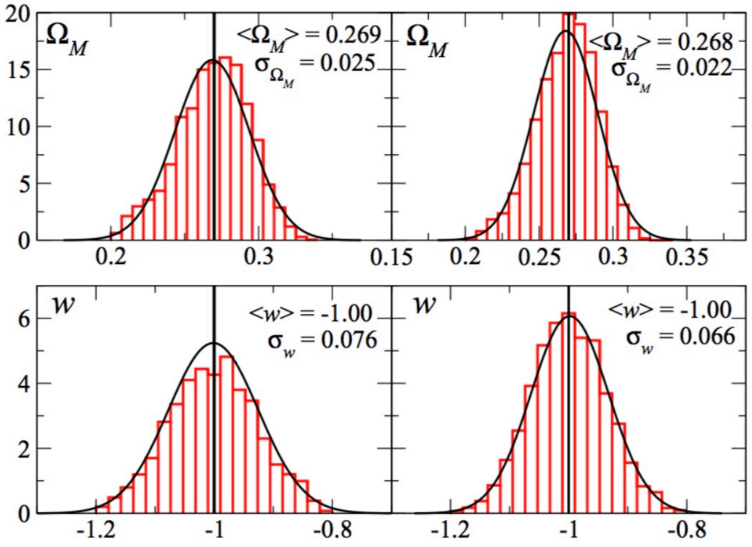
- **peculiar velocities may be important for z << 1**
- **weak lensing will be important for z > 1**

Lensing become important

Sathyaprakash et al. 2009



with weak lensing errors in D_L



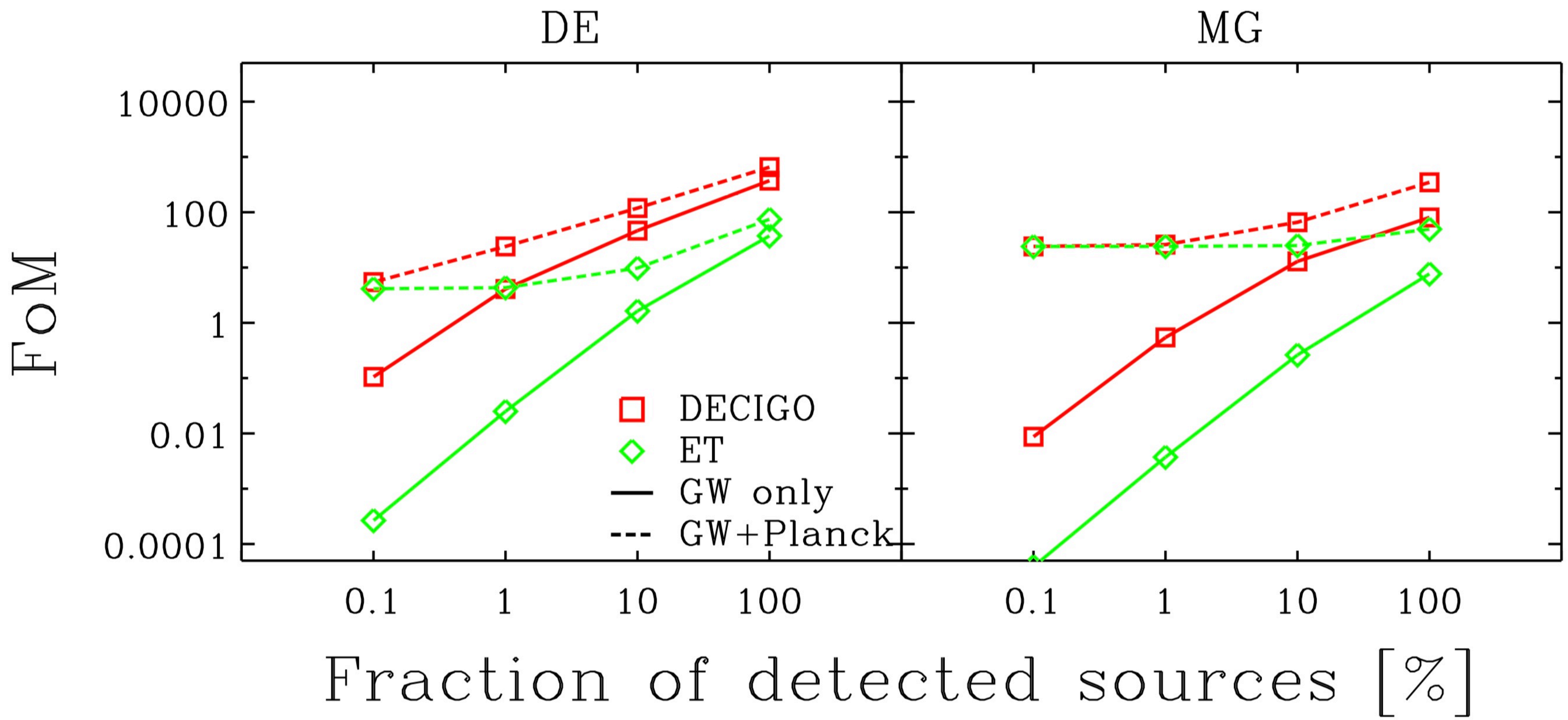
if weak lensing errors can be corrected

- These sources are however affected by weak gravitational lensing by intervening inhomogeneities in the cosmic mass distribution.

- This introduces changes of typically a few percent (but occasionally much larger) in the flux, while not significantly affecting the redshift, and thus provides a source of noise in the $D_L(z)$ relation (Hirata, Holtz & Cutler 2010)

Making use of the weak-lensing magnification effect on a GW from a compact binary object

In [Camera & Nishizawa, 2013] they showed that it is possible to discriminate the concordance ΛCDM cosmological model and up-to-date competing alternatives as DE or MG theories



FoM (w_0, w_a) (left panel) and FoM (μ_0, η_0) (right panel) as a function of the fraction of detected sources for DECIGO (squares, red) and ET (diamonds, green). Solid lines refer to GW detectors only, whilst dashed lines show the results in combination with Planck priors

GW luminosity distance in the context of scalar-tensor theories of gravity

- In presence of DE/MG the GW luminosity distance generally differs from the one traced by electromagnetic (EM) signals, both at the unperturbed, background level [E. Belgacem et al. (2017, 2018)]

$$h''_\lambda(\mathbf{k},\eta) + 2\mathcal{H}[1-\delta(\eta)]h'_\lambda(\mathbf{k},\eta) + k^2 h_\lambda(\mathbf{k},\eta) = 0\,,$$

with some function $\delta(\eta)$ which modifies the friction term in the propagation equation.

- This affects the amplitude of a GW propagating across cosmological distances, giving rise to a notion of "gravitational-wave luminosity distance"

$$d_L^{\rm gw}(z) = d_L^{\rm em}(z)\exp\left\{-\int_0^z \frac{{\rm d}z'}{1+z'}\,\delta(z')\right\}$$

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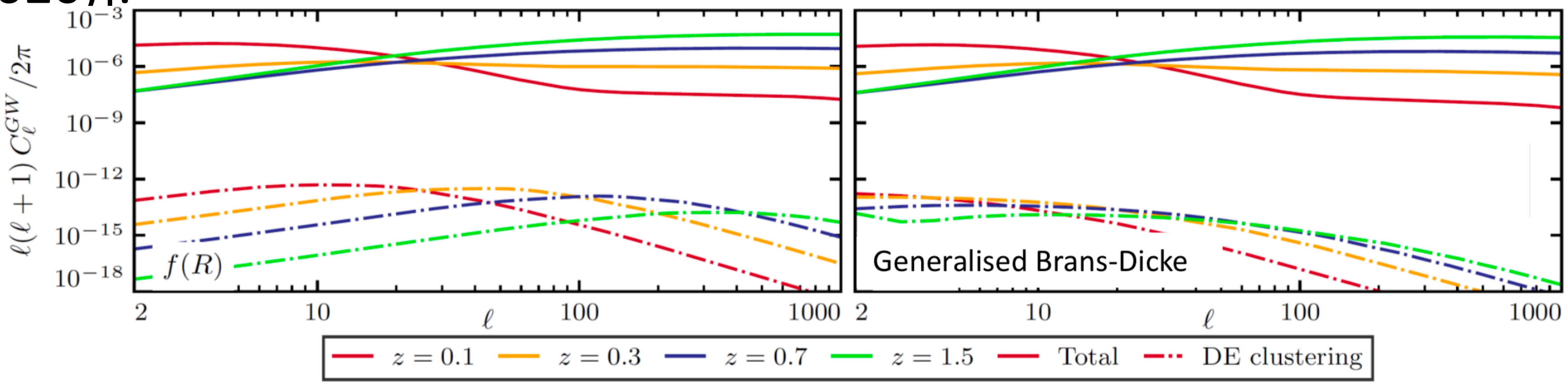
- Fluctuations in the EM luminosity distance are affected by the DE field only indirectly while linearized fluctuations of the GW luminosity distance contain contributions directly proportional to the clustering of the DE field [Garoffolo +DB et al. (2019, 2020)].



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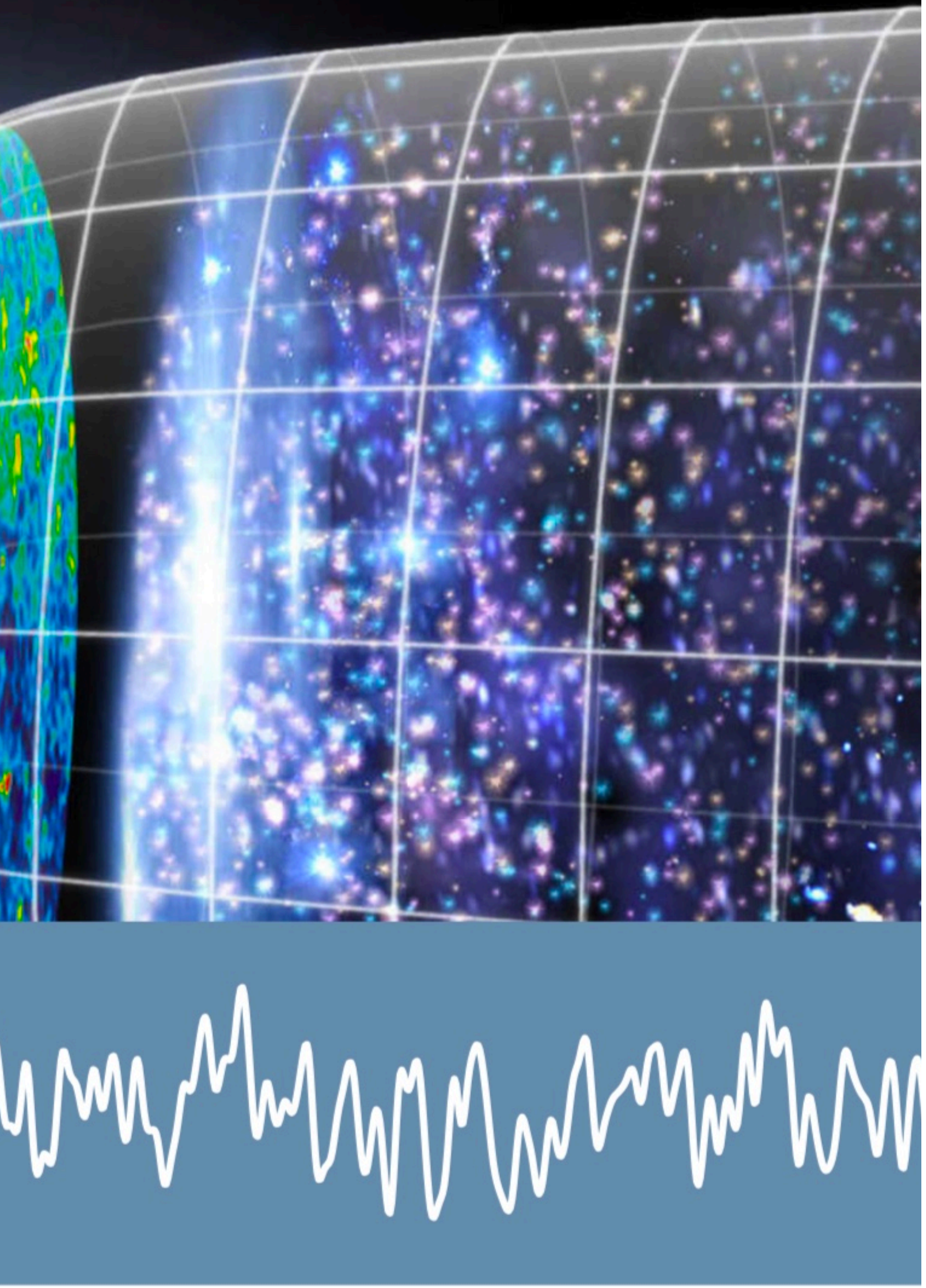
- In [Garoffolo +DB et al. (2020)] exploiting the synergy in supernovae and gravitational wave distance measurements, we build a joint estimator that directly probes DE/MG fluctuations, providing a conclusive evidence for their existence in case of detection.

Figure credit: NASA



Stochastic Gravitational Wave Background (SGWB)

- A SGWB radiation is a superposition of gravitational-wave signals that are either too weak or too numerous to individually detect.
- The SGWB is expected to be a key observable for GW interferometry.
- Its detection will open a new window on early universe cosmology and on the astrophysics of compact objects.
- But despite the fact that one cannot resolve the individual signals that comprise the background, the detection of a gravitational-wave background (GWB) will provide information about the statistical properties (or population properties) of the source.
- The detection and characterisation of the SGWB is one of the main goal of the Gravitational Waves (GW) search.

Different types of SGWB

The combined GW signal from such events occurring throughout the Universe will produce:

- A Cosmological GW background signature of the early Universe

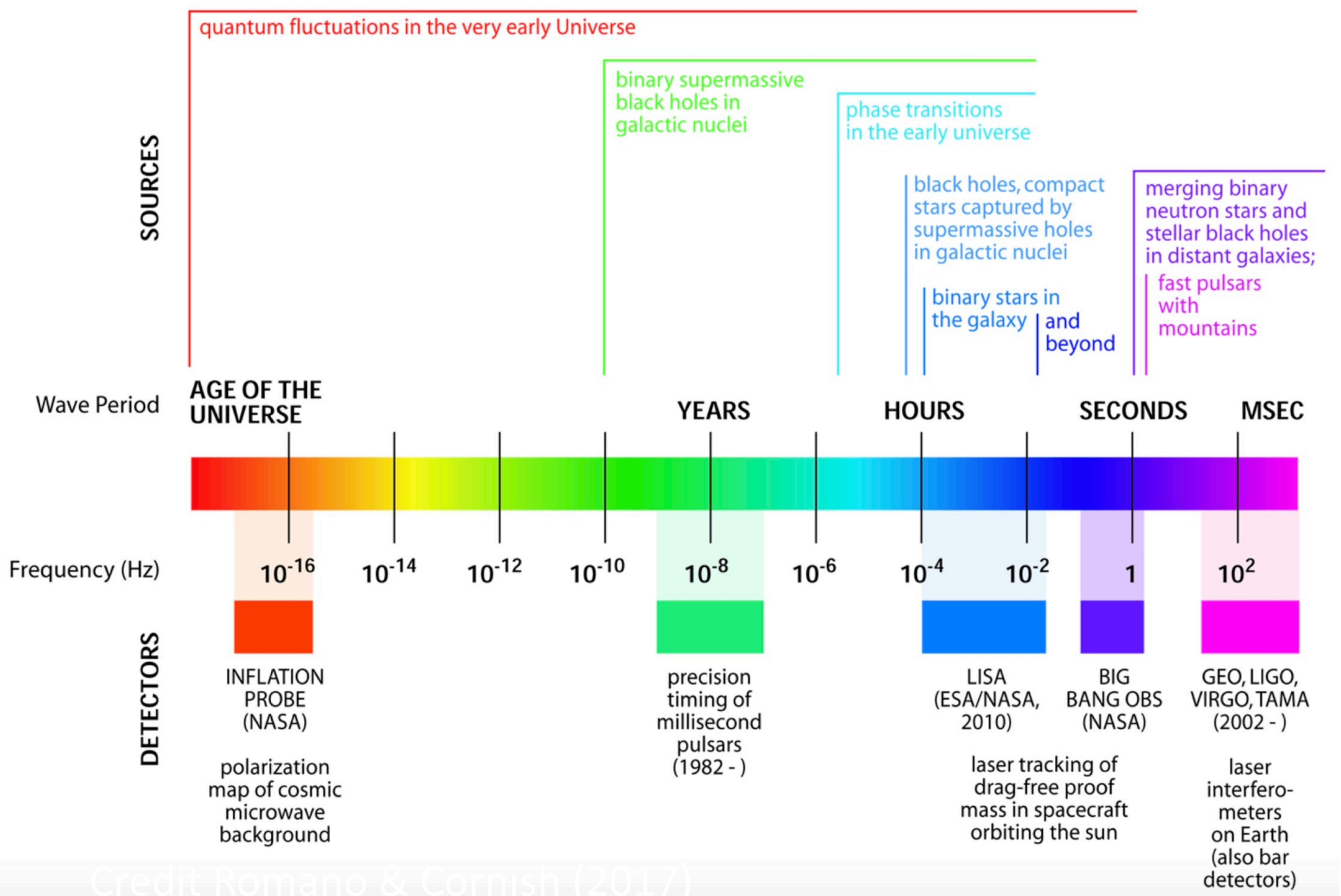
- **Inflationary epoch, preheating, reheating** ($10^{-18} - 10^{8}$ Hz)
- **Phase transitions** (the spectrum has a narrow band feature peaking at 10^{-12} Hz and a broad band component in the $10^{-5} - 1$ Hz band.)
- **Cosmic strings** ($10^{-10} - 10^{10}$ Hz)
- **Alternative cosmologies…**

(e.g. see Maggiore 2000, Guzzetti et al. 2016, Caprini & Figueroa 2018)

- An astrophysical GW background (ASGWB)

- **Dominated by compact binary coalescences**: BBHs, BNSs, BH-NSs
- **Supernova**

Detectors and potential sources of GWBs across the GW spectrum.



SGWB anisotropies:

- Relaxing the assumption of isotropy and generalizing the search for a stochastic signal to the case of arbitrary angular distribution, we find a direction-dependent contribution

$$\Omega_{\mathrm{GW}}\left(f_{\mathrm{o}}, \Omega_o\right)=\frac{f_{\mathrm{o}}}{\rho_{\mathrm{c}}} \frac{\mathrm{d} \rho_{\mathrm{GW}}}{\mathrm{d} f_{\mathrm{o}} \mathrm{d} \Omega_o}$$

- Such a quantity will have both a background (monopole) contribution in the observed frame, which is, by definition, homogeneous and isotropic, i.e. $\overline{\Omega_{\mathrm{GW}}}/4\pi$ and a direction-dependent contribution $\Delta\Omega_{\mathrm{GW}}(f_o, \Omega_o)$

$$\Omega_{\mathrm{GW}}=\frac{f_{\mathrm{o}}}{\rho_{\mathrm{c}}} \frac{\mathrm{d} \rho_{\mathrm{GW}}}{\mathrm{d} f_{\mathrm{o}} \mathrm{d} \Omega_{\mathrm{o}}}=\frac{\bar{\Omega}_{\mathrm{GW}}}{4 \pi}+\Delta \Omega_{\mathrm{GW}}$$

Anisotropies of SGWB from inflation

A derivation of the angular power spectrum of cosmological anisotropies, using a Boltzmann approach, has been obtained in [Alba & Maldacena 2016 (1512.01531), Contaldi 2017 (1609.08168) , Bartolo et al. 2019 (1908.00527)]

Why important?

- to disentangle cosmological from astrophysical SGWB
- to probe evolution of cosmological perturbations
- to provide a new observable to probe primordial non-Gaussianity, in particular primordial non-Gaussianity of GWs!!
- to provide a new way to characterize various generation mechanisms of primordial SGWB

Angular anisotropies of the GW energy density

- Angular anisotropies of the GW energy density

$$\Omega_{\rm GW} = \int d^2\hat{n}\, \omega_{\rm GW}(\eta,\, \vec{x},\, q,\, \hat{n})/4\pi$$

$$\downarrow$$

$$\delta_{\rm GW}\left(\eta,\, \vec{x},\, q,\, \hat{n}\right) \equiv \frac{\delta\omega_{\rm GW}\left(\eta,\, \vec{x},\, q,\, \hat{n}\right)}{\bar{\omega}\left(\eta,\, q\right)}$$

- Imprinted both at production of GWs and by propagation through the large-scale cosmological perturbations

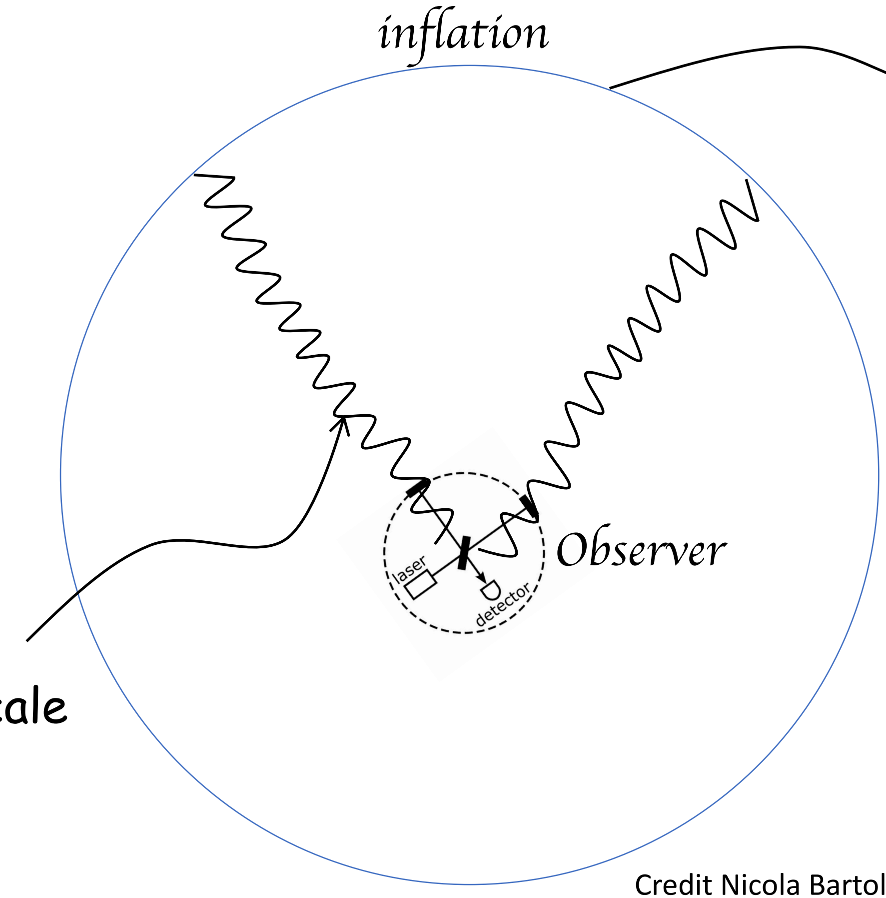
Angular anisotropies of the GW energy density from inflation

"Emission surface" of gravitons at the end of inflation!!!

Two contributions to anisotropies of SGWB:

1. At production

2. by GW propagation up to the observer



Propagation through large-scale cosmological perturbations (both scalar and tensor!!)

similar approach of C. Contaldi, Phys.Lett. B771 (2017)

Credit Nicola Bartolo

Angular anisotropies of the GW energy density from inflation

Boltzmann equation approach

$$\frac{\partial f}{\partial \eta} + n^i \frac{\partial f}{\partial x^i} + \left[\frac{\partial \Psi}{\partial \eta} - n^i \frac{\partial \Phi}{\partial x^i} + \frac{1}{2} n^i n^j \frac{\partial h_{ij}}{\partial \eta} \right] q \frac{\partial f}{\partial q} = 0$$

Free streaming:
keeps memory of initial conditions!!!

Gravitational effects that imprint anisotropies during propagation

$$\delta f \equiv -q \frac{\partial \bar{f}}{\partial q} \Gamma\left(\eta, \vec{x}, q, \hat{n}\right)$$

In the case of CMB
$\Gamma_{\rm CMB} = \delta T / T$

$$\delta_{\rm GW} = \left[4 - \frac{\partial \ln \bar{\Omega}_{\rm GW}\left(\eta, q\right)}{\partial \ln q} \right] \Gamma\left(\eta, \vec{x}, q, \hat{n}\right)$$

N.B.: fixed by Planckian distribution in the case of CMB

Angular anisotropies of the GW energy density from inflation

$$\Gamma_I = \mathrm{e}^{ik\mu(\eta_{\rm in}-\eta)}\Gamma\left(\eta_{\rm in},\,\vec{k},\,q,\,\hat{n}\right)$$

- $$\Gamma\left(\eta,\,\vec{k},\,q,\,\hat{n}\right) = \int_{\eta_{\rm in}}^{\eta} d\eta'\,\mathrm{e}^{ik\mu(\eta'-\eta)}\left\{\Gamma\left(\eta',\,\vec{k},\,q,\,\hat{n}\right)\delta\left(\eta'-\eta_{\rm in}\right) + \Phi\left(\eta',\,\vec{k}\right)\delta\left(\eta'-\eta_{\rm in}\right) + \frac{\partial\left[\Psi\left(\eta',\,\vec{k}\right)+\Phi\left(\eta',\,\vec{k}\right)\right]}{\partial\eta'} - \frac{1}{2}n_i n_j\frac{\partial h_{ij}\left(\eta',\,\vec{k}\right)}{\partial\eta'}\right\}$$

Anisotropies at production: O(1)-dependence on frequency q

Anisotropies from propagation through scalar perturbations

Anisotropies from propagation through tensor perturbations

- $$\Gamma\left(\hat{n}\right) = \sum_{\ell}\sum_{m=-\ell}^{\ell}\Gamma_{\ell m}\,Y_{\ell m}\left(\hat{n}\right) \longrightarrow \Gamma_{\ell m} = \Gamma_{\ell m,I}(q) + \Gamma_{\ell m,S} + \Gamma_{\ell m,T}$$

$$\Gamma_{\ell m} \sim \int d^3k\,X\,\mathcal{T}_\ell^{(X)}(k,\eta_0,\eta_{\rm in})$$

Seed: can be $\Gamma_I(\eta_{\rm in}, k, q)$, **$\zeta$ or h_{ij}**

Transfer function

Angular anisotropies of the GW energy density from inflation

$$\Gamma\left(\hat{n}\right) = \sum_{\ell} \sum_{m=-\ell}^{\ell} \Gamma_{\ell m} \, Y_{\ell m}\left(\hat{n}\right)$$

$$\Gamma_{\ell m} = \Gamma_{\ell m, I} + \Gamma_{\ell m, S} + \Gamma_{\ell m, T}$$

$$\Gamma_{\ell m, I}\left(q\right) = 4\pi \left(-i\right)^{l} \int \frac{d^3k}{\left(2\pi\right)^3} \, \mathrm{e}^{i\vec{k}\cdot\vec{x}_0} \, \Gamma\left(\eta_{\rm in}, \, \vec{k}, \, q\right) Y^*_{\ell m}\left(\hat{k}\right) \, j_\ell\left(k\left(\eta_0 - \eta_{\rm in}\right)\right)$$

$$\Gamma_{\ell m, S} = 4\pi \left(-i\right)^{l} \int \frac{d^3k}{\left(2\pi\right)^3} \, \mathrm{e}^{i\vec{k}\cdot\vec{x}_0} \, \zeta\left(\vec{k}\right) \, Y^*_{\ell m}\left(\hat{k}\right) \, \mathcal{T}_\ell^S\left(k, \, \eta_0, \, \eta_{\rm in}\right)$$

$$\Gamma_{\ell m, T} = 4\pi \left(-i\right)^{l} \int \frac{d^3k}{\left(2\pi\right)^3} \, \mathrm{e}^{i\vec{k}\cdot\vec{x}_0} \, \left[{}_{-2}Y^*_{\ell m}\left(\Omega_k\right) \, \xi_R\left(\vec{k}\right) + {}_{2}\,Y^*_{\ell m}\left(\Omega_k\right) \, \xi_L\left(\vec{k}\right)\right] \mathcal{T}_\ell^T\left(k, \, \eta_0, \, \eta_{\rm in}\right)$$

Angular anisotropies of the GW energy density from inflation

- **Angular power spectra:**

$$\langle \Gamma_{\ell m} \Gamma^*_{\ell' m'} \rangle \equiv \delta_{\ell\ell'}\, \delta_{mm'}\, \widetilde{C}_\ell = \delta_{\ell\ell'}\, \delta_{mm'} [\widetilde{C}_{\ell,I}(q) + \widetilde{C}_{\ell,S} + \widetilde{C}_{\ell,T}]$$

$$\widetilde{C}_\ell \sim \sum_X \int \frac{dk}{k} P^{(X)}(k,q)\, \mathcal{T}_\ell^{(X)2}(k,\eta_0,\eta_{\rm in})$$

Primordial power spectra of:

- **initial anisotropies** $\left\langle \Gamma\left(\eta_{\rm in}, \vec{k}, q\right) \Gamma^*\left(\eta_{\rm in}, \vec{k}', q\right)\right\rangle = \frac{2\pi^2}{k^3} P_\Gamma\left(q, k\right) \left(2\pi\right)^3 \delta\left(\vec{k} - \vec{k}'\right)$

- **large-scale curvature perturbation from inflation** $\left\langle \zeta\left(\vec{k}\right) \zeta^*\left(\vec{k}'\right)\right\rangle = \frac{2\pi^2}{k^3} P_\zeta\left(k\right) \left(2\pi\right)^3 \delta\left(\vec{k} - \vec{k}'\right)$

- **large-scale gravitational waves** $\left\langle h_\lambda\left(\vec{k}\right) h^*_{\lambda'}\left(\vec{k}'\right)\right\rangle = \frac{2\pi^2}{k^3} P_\lambda\left(k\right) \delta_{\lambda,\lambda'} \left(2\pi\right)^3 \delta\left(\vec{k} - \vec{k}'\right)$

Angular anisotropies of the GW energy density from inflation

Bispectra of anisotropies: will be generated by a non-vanishing primordial (inflationary) non-Gaussianity of the large-scale curvature perturbations and of the large-scale primordial gravitational waves

$$\langle \Gamma_{\ell_1 m_1} \Gamma_{\ell_2 m_2} \Gamma_{\ell_3 m_3} \rangle \sim \left[\prod_{i=1}^{3} \int \frac{k_i^2 \, dk_i}{(2\pi)^3} \mathcal{T}_{\ell,i}^{T}(k_i,\, \eta_0,\, \eta_{\rm in}) \right] \sum_{\lambda=\pm R,L} \langle h_\lambda(k_1) h_\lambda(k_2) h_\lambda(k_3) \rangle$$

3-point function of SGWB anisotropies at interferometer scales can be a sensitive probe of inflationary non-Gaussianity in the gravitational wave sector!

Frequency dependence of initial anisotropies

$$\Gamma\left(\eta_0, \vec{x}_0,\, q,\, \hat{n}\right) = \Gamma_I\left(\eta_0, \vec{x}_0,\, q,\, \hat{n}\right) + \Gamma_S\left(\eta_0, \vec{x}_0,\, \hat{n}\right) + \Gamma_T\left(\eta_0, \vec{x}_0,\, \hat{n}\right)$$

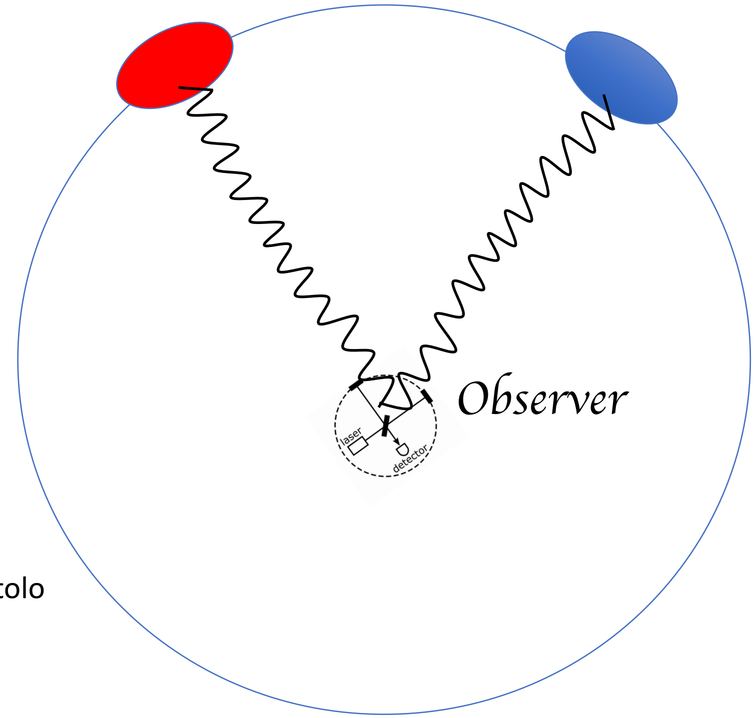
What about the initial conditions?
What about their frequency dependence?

for the case of a primordial SGWB visible at interferometers scales this term is present and can lead to anisotropies with large (order-one) frequency dependence.

NOTE that, in the case of CMB anisotropies, the universe is not transparent to photons before recombination and this contribution is completely erased by collisions at linear order
The frequency dependence in CMB arises only at second-order in the perturbations collision.

Frequency dependence of initial anisotropies

When the initial anisotropies $\Gamma_I\left(\eta_0, \vec{x}_0,\, q,\, \hat{n}\right)$ have a non-trivial q-dependence?



Credit Nicola Bartolo

$$\Omega_{\rm GW}\left(\eta_0,\, q\right) = \text{constant} \times \sum_\lambda P_\lambda\left(\eta_{\rm in},\, q\right)$$

Suppose the GW energy density that arrives at $\mathbf{x}_0$ from direction $\mathbf{n}$ depends on

$$\xi = \bar{\xi} + \delta\xi\left(\vec{x}_0 + d\,\hat{n}\right)$$

$$\omega_{\rm GW}\left(\eta_0,\, \vec{x}_0,\, q,\, \hat{n}\right) \propto \sum_\lambda P_\lambda\left(q,\, \xi\left(\eta_0,\, \vec{x},\, \hat{n}\right)\right)$$

q-dependence of initial anisotropies if q-dependence of

$$\mathcal{F}\left(q,\, \bar{\xi}\right) \equiv \frac{1}{4 - n_T} \frac{\partial \sum_\lambda \left[\ln P_\lambda\left(q,\, \bar{\xi}\right)\right]}{\partial \bar{\xi}}$$

where

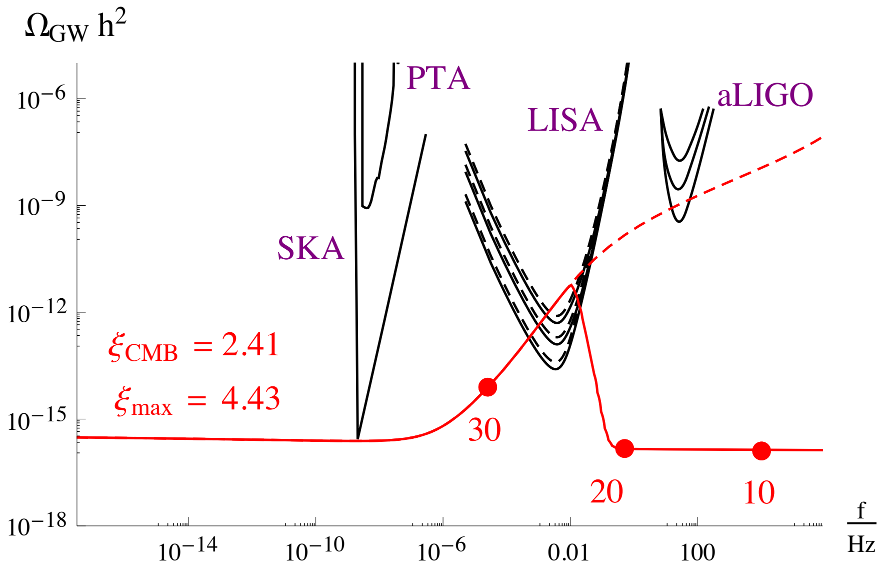
$$n_T = \frac{\partial \ln \sum_\lambda [P_\lambda(q, \bar{\xi})]}{\partial q}$$

Frequency dependence of initial anisotropies

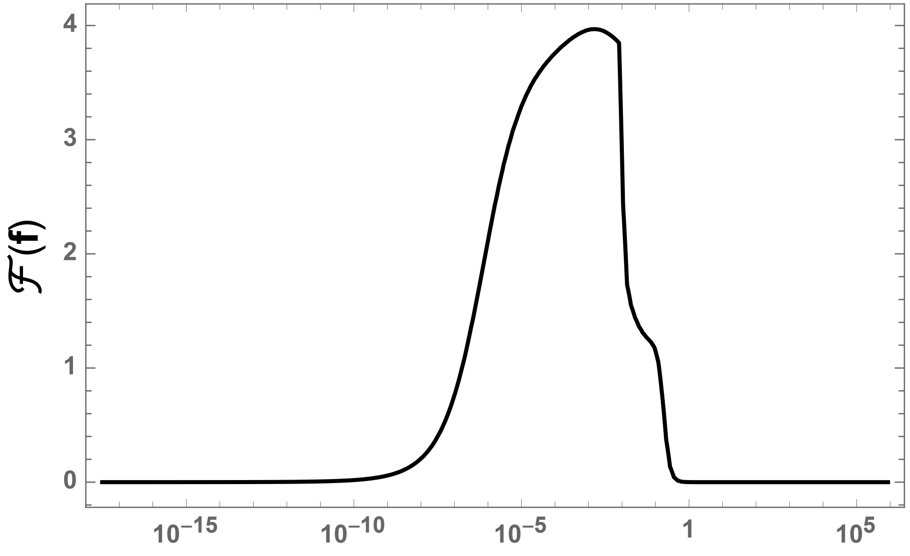
Example: sourced SGWB from axion inflation

$$P_{\rm GW} \simeq \frac{2H^2}{\pi^2 M_p^2}\left[1+\frac{H^2}{M_p^2}\, f_L(\xi)\; {\rm e}^{4\pi\xi}\right]\Bigg|_{q=aH}$$

$$\xi \equiv \frac{\dot\phi}{2fH} \quad \text{thus} \quad \delta\phi \rightarrow \delta\xi$$



From Bellido, Peloso, Unal



N.B, D. Bertacca et al 19 $\frac{\mathbf{f}}{\mathbf{Hz}}$

Frequency dependence of initial anisotropies can be a new observable to probe the origin of a primordial SGWB

Astrophysical Stochastic gravitational background (ASGWB)

- A gravitational wave stochastic background of astrophysical origin may have resulted from the superposition of a large number of unresolved sources since the beginning of stellar activity.
- Its detection would put very strong constrains on the physical properties of compact objects, the initial mass function or the star formation history.

Astrophysical Stochastic gravitational background (ASGWB)

- It could potentially be used to probe the Universe at redshifts $z \sim 0.02$–10 and be used as a tool to study the evolving star formation rate, supernova rates and the mass distribution of black-hole births.

- However, from the point of view of detecting the cosmological background produced in the primordial Universe, **the astrophysical background is a 'noise', which could possibly mask the relic cosmological signal**.

- Hence, an understanding of ASGWBs is important on two fronts:

- first to provide fundamental knowledge of astrophysical source evolution on a cosmological scale
- second to differentiate this background from the early-Universe background.

ASGWB: energy density of the background

The above relationship can also be written in terms of the comoving rate density $R(z)$

$$\Omega_{\rm gw}(f) = \frac{f}{\rho_{\rm c} H_0} \int_{z_{\rm inf}}^{z_{\rm sup}} {\rm d}z \, R(z) \frac{1}{(1+z)E(z)} \left(\frac{{\rm d}E_{\rm gw}}{{\rm d}f_{\rm s}} \right) \Big|_{f_{\rm s}=f(1+z)}$$

- the energy spectrum of an individual source ${\rm d}E_{\rm gw}/{\rm d}f_{\rm s}$ is measured in its rest frame.
- $E(z)$ is a cosmological factor

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- the energy spectrum of an individual source dE_{gw}/df_s is measured in its rest frame.
- $E(z)$ is a cosmological factor
- Progenitors are most likely of a stellar origin it is expected that:

$$R(z) \propto \int_z^{\infty} \mathrm{SFR}(z') f\big(t(z) - t(z')\big) \frac{dt}{dz'} dz'$$

where $\mathrm{SFR}(z)$ is the star formation rate at redshift z (per unit comoving volume and comoving time), $t(z)$ is the age of the universe at redshift z, and f(τ)dτ is the fraction of progenitors that are born with a lifetimes between τ and τ + dτ .

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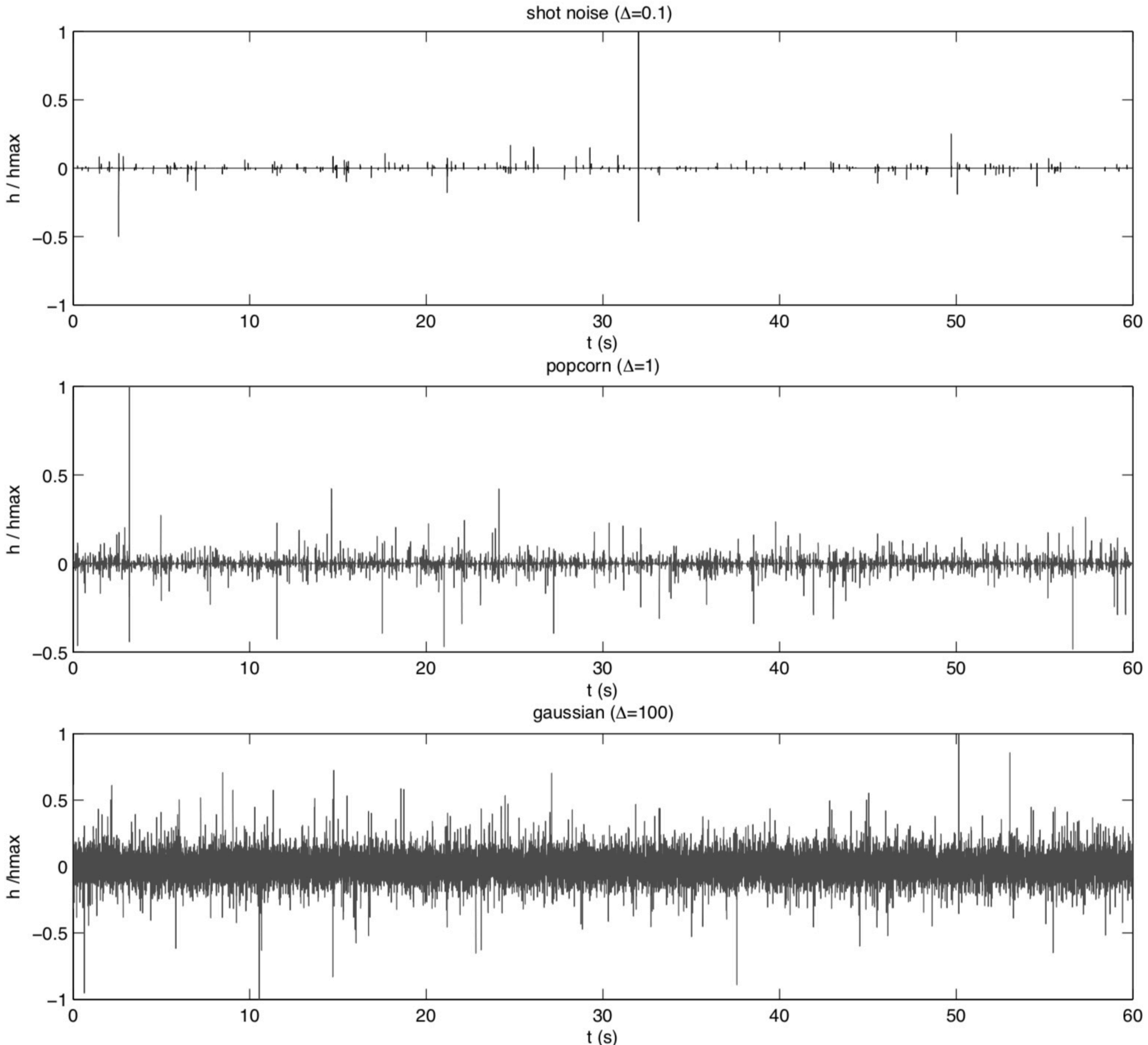
- The limits of the integral over z depend on both the emission frequency range in the source frame, and the redshift interval, where the source can be located
- **Signals** may have very **different statistical behaviour** which depends on the ratio between the duration of the events and the time interval between successive events, the duty cycle:

$$\Delta = \int_0^z \bar{\tau}(1+z) R {\rm d}z'$$

Stochastic backgrounds can also differ from one another in temporal distribution and amplitude.

The duty cycle: $\Delta = \int_0^z \bar{\tau}(1+z)R\mathrm{d}z'$



Regimbau 2011

Stochastic backgrounds can also differ from one another in temporal distribution and amplitude.

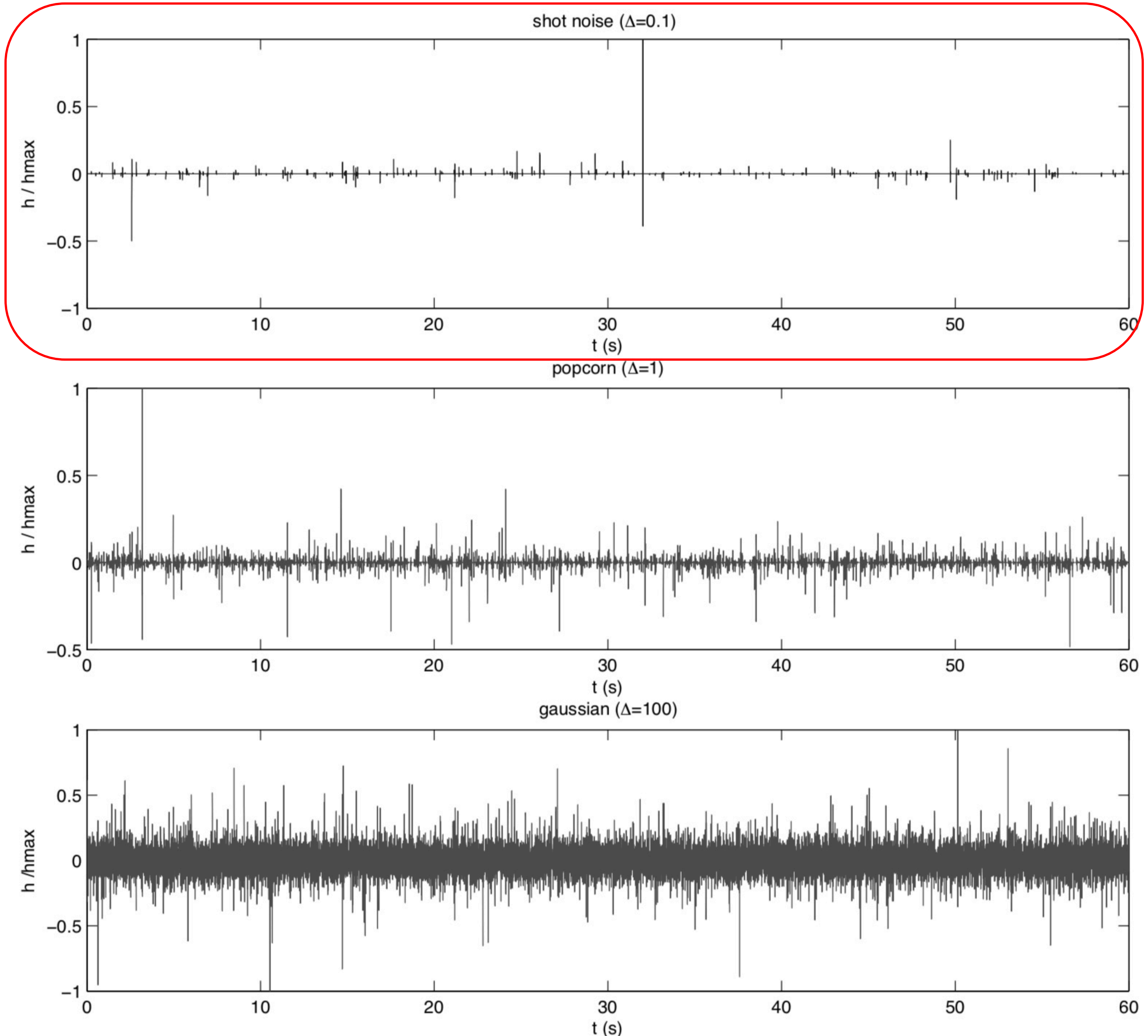
The duty cycle: $$\Delta = \int_0^z \bar{\tau}(1+z)R\mathrm{d}z'$$

Shot noise $\boldsymbol{\Delta < 1}$: the number of sources is small enough for the time interval between events to be long compared to the duration of a single event.

The waveforms are separated by long stretches of silence and the closest sources may are resolved and can be detected by data analysis techniques in the time domain (or the time frequency domain) such as match filtering (Arnaud et al. 1999; Pradier et al. 2001).

The optimal method is matched filtering but this filter can only be optimal if the exact shape of the signal is known.

Regimbau 2011



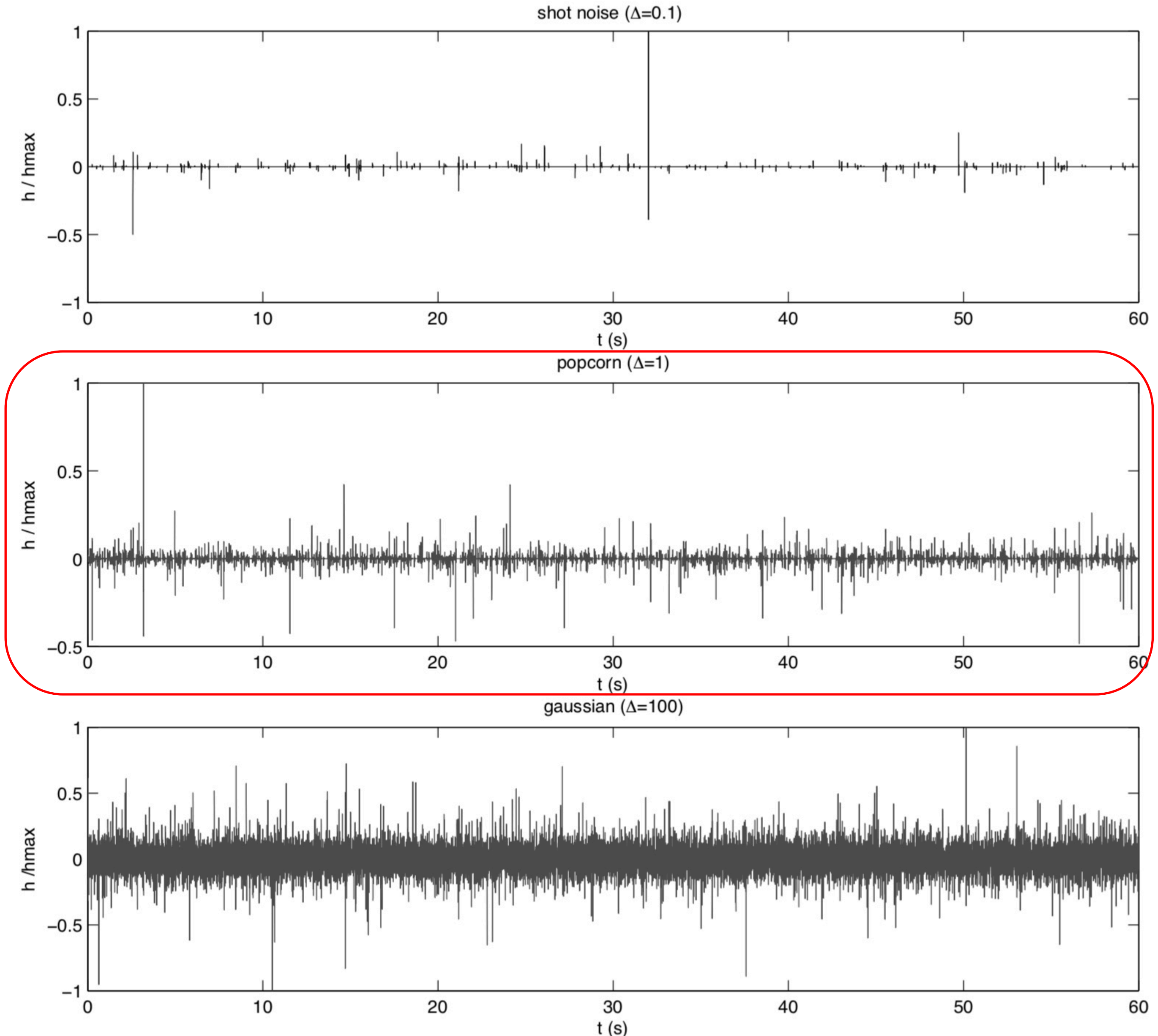
Stochastic backgrounds can also differ from one another in temporal distribution and amplitude.

The duty cycle: $\Delta = \int_0^z \bar{\tau}(1+z)R\mathrm{d}z'$

- **Popcorn noise $\boldsymbol{\Delta \sim 1}$** : the time interval between events is of the same order of the duration of a single event. These signals, which sound like crackling popcorn.

- The waveforms may overlap but the statistic is not Gaussian and the amplitude on the detector at a given time is unpredictable.

- The amplitude distribution of this background signal will contain information on the spatial distribution of the sources.

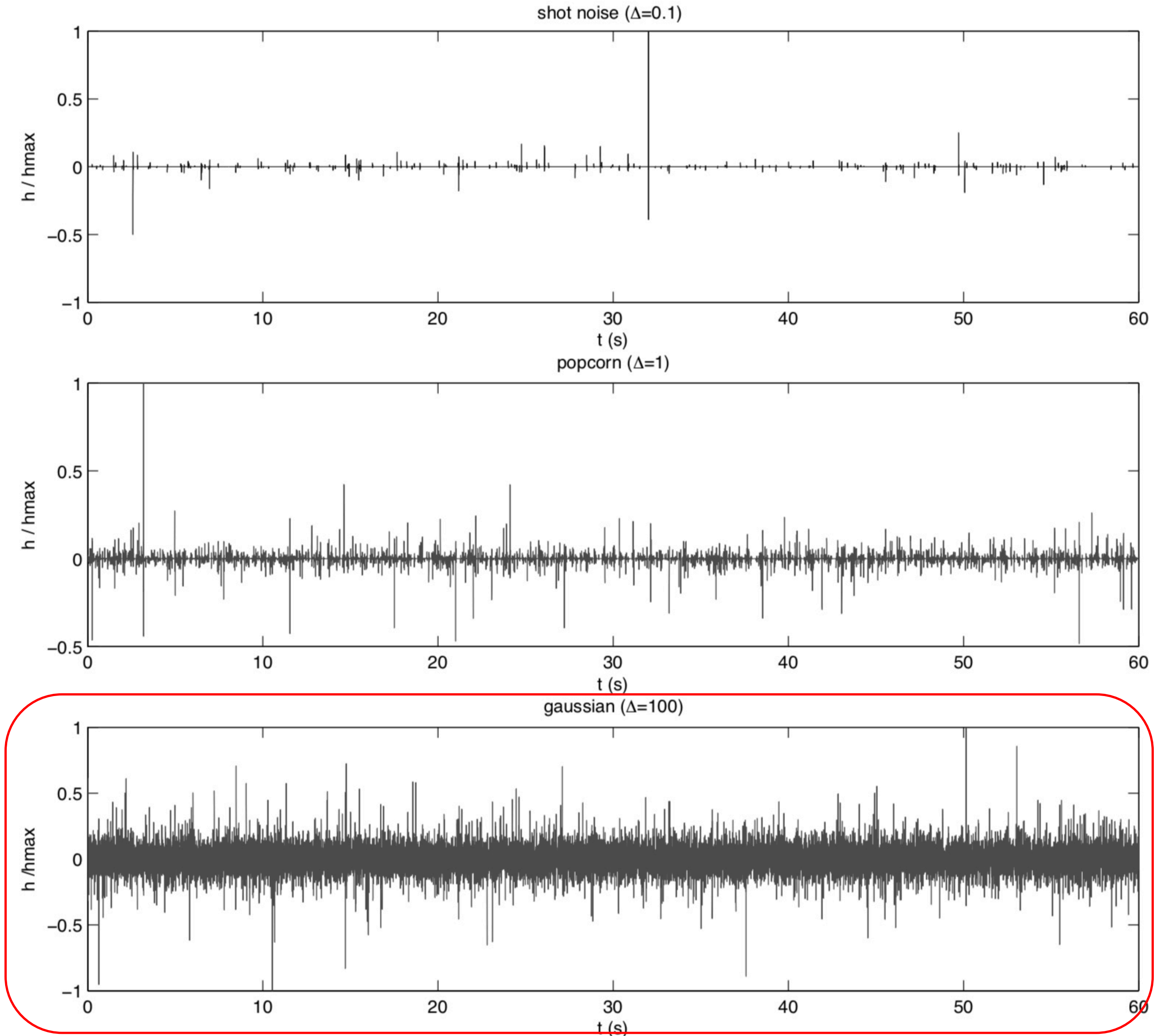


Regimbau 2011

Stochastic backgrounds can also differ from one another in temporal distribution and amplitude.

The duty cycle: $\Delta = \int_0^z \bar{\tau}(1+z)R\mathrm{d}z'$

- **Continuous $\Delta \gg 1$**: the number of sources is large enough for the time interval between events to be small compared to the duration of a single event.
- The waveforms overlap to create a continuous background and due to the central limit theorem, such backgrounds obey the Gaussian statistic.
- They are completely determined by their spectral properties and could be detected by data analysis methods in the frequency domain such as the standard cross correlation statistic (Allen & Romano 1999).

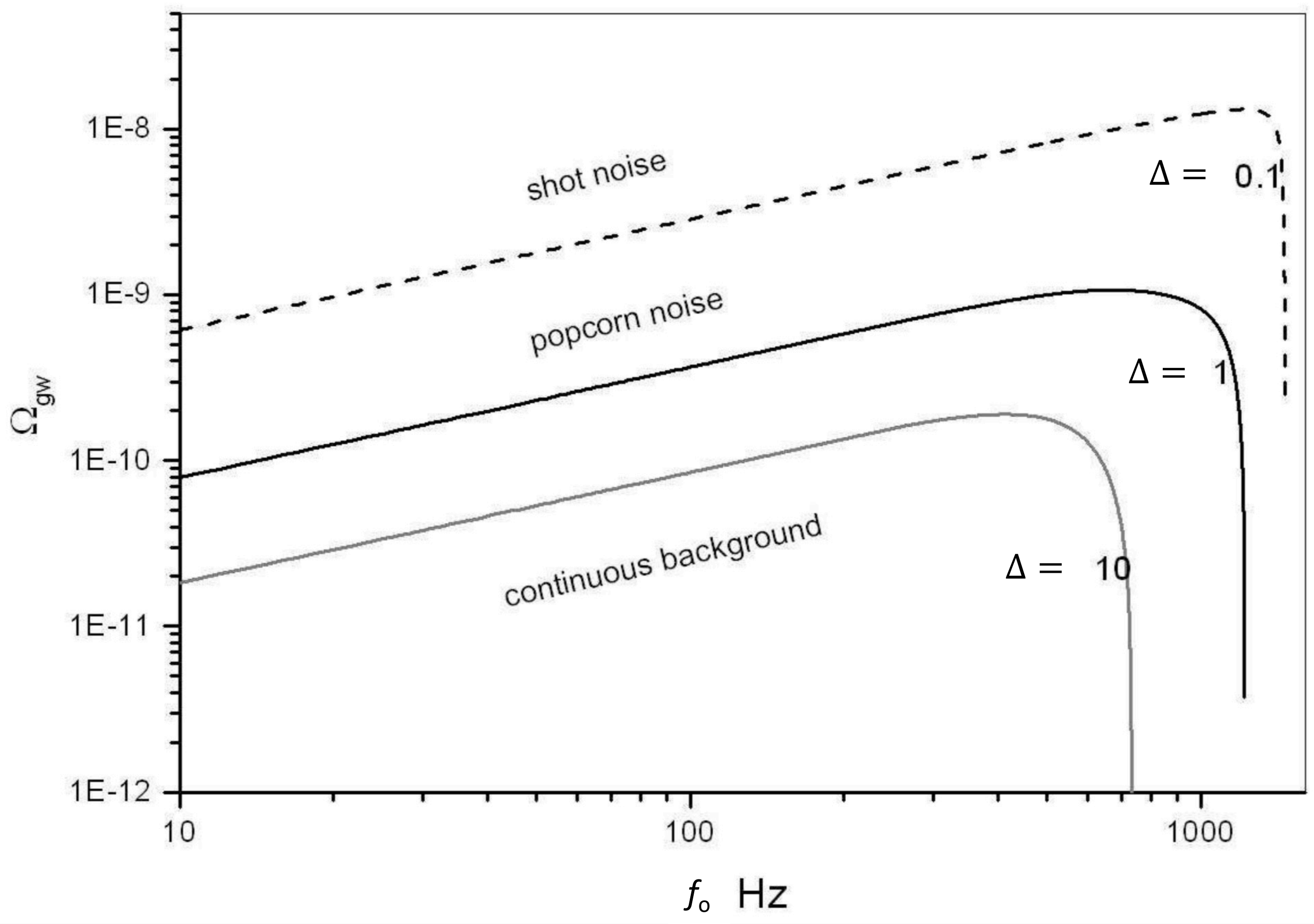


Regimbau 2011

Nature of the background.

Coward & Regimbau 2006

- Events located beyond the critical redshift $z^* = 0.23$ produced a continuous background;
- in the redshift interval $0.027 < z < 0.23$ produce a nearly continuous (popcorn noise) signal;
- The signal is discrete, i.e. shot noise like, for events occurring closer than $z = 0.027$.



Ω_{gw} ,for the continuous component, has a maximum at 670 Hz with an amplitude of 1.1×10^{-9}, while the popcorn noise component has an amplitude about one order of magnitude higher with maximum at 1.2 kHz. That study highlights the importance of the popcorn noise regime to the ASGWB (Coward & Regimbau 2006).

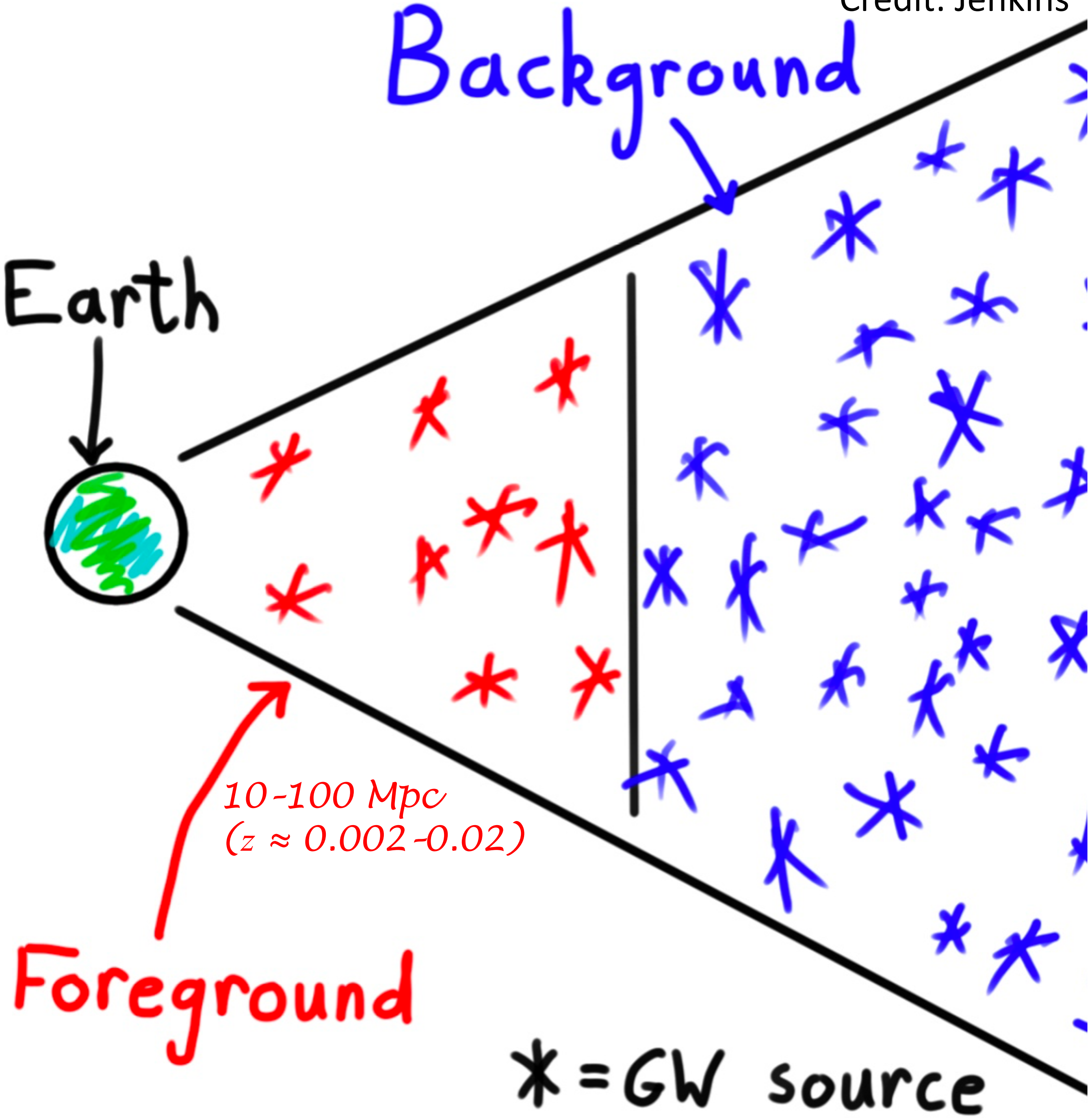
Compact white-dwarf binaries in the Milky Way

- Combined GW signal produced by compact white-dwarf binaries in the Milky Way, producing a “confusion-limited” GWB in the frequency band $\sim 10^{-4}$ Hz to 10^{-1} Hz [Bender and Hils (1997)].

- This is a guaranteed signal for the proposed space-based interferometer LISA

- The confusion-limited white-dwarf binary signal is expected to be so strong that it will dominate the instrumental noise at low frequencies, forming a GW “**foreground**” that will have to be contended with when searching for other gravitational sources in the LISA band [e.g. see Adams et al. (2014)]

Credit: Jenkins



Regimbau 2011

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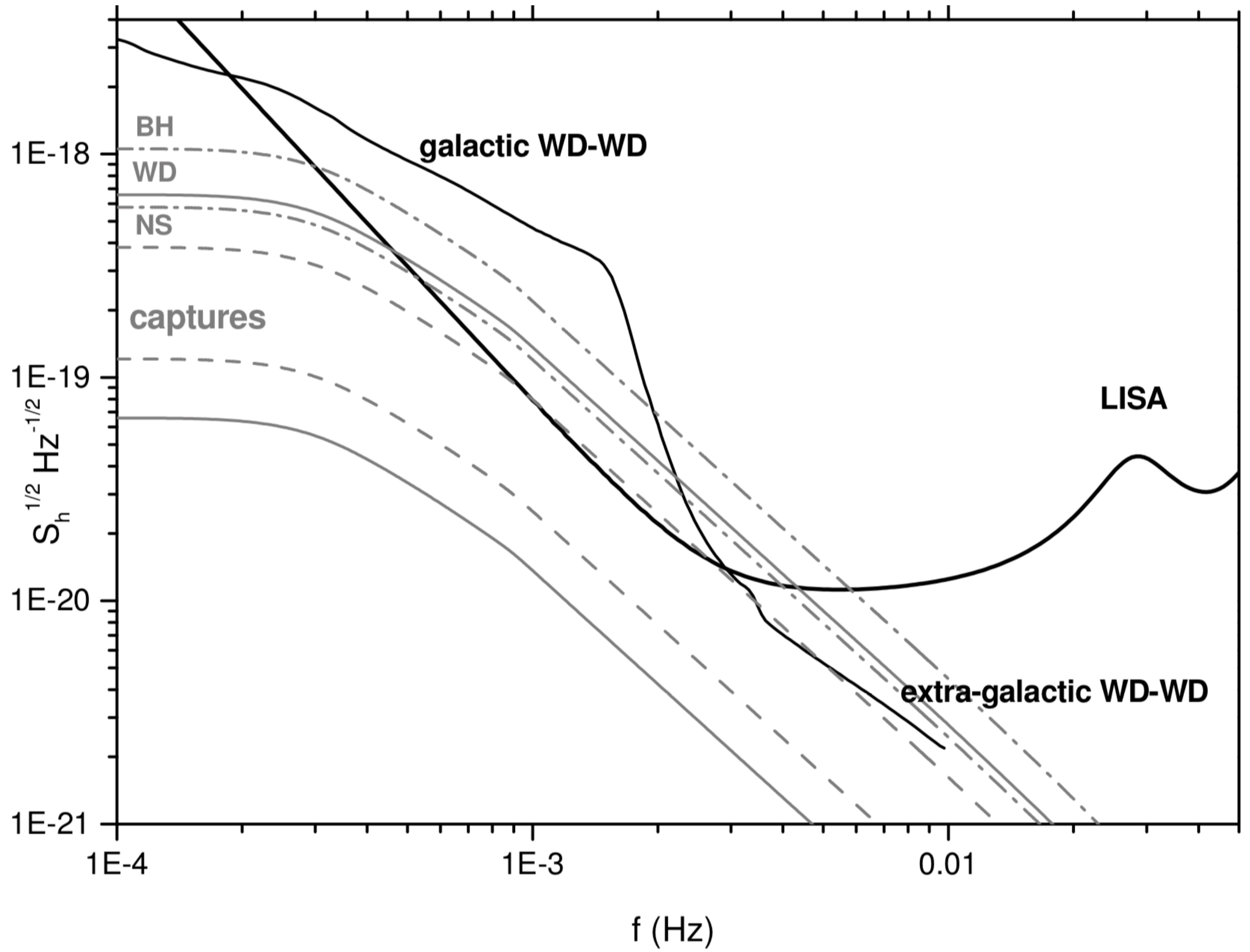
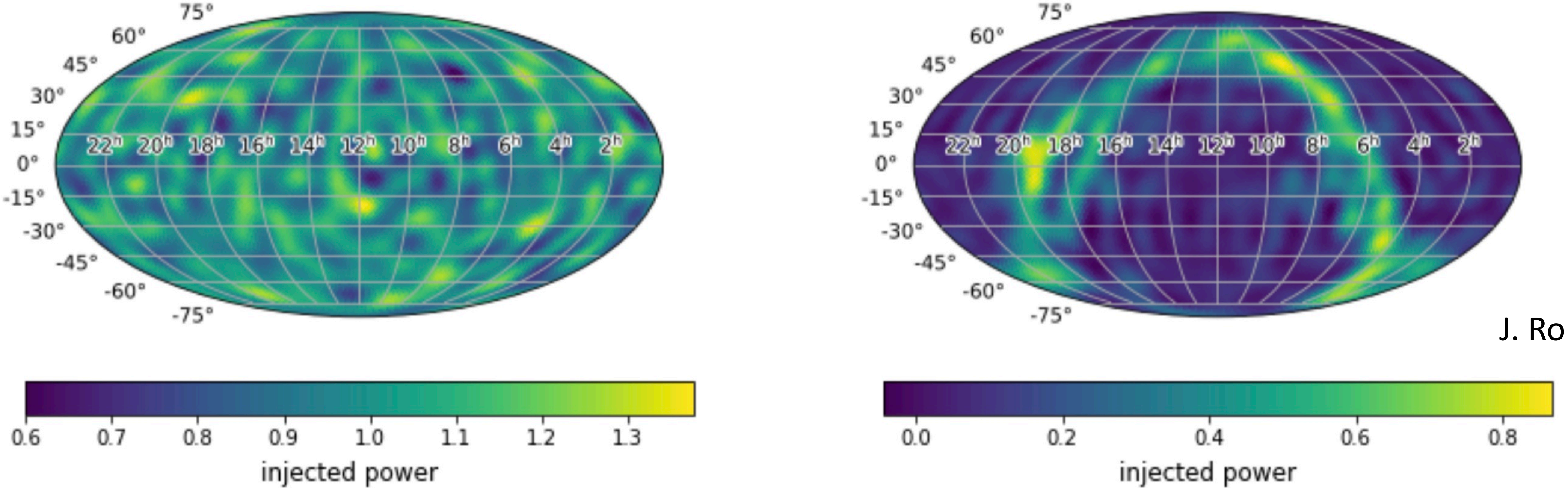


Gravitational strain in $Hz^{-1/2}$, corresponding to optimistic (grey continuous curve) and pessimistic (grey dashed curve) compact object captures (Barack & Cutler 2004), along with the LISA instrumental noise (black) and the WD-WD foreground (black).

SGWB anisotropies:

Statistically isotropic backgrounds are to be contrasted with statistically anisotropic backgrounds, whose distribution of power on the sky has preferred directions, even when averaged over different realizations of the sources that produce it. For example, **the "confusion-limited" foreground** that LISA will see from the population of close **white-dwarf binaries in the Milky Way** will have its GW power concentrated in the direction of the Milky Way.



J. Romano 2019

Figure shows simulated sky-maps for a statistically isotropic background (left panel) and a statistically anisotropic background (right panel). The anisotropic background in that figure follows the galactic plane in equatorial coordinates.

ASGWB anisotropies:

- Angular power spectrum by [Cusin et al. (2017, 2018a,b)] considering the presence of inhomogeneities in the matter distribution and working with a coarse graining approach which allow to probe GW sources on cosmological, galactic and sub-galactic scales.
- Other predictions for the GW angular power spectrum have been derived in [Jenkins et a. (2018a,b)] where both analytical expression and numerical study, using galaxy catalogue from the Millennium simulation, are presented.
- Very recently, [Cusin et al. (2019a,b)] analyses the astrophysical dependence of the angular power spectrum for different stellar models.

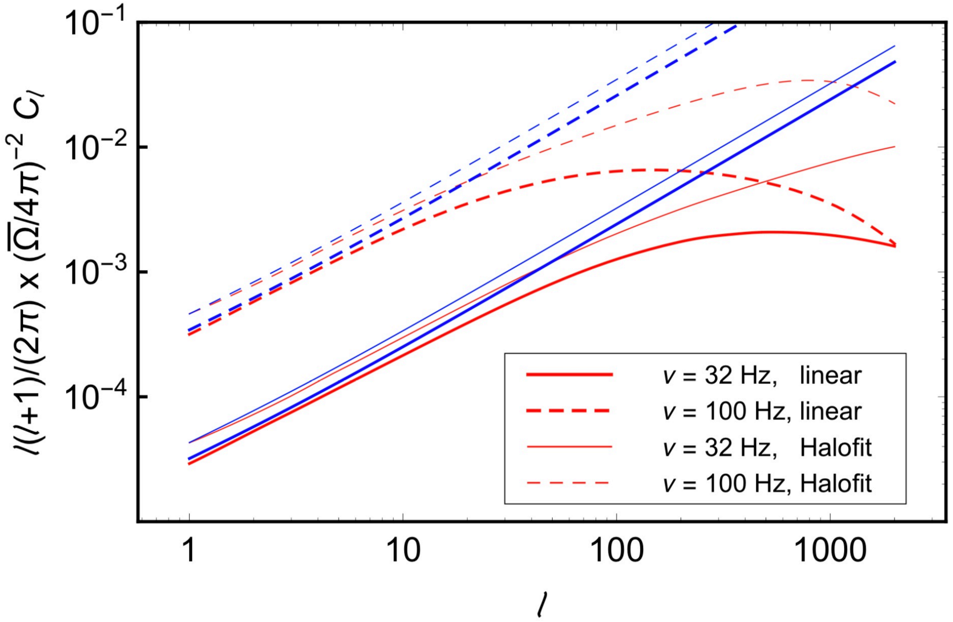
$$C(\nu_o,\theta) = \langle \delta\Omega_{GW}(\nu_o, \mathbf{e}_1)\delta\Omega_{GW}(\nu_o, \mathbf{e}_2)\rangle$$

$$\sum_\ell \frac{2\ell+1}{2\pi} C_\ell(\nu_o) P_\ell(\mathbf{e}_1\cdot\mathbf{e}_2)$$

$$C_\ell(\nu_o) = \frac{2}{\pi}\int \mathrm{d}k\, k^2 |\delta\hat{\Omega}_\ell(k,\nu_o)|^2$$

Cl ∝ 1/l on large scales

- relative fluctuations of the signal are of order 30% at 100 Hz.



Cusin, Dvorkin, Pitrou, Uzan 2017,2018 ,2019

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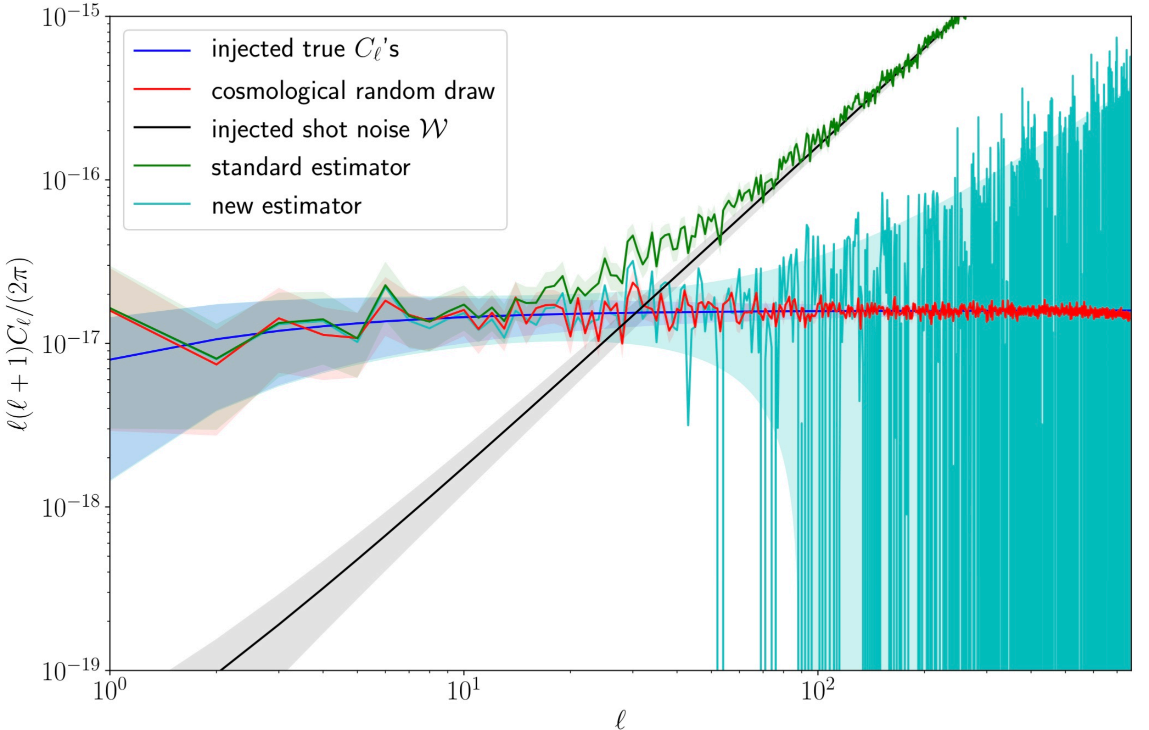
- In [Jenkins et al. (2019a,b)]] they pointed out that ASGWB is obscured by shot noise, caused by the finite number of GW sources at non-linear scales (precisely for scales less than 100Mpc, i.e. z<0.023). They developed a new method for estimating the angular spectrum of anisotropies, based on the principle of combining statistically-independent data segments.



Bertacca et al 2019

AGWB in a general covariant setting

The total gravitational energy density in a direction n is the sum of the all unresolved astrophysical contributions along the line of sight contained in a given volume dV_e(***n***) [see also Cusin et al. 2017]

$$\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}f_{\mathrm{o}}\mathrm{d}\Omega_o} = \frac{\mathrm{d}\mathcal{E}^{\mathrm{tot}}_{\mathrm{GW}}}{\mathrm{d}f_o\mathrm{d}\mathcal{T}_{\mathrm{o}}\mathrm{d}A_{\mathrm{o}}\mathrm{d}\Omega_{\mathrm{o}}} = \sum_{[i]}\int n_{\mathrm{h}}^{[i]}(x^\alpha_{\mathrm{e}},\vec{\theta})\frac{\mathrm{d}\mathcal{E}^{[i]}_{\mathrm{GW}}(x^\mu_{\mathrm{e}}\to x^\mu_{\mathrm{o}},\vec{\theta})}{\mathrm{d}f_o\mathrm{d}\mathcal{T}_{\mathrm{o}}\mathrm{d}A_{\mathrm{o}}}\left|\frac{\mathrm{d}V_{\mathrm{e}}}{\mathrm{d}\Omega_{\mathrm{o}}\mathrm{d}\chi}\right|\mathrm{d}\chi\,\mathrm{d}\vec{\theta}$$

where

- $n_{\mathrm{h}}^{[i]}$ is the (physical) GW number density in a halo located at x_e^μ,
- [*i*] is the index of summation over all unresolved astrophysical sources that produce the background of GWs
- $\vec{\theta} = \{M_{\mathrm{h}}, M^*, \vec{m}, \vec{\theta}^*\}$, where M_{h} is the halo mass, M^* the masses of the stars that give origin of the sources, $\vec{m}$ are the masses of the compact objects and $\vec{\theta}^*$ includes the astrophysical parameters related to the model (i.e. spin, orbital parameters, star formation rate).
- The letter "e" stands for "evaluated at the emission (source) position" while "o" for "evaluated at the observers position".
- The physical volume dV_e(***n***) at emission is defined as

$$\mathrm{d}V_e \equiv \mathcal{D}^2_{\mathrm{A}}(z)(-u_\mu p^\mu_{\mathrm{GW}})\left|\frac{\mathrm{d}\lambda}{\mathrm{d}\chi}\right|\mathrm{d}\Omega_{\mathrm{o}}\mathrm{d}\chi$$

- u_μ is the four velocity vector as a function of comoving location
- $\mathcal{D}_{\mathrm{A}}(z)$ the angular diameter distance
- p^μ_{GW} GW four-momentum

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AGWB in a general covariant setting

The total gravitational energy density in a direction n is the sum of the all unresolved astrophysical contributions along the line of sight contained in a given volume dV_e(***n***) [see also Cusin et al. 2017]

$$\frac{\mathrm{d}\rho_{\rm GW}}{\mathrm{d}f_{\rm o}\mathrm{d}\Omega_o}=\frac{\mathrm{d}\mathcal{E}^{\rm tot}_{\rm GW}}{\mathrm{d}f_o\mathrm{d}\mathcal{T}_{\rm o}\mathrm{d}A_{\rm o}\mathrm{d}\Omega_{\rm o}}=\sum_{[i]}\int n_{\rm h}^{[i]}(x^\alpha_{\rm e},\vec{\theta})\frac{\mathrm{d}\mathcal{E}^{[i]}_{\rm GW}(x^\mu_{\rm e}\to x^\mu_{\rm o},\vec{\theta})}{\mathrm{d}f_{\rm o}\mathrm{d}\mathcal{T}_{\rm o}\mathrm{d}A_{\rm o}}\left|\frac{\mathrm{d}V_{\rm e}}{\mathrm{d}\Omega_{\rm o}\mathrm{d}\chi}\right|\mathrm{d}\chi\,\mathrm{d}\vec{\theta}$$

- Here we consider the local wave zone approximation to define the tetrads at source position (i.e. the observer "at the emitted position" is a region with a comoving distance to the source sufficiently large so that the gravitational field is "weak enough" but still "local", i.e. its wavelength is small w.r.t. the comoving distance from the observer
- Using

$$p^\mu_{\rm GW}=\frac{\mathrm{d}\chi}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\mu}{\mathrm{d}\chi}=-\frac{2\pi f_o}{a^2}k^\mu \qquad (-u_\mu p^\mu_{\rm GW})=2\pi f_e \qquad \frac{\mathrm{d}\mathcal{E}^{[i]}_{\rm GW_o}}{\mathrm{d}f_{\rm o}\mathrm{d}\mathcal{T}_{\rm o}\mathrm{d}A_{\rm o}}=\frac{1}{(1+z)^3\mathcal{D}^2_{\rm A}(z)}\frac{\mathrm{d}\mathcal{E}^{[i]}_{\rm GW_e}}{\mathrm{d}f_{\rm e}\mathrm{d}\mathcal{T}_{\rm e}\mathrm{d}\Omega_{\rm e}}$$

We find

$$\frac{\mathrm{d}\rho_{\rm GW}}{\mathrm{d}f_{\rm o}\mathrm{d}\Omega_{\rm o}}=\sum_{[i]}\int a(x^0)^2\frac{n^{[i]}(x^\alpha_{\rm e},\vec{\theta})}{(1+z)^2}\mathrm{d}\chi\mathrm{d}\vec{\theta}\,,$$

where we define

$$n^{[i]}(x^\alpha_{\rm e},\vec{\theta})\equiv n_h^{[i]}(x^\alpha_{\rm e})\frac{\mathrm{d}\mathcal{E}^{[i]}_{\rm GW_e}}{\mathrm{d}f_{\rm e}\mathrm{d}\mathcal{T}_{\rm e}\mathrm{d}\Omega_{\rm e}}(z,f_{\rm e},x^\mu_{\rm e},\vec{\theta})$$

The total GW density

Energy of gravitational waves

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[see also Cusin et al. 2017]

$$\frac{\mathrm{d}\mathcal{E}^{[i]}_{\mathrm{GW\,e}}}{\mathrm{d}f_{\mathrm{e}}\mathrm{d}\mathcal{T}_{\mathrm{e}}\mathrm{d}\Omega_{\mathrm{e}}}$$

- **Energy at emission has a specific distribution function** characterised by local parameters of the source which **depends on the mass, environment, distribution of matter, velocity dispersion of the matter and source, and the type of galaxies within the host halo.**
- We can thus distinguish two cases:

(I) events with short emission (burst sources), e.g. merging binary sources (BH-BH, NS-NS and/or NS-BH) and SNe explosions;

(II) events with inspiral binary sources which have not merged during a Hubble time, and hence GW emission is averaged over several periods of the slow evolution of the orbitals parameters (continuous sources). The resulting energy in the two cases reads

$$\frac{\mathrm{d}\mathcal{E}^{[i]}_{\mathrm{GW}\,e}}{\mathrm{d}f_e\mathrm{d}\mathcal{T}_e\mathrm{d}\Omega_e} = \begin{cases} \dfrac{\mathrm{d}\mathcal{N}^{[i]}_{\mathrm{GW\,e}}}{\mathrm{d}\mathcal{T}_{\mathrm{e}}} \dfrac{\mathrm{d}\mathcal{E}^{[i]}_{\mathrm{GW\,e}}}{\mathrm{d}f_{\mathrm{e}}\mathrm{d}\Omega_{\mathrm{e}}} & \text{for} \quad (\mathrm{I}) \\ \mathcal{N}^{[i]}_{\mathrm{GW}\,e} \dfrac{\mathrm{d}A_{\mathrm{e}}}{\mathrm{d}\Omega_{\mathrm{e}}} \dfrac{\mathrm{d}\mathcal{E}^{[i]}_{\mathrm{GW\,e}}}{\mathrm{d}f_{\mathrm{e}}\mathrm{d}\mathcal{T}_{\mathrm{e}}\mathrm{d}A_{\mathrm{e}}} & \text{for} \quad (\mathrm{II}) \end{cases}$$

ASGWB anisotropies in the observed frame

Using the COSMIC LABORATORY (cosmic rulers)

$$\Omega_{\rm GW} = \frac{f_{\rm o}}{\rho_{\rm c}} \frac{{\rm d}\rho_{\rm GW}}{{\rm d}f_{\rm o}{\rm d}\Omega_{\rm o}} = \frac{\bar{\Omega}_{\rm GW}}{4\pi} + \Delta\Omega_{\rm GW},$$

where

$$\frac{\bar{\Omega}_{\rm GW}}{4\pi} = \frac{f_{\rm o}}{\rho_{\rm c}} \frac{{\rm d}\bar{\rho}_{\rm GW}}{{\rm d}f_{\rm o}{\rm d}\Omega_{\rm o}} = \frac{f_{\rm o}}{\rho_{\rm c}} \sum_{[i]} \int \frac{N^{[i]}(z, f_{\rm e}, \vec{\theta})}{(1+z)} \, {\rm d}\bar{\chi} \, {\rm d}\vec{\theta} \,,$$

with $N^{[i]}(z, f_{\rm e}, \vec{\theta}) = \bar{n}^{[i]}(z, f_{\rm e}, \vec{\theta})/(1+z)^3$ the *total* comoving number density at a given redshift, and

$$\Delta\Omega_{\rm GW} = \frac{f_o}{\rho_c} \sum_{[i]} \int \frac{N^{[i]}(z, f_{\rm e}, \vec{\theta})}{(1+z)} \left\{ \delta^{[i]} + \frac{{\rm d}\ln N^{[i]}}{{\rm d}\ln \bar{a}} \Delta \ln a - \left(1 + \frac{\mathcal{H}'}{\mathcal{H}^2}\right) \Delta \ln a + \delta f - \frac{1}{\mathcal{H}} \frac{{\rm d}\Delta \ln a}{{\rm d}\bar{\chi}} \right\} {\rm d}\bar{\chi} \, {\rm d}\vec{\theta} \,.$$

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ASGWB anisotropies in the observed frame

Using the COSMIC LABORATORY (cosmic rulers)

When the integration along the line of sight is performed, one should also consider the normalized selection window $w(z)$ function, whose form depends, besides redshift, on the sensitivity/characteristics of the detector (e.g. interferometers in the case of GW). Then we have

$$\Omega_{\rm GW} = \frac{f_{\rm o}}{\rho_{\rm c}} \frac{{\rm d}\rho_{\rm GW}}{{\rm d}f_{\rm o}{\rm d}\Omega_{\rm o}} = \frac{\bar{\Omega}_{\rm GW}}{4\pi} + \Delta\Omega_{\rm GW},$$

$$\frac{\bar{\Omega}_{\rm GW}}{4\pi} = \frac{f_{\rm o}}{\rho_{\rm c}} \frac{{\rm d}\bar{\rho}_{\rm GW}}{{\rm d}f_{\rm o}{\rm d}\Omega_{\rm o}} = \frac{f_{\rm o}}{\rho_{\rm c}} \sum_{[i]} \int w(z) \frac{N^{[i]}(z, f_{\rm e}, \vec{\theta})}{(1+z)} \, {\rm d}\bar{\chi} \, {\rm d}\vec{\theta} \,,$$

with $N^{[i]}(z, f_{\rm e}, \vec{\theta}) = \bar{n}^{[i]}(z, f_{\rm e}, \vec{\theta})/(1+z)^3$ the *total* comoving number density at a given redshift or $\bar{\chi}$, and, at linear order, we obtain

$$\Delta\Omega_{\rm GW} = \frac{f_{\rm o}}{\rho_c} \sum_{[i]} \int w(z) \frac{N^{[i]}(z, f_{\rm e}, \vec{\theta})}{(1+z)} \left\{ \delta^{[i]} + \frac{{\rm d}\ln N^{[i]}}{{\rm d}\ln \bar{a}} \Delta \ln a - \left(1 + \frac{\mathcal{H}'}{\mathcal{H}^2}\right) \Delta \ln a + \delta f \right.$$
$$\left. - \frac{1}{\mathcal{H}} \frac{{\rm d}\Delta \ln a}{{\rm d}\bar{\chi}} \right\} {\rm d}\bar{\chi} \, {\rm d}\vec{\theta} \,.$$

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ASGWB anisotropies in the observed frame

Where the AGWB energy density fluctuation is

$$\Delta\Omega_{\rm AGWB}(f_o,\hat{\mathbf{n}},\theta) = \sum_{[i]}\int dz\, w(z)\,\mathcal{F}^{[i]}(f_o,z,\theta)\times$$

$$\times\Big\{ b^{[i]} D$$ → density contribution

$$+\left(b^{[i]}_{\rm evo}-2-\frac{\mathcal{H}'}{\mathcal{H}^2}\right)\hat{\mathbf{n}}\cdot\mathbf{V}-\frac{1}{\mathcal{H}}\partial_\parallel(\hat{\mathbf{n}}\cdot\mathbf{V})-(b^{[i]}_{\rm evo}-3)\mathcal{H}V$$ → velocity contribution

$$+\left(3-b^{[i]}_{\rm evo}+\frac{\mathcal{H}'}{\mathcal{H}^2}\right)\Psi+\frac{1}{\mathcal{H}}\Phi'+\left(2-b^{[i]}_{\rm evo}+\frac{\mathcal{H}'}{\mathcal{H}^2}\right)\int_0^{\chi(z)}d\tilde{\chi}\,(\Phi'+\Psi')$$ → gravity contribution

$$+\left(b^{[i]}_{\rm evo}-2-\frac{\mathcal{H}'}{\mathcal{H}^2}\right)\left(\Psi_o-\mathcal{H}_0\int_0^{\tau_0}d\tau\,\frac{\Psi(\tau)}{1+z(\tau)}\bigg|_o-(\hat{\mathbf{n}}\cdot\mathbf{V})_o\right)\Big\},$$ → observer contribution

$\mathcal{F}^{[i]}(f_o,z,\theta)$ → contains all the astrophysical dependencies (mass and spin distribution of the binary, emitted GW energy spectrum, clustering properties of GW events and details of the GW detectors)

$b^{[i]}(z,\theta)$ → Bias of the i-th type of GW source

$b^{[i]}_{\rm evo}(z,\theta) = -\dfrac{d\log\left[(1+z)\mathcal{F}^{[i]}\right]}{d\log(1+z)}$ → Evolution bias of the i-th type of GW source

NB: normalized selection window $w(z)$ function depends on redshift, sensitivity/characteristics of the detector/detectors

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ANGULAR POWER SPECTRUM

Rewriting the background energy density in the following way

$$\frac{\bar{\Omega}_{\rm GW}}{4\pi} = \sum_{[i]} \frac{\bar{\Omega}^{[i]}_{\rm GW}}{4\pi}$$

the GW energy-density overdensity

$$\Delta_{\rm GW} = \frac{\Delta\Omega_{\rm GW}}{\bar{\Omega}_{\rm GW}/4\pi} = \sum_{[i]} f^{[i]}_{\rm GW}\Delta^{[i]}_{\rm GW}$$

where

$$f^{[i]}_{\rm GW} \equiv \frac{\bar{\Omega}^{[i]}_{\rm GW}}{\bar{\Omega}_{\rm GW}}$$

is the weight of the relative contribution of the sources which is bounded to be

$$f^{[i]}_{\rm GW} \in [0,1]$$

Here $\Delta^{[i]}_{\rm GW}$ is the GW energy-density contrast for each contribution. Note that, using this definition, it is possible to describe quickly both the ASGW and Cosmological SWG, and compute the angular power spectrum of the GW energy density contrast.

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ANGULAR POWER SPECTRUM of the GW energy density contrast

$$\mathcal{D}_\ell^{\rm GW} = \sum_{i,j;\alpha,\beta} \mathcal{D}_\ell^{[ij]\alpha\beta}$$

where

$$\mathcal{D}_\ell^{[ij]\alpha\beta} = f_{\rm GW}^{[i]} f_{\rm GW}^{[j]} \sum_{\ell=-m}^{\ell=m} \frac{\left\langle \beta_{\ell m}^{[i]\alpha *} \;\; \beta_{\ell m}^{[j]\beta} \right\rangle}{2\ell+1} = f_{\rm GW}^{[i]} f_{\rm GW}^{[j]} \int \frac{k^2 {\rm d}k}{(2\pi)^3} \tilde{\mathcal{S}}_\ell^{[i]\alpha *}(k) \tilde{\mathcal{S}}_\ell^{[j]\beta}(k) P_{\rm m}(k)$$

with

$$\beta^{[i]\alpha}{}_{\ell m} = \int \frac{{\rm d}^3\mathbf{k}}{(2\pi)^3} \, Y_{\ell m}^*(\hat{\mathbf{k}}) \tilde{\mathcal{S}}_\ell^{[i]\alpha}(k) \delta_{\rm m}(\mathbf{k}, \eta_0)$$

and

$$\tilde{\mathcal{S}}^{[i]\alpha}(k) \equiv 4\pi i^\ell \int {\rm d}\bar{\chi} \; \tilde{\mathcal{W}}^{[i]}(\bar{\chi}) \int_0^{\bar{\chi}} {\rm d}\tilde{\chi} \; \left[\mathbb{W}^\alpha \left(\bar{\chi}, \tilde{\chi}, \eta, \tilde{\eta}, \frac{\partial}{\partial \tilde{\chi}}, \frac{\partial}{\partial \tilde{\eta}} \right) \Upsilon^\alpha(\mathbf{k}, \tilde{\eta}) j_\ell(k\tilde{\chi}) \right].$$

Here we have defined a weight function

$$\tilde{\mathcal{W}}^{[i]}(\bar{\chi}) = \frac{f_{\rm o}}{\rho_c} \frac{4\pi}{\bar{\Omega}_{\rm GW}^{[i]}} w(z) \frac{N^{[i]}[z, f_{\rm o}(1+z)]}{(1+z)}$$

- $\Upsilon^\alpha(\mathbf{k}, \tilde{\eta})$ is a generalised transfer function which relates the linear primordial potential with a generic perturbation term (labeled with b);
- $\mathbb{W}^b$ is a generic operator that depends on χ ,η , $\partial/\partial\chi$ and $\partial/\partial\eta$

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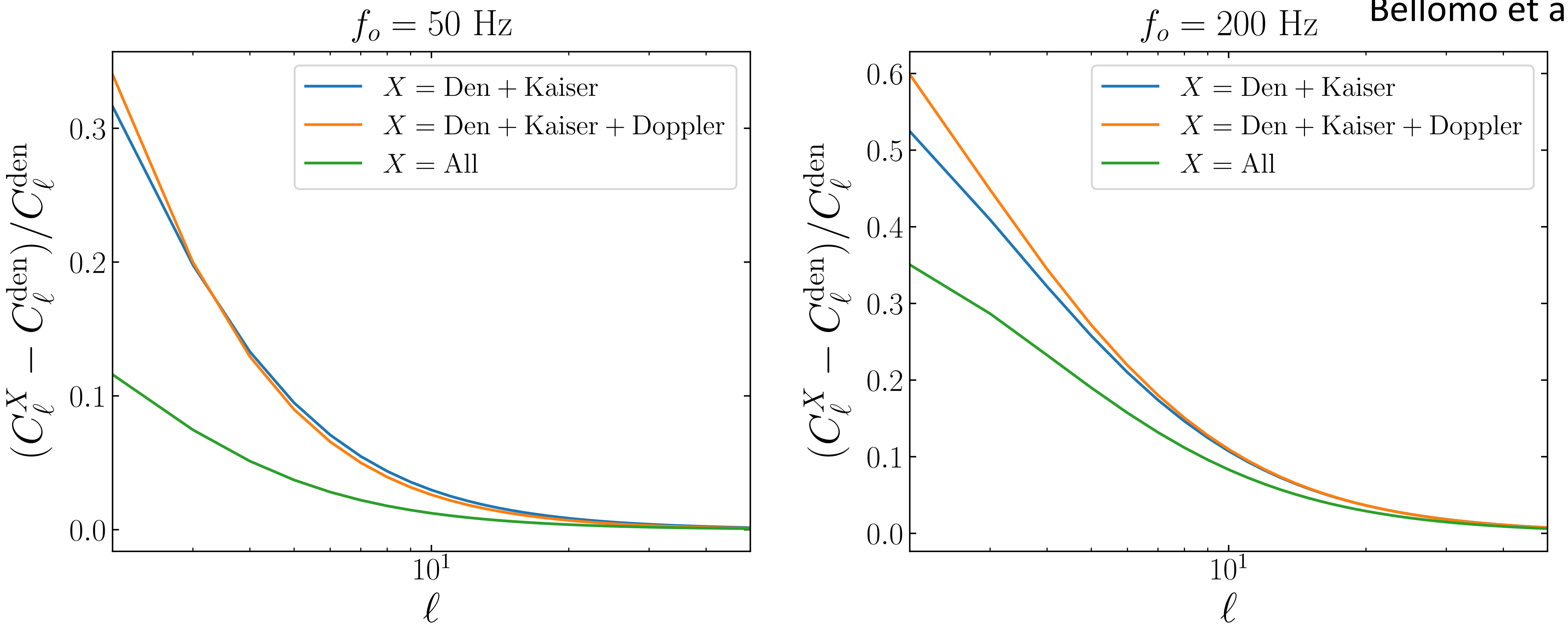
SGWB generated by black holes mergers in the frequency range of LIGO-Virgo.

Bertacca et al. 2019
Bellomo et al. +DB in prep.



- Given that only **unresolved sources contribute to the SGWB, the merger rate of black holes binaries** has to be corrected with the detector efficiency.
- We consider the GWs emission in the f_o = 50 Hz and f_o = 200 Hz channels, assuming that all black holes binaries have members with masses (MBH_1, MBH_2) = (15 $M_\odot$, 15 $M_\odot$) and zero spin.
- On the cosmological side, we compute the halo bias using the fitting formula calibrated on numerical simulations provided in [Tinker et al 2010]. The evolution bias is computed using the halo number density distribution of [Tinker et al 2008], also calibrated on numerical simulations.
- For simplicity, in the following we assume that all the events come from halos with mass $M_h = 10^{12}\ M_\odot$.

Thank You !