Symmetries in Physics Historical-Philosophical Reflections

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• History

- 1. Two notions of symmetry
- 2. Symmetry, equality, equivalence
- 3. Symmetry principles
- 4. Symmetry arguments

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• History

- 1. Two notions of symmetry
- 2. Symmetry, equality, equivalence
- 3. Symmetry principles
- 4. Symmetry arguments
- Philosophical reflections
 - 5. Symmetries and their functions in science
 - 6. Interpretations of physical symmetries

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1. Two notions of symmetry

Claude Perrault (1613-1688)





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C. PERRAULT (1673)

Distinction between two notions of symmetry:

• the *ancient notion*, grounded on proportions relations (Greeks, Latins, Renaissance)

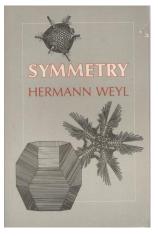
• the *modern notion*, defined as a relation of equality between parts that are opposed

C. Perrault, *Les dix livres d'Architecture de Vitruve, corrigez et traduits nouvellement en françois, avec des notes et des figures,* Paris 1673.

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Hermann Weyl (1885-1955)





(1952)

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H. WEYL (Symmetry, 1952)

"If I am not mistaken the word *symmetry* is used in our everyday language in **two meanings**. In the one sense symmetric means something like well-proportioned, well-balanced, and symmetry denotes that sort of concordance of several parts by which they integrate into a whole. ... The image of the balance provides a natural link to the second sense in which the word symmetry is used in modern times: *bilateral symmetry*, the symmetry of left and right ..."

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"La symétrie des anciens":

• Meaning of the **term** \rightarrow from the Greek $\sigma \dot{\nu} \nu$ (with, together) and $\mu \dot{\epsilon} \tau \rho \nu$ (measure), yielding $\sigma \nu \mu \mu \epsilon \tau \rho \dot{\alpha}$, originally indicating a relation of *commensurability* (in **Euclid**'s *Elements*, Book X: "Those magnitudes are said to be commensurable ($\sigma \dot{\nu} \mu \mu \epsilon \tau \rho \alpha$) which are measured by the same measure, and those incommensurable ($\dot{\alpha} \sigma \dot{\nu} \mu \mu \epsilon \tau \rho \alpha$) which cannot have any common measure").

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- It quickly acquired a further, more general, meaning → that of (a system of) proportion relations, grounded on (integer) numbers, and with the function of *harmonizing the different elements into a unitary whole*. (Ex: Vitruvius' *De architectura*).

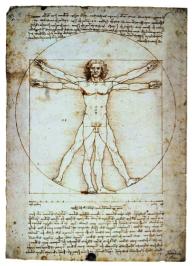
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Leonardo da Vinci (1452-1519)

Vitruvian Man (around 1460):

Leonardo's drawing of the *Vitruvian* proportions of a man's body inscribed in a square and with feet and arms outspread inscribed in a *circle*: a paradigmatic example of an *harmonic ensemble of proportions*.

NOTE: the picture of the human body as a "cosmografia del minor mondo" (cosmography of the microcosm) \rightarrow the proportions of the human body as an analogy for the proportions of the universe.



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• the *modern notion*, defined as a relation of equality between parts that are opposed

The modern notion (Perrault, 1673):

"a relation of equality between parts that are opposed"



Alhambra, Granada (Spain)

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The modern notion (Weyl, 1952):

"The image of the balance provides a natural link to the second sense in which the word symmetry is used in modern times: **bilateral symmetry**"



Symmetry of a figure: history

From Perrault's modern notion to the (late 18th and early 19th century) crystallographic notion:

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Symmetry of a figure: history

From Perrault's modern notion to the (late 18th and early 19th century) crystallographic notion:

- Interpretation of the equality of the parts with respect to the figure as a whole in the sense of their **interchangeability**: equal parts can be exchanged with one another, while preserving the whole.
- Introduction of specific **mathematical operations** (*reflections*, *rotations*, *translations*, and their combinations) in order to describe with precision how the parts are to be exchanged.
- Definition of the symmetry of a figure in terms of its *invariance* when equal parts component parts are *exchanged* according to one of these "symmetry operations" or *symmetry transformations* (the kind of symmetry – *reflection symmetry, rotation symmetry*, etc. – corresponds to the kind of transformation)

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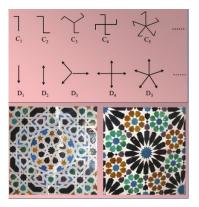
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 \Rightarrow crystallographic notion of symmetry (since it was in the context of early developments in crystallography that symmetry was first so defined and applied)

Reflections+Rotations



Snow crystal

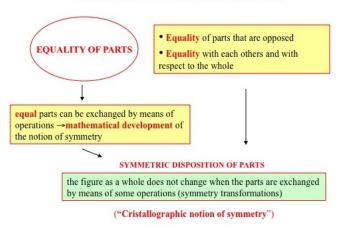


Alhambra's rosettes

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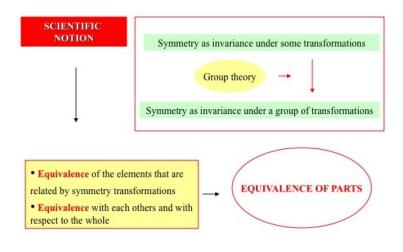
History: "modern notion" \rightarrow "crystallographic notion"

MODERN NOTION OF SYMMETRY



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Generalization to the group-theoretic notion



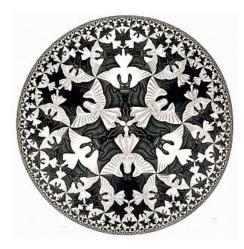
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Group-theoretic notion: first steps

- Introduction of the algebraic concept of a group (Évariste Galois, 1811-1832) and the following 19th century development of group theory (Camille Jordan, Felix Klein, Sophus Lie,..)
- The symmetry transformations of a figure were found to satisfy the conditions for forming a group (symmetry group of the figure).
- ⇒ Generalization of the theory of symmetry (a systematic classification of all possibile symmetric figures) to other kinds of "spaces" and other kinds of transformations.

Ref.: **A. V. Shubnikov - V. A.Koptsik** (1974) *Symmetry in Science and Art* (London, Plenum Press)

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M. C. Escher (1898-1972)

Yin-Yang

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Symmetry in science

 \rightarrow the group-theoretic notion of symmetry is the one that has proven so *successful* in modern science:

 Symmetry as invariance under a specified group of transformations allowed the concept to be applied much more widely, not only to *figures* (of external or internal spaces) but also to *abstract objects* such as **mathematical expressions** → in particular, expressions of physical relevance such as **dynamical** equations.

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- Symmetry as invariance under a specified group of transformations allowed the concept to be applied much more widely, not only to *figures* (of external or internal spaces) but also to *abstract objects* such as **mathematical expressions** → in particular, expressions of physical relevance such as **dynamical** equations.
- Moreover: the technical apparatus of group theory could be transferred and used to great advantage within physical theories.

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Note: symmetry remains linked to beauty (regularity) and unity.

- Distinct (but equivalent) elements are related to each other and to the whole, thus forming a regular unity → in the case of the scientific notion, the way in which the regularity of the whole emerges is dictated by the nature of the specified transformation group;
- \rightarrow thus, a *unity* of *different* and *equal* elements is always associated with symmetry, in its ancient or modern sense:
- → the way in which this *unity* is realized, on the one hand, and how the equal and different elements are chosen, on the other hand, determines the resulting symmetry and in what exactly it consists.

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3. Symmetry principles

Physical symmetries:

- symmetries of laws
 - \rightarrow symmetries of physical equations
- symmetries of objects (physical systems)
 - \rightarrow symmetries of physical states (solutions of physical equations)

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Physical symmetries:

- symmetries of laws
 - \rightarrow symmetries of physical equations
- symmetries of objects (physical systems)
 - → symmetries of physical states (solutions of physical equations)
- \Rightarrow Symmetry principles (Invariance principles):
 - principles postulating the symmetry properties of physical laws
 - \rightarrow symmetry transformations of physical laws (/equations):
 - transformations (applied to the equation's variables) that leave the *form of the laws* unchanged.
 - transformations that map *solutions into solutions*.

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History

First *explicit* study and use of the *invariance properties of equations* in physics → transformational approach to the problem of motion in the *analytic mechanics* of the first half of the nineteenth century (W.R. Hamilton, C. G. Jacobi, ...).

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- First principle explicitly formulated as an invariance principle →
 Einstein's 1905 principle of special relativity (*invariance of physical laws under the transformation group of inertial reference frames*).
 - Einstein's work on special relativity marks "the reversal of a trend: until then, the principles of invariance were derived from the laws of motion [...] It is now natural for us to derive the laws of nature and to test their validity by means of the laws of invariance, rather than to derive the laws of invariance from what we believe to be the laws of nature" [Wigner (1967, 5)]

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- Afterwards: GR, application of symmetry group to quantum mechanics and QFT, ... string theory → the history of the application of symmetry groups (and algebras) to physics coincides, in large part, with the history of 'fundamental' theoretical physics

Varieties of physical symmetries

- Space-time symmetries (continuous, discrete, global, and local)
- Permutation symmetry (symmetry under exchange of "identical" quantum particles)
- Charge conjugation (particle-antiparticle symmetry)
- Internal symmetries (global, local (\rightarrow gauge symmetries))
- Supersymmetry (fermion-boson symmetry → global [SUSY], local [SUGRA])
- Duality symmetries (electric-magnetic duality, S-duality, T-duality, AdS/CFT, ..)

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Symmetry arguments

Symmetry arguments (s.a.) \rightarrow arguments drawing specific consequences on the grounds of the symmetry properties involved.

Elena Castellani Symmetries in Physics

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Symmetry arguments

Symmetry arguments (s.a.) \rightarrow arguments drawing specific consequences on the grounds of the symmetry properties involved.

Old examples (of an *implicit use* of s.a.):

- Anaximander's argument for the immobility of the Earth (as reported by Aristotle): the Earth remains at rest since, being at the centre of the spherical cosmos (and in the same relation to the boundary of the cosmos in every direction), there is *no reason why* it should move in one direction rather than another.
- Archimedes's equilibrium law for the balance: if equal weights are hung at equal distances along the arms of a balance, then it will remain in equilibrium since there is *no reason for it to* rotate one way or the other about the balance point.
- **Buridan's ass**: situated between what are, for him, two completely equivalent bundles of hay, the ass has *no reason to* choose the one located to his left over the one located to his right, and so he is not able to choose and dies of starvation.

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Typical form of s.a.: a situation with a certain symmetry evolves in such a way that, *in the absence of an asymmetric cause*, the initial symmetry is preserved.

→ In other words: a breaking of the initial symmetry *cannot happen without a reason* (suggestion: Leibniz's principle of sufficient reason), i.e. *an asymmetry cannot originate spontaneously*.

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• Historically, the first *explicit use* of a symmetry argument: the so-called **Curie's principle (CP)**.

 \rightarrow Curie's CP (slogan): the symmetries of the causes are to be found in the effects.

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P. Curie, 1894

Pierre Curie (1859-1906)



CURIE. - SYMÉTRIE DANS LES PHÉNOMÈNES PHYSIQUES. 143

SUR LA SYMÉTRIE DANS LES PHÉNOMÈNES PHYSIQUES, SYMÉTRIE D'UN CHAMP ÉLECTRIQUE ET D'UN CHAMP MAGNÉTIQUE;

PAR M. P. CURIE.

I. Je pense qu'il y aurait intérêt à introduire dans l'étude des phénomènes physiques les considérations sur la symétrie familières aux cristallographes.

In onesis interior, pur exemple, part fare animi d'un mouvement recellique on de contains, lipidad, al part fare la sing de tenter de la seconda de la seconda de la seconda de la seconda de tender, il parta tenvere d'ans en cham d'encrique on aparitajen; el pent fare traverdo pro un construit dettaripo on charisfique el pent d'est proved prova o mante d'activitation de la seconda de la pent de traverse por un espont de la seconda de la seconda de pent, el pent de la seconda de la seconda de la seconda de la pent, el pent de la seconda de la

Les physiciens utilisent souvent les conditions données par la symétrie, mais négligent généralement de définir la symétrie dans un phénomène, parce que, assez souvent, les conditions de symétrie sont simples et presque évidentes a priori (').

Dans l'enseignement de la Physique, il vandrait cependant mieux exposer franchement ces questions : dans l'étude de l'électricité, par exemple, énoncer presque au début la symétrie caractéristique du champ électrique et du champ magnétique ; on pourrait ensuite servir de ces notions pour simplifier bien des démonstrations.

Au point de vue des idées générales, la notion de symétrie peut être rapprochée de la notion de *dimension* : ces deux notions fondamentales sont respectivement caractéristiques pour le *milieu*

(1) Les cristallographes qui ont à considérer des cas plus complexes ont établi la théorie guiariza de la synatric. Dans les traités de Gristallographis phylique (qui sont en mine temps de vérilables traités de l'Phylique), les quotisons de symétrie sont exposées avec le plus grand soin. Foir les traités de Mallard, de Libinisa, de Sverti.

J. de Phys., 3ª série, t. III. (Septembre (896.)

Sur la symétrie dans les phénomènes physiques, 1894

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First explicit formulation in **P. Curie**, "Sur la symétrie dans les phénomènes physiques. Symétrie d'un champ électrique et d'un champ magnétique", *Journal de Physique* (1894), 3^e série, vol. 3: 393-417.

Curie's motivating question: Which phenomena are allowed to occur in a given physical medium having specified symmetry properties?

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First explicit formulation in **P. Curie**, "Sur la symétrie dans les phénomènes physiques. Symétrie d'un champ électrique et d'un champ magnétique", *Journal de Physique* **(1894)**, 3^{*e*} série, vol. 3: 393-417.

Curie's motivating question: Which phenomena are allowed to occur in a given physical medium having specified symmetry properties?

Background:

- Curie's studies on the thermal, electric and magnetic properties of crystals (properties related to the structure, and hence the symmetry, of the crystals).
- The methods and results of the theory of symmetry groups used in the crystallography of his time.

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- (a₁) "When certain causes produce certain effects, *the symmetry elements of the causes must be found in their effects.*" [CP slogan].
- (a_2) "When certain effects show a certain dissymmetry, this dissymmetry must be found in the causes which gave rise to them." [logically equivalent to (a_1)].
- (a₃) "In practice, the converses of these two propositions are not true, i.e., *the effects can be more symmetric than their causes.*"
 [asymmetry of CP].

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(*b*) "A phenomenon may exist in a medium having the same characteristic symmetry or the symmetry of a subgroup of its characteristic symmetry.

In other words, certain elements of symmetry can coexist with certain phenomena, but they are not necessary.

What is necessary, is that certain elements of symmetry do not exist. **Dissymmetry is what creates the phenomenon**."

 \Rightarrow relevance of *symmetry breaking* in physics

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Curie's examples:

cases where, by applying CP, conditions are derived for a given phenomenon (the 'effect') to obtain in a medium (the 'cause').

In such terms \rightarrow CP reads:

the symmetry of the medium cannot be higher than the symmetry of the phenomenon.

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Curie's examples:

cases where, by applying CP, conditions are derived for a given phenomenon (the 'effect') to obtain in a medium (the 'cause').

In such terms \rightarrow CP reads:

the symmetry of the medium cannot be higher than the symmetry of the phenomenon.

Example:

 \rightarrow for a **magnetic field** (the effect) to obtain, the *medium* (the cause) must have a symmetry lower or equal to that of a **rotating cylinder**.

(The *characteristic symmetry of a magnetic field* being that of a cylinder rotating about its axis.)

CP (in terms of the symmetries of *media* and *phenomena*):

for the **occurrence of a phenomenon in a medium**, the symmetry of the medium cannot be of higher than the symmetry of the phenomenon.

 \Rightarrow If not, the symmetry of the medium it must be lowered to the symmetry of the phenomenon

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In today's terminology: the *symmetry group* of the medium must be *broken* to the phenomenon's *symmetry group*.

 \Rightarrow In this sense, "symmetry breaking is what creates the phenomenon".

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Conditions for applying CP

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In Curie's terminology:

- the cause and the effect must be well-defined;
- the causal connection between them must hold good;
- the symmetries of both the cause and the effect must also be well-defined (this involves both the physical and the geometrical properties of the physical systems considered).

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Modulo the conditions for its application \rightarrow CP provides a necessary condition for given phenomena to occur:

only those phenomena can happen that are compatible with the symmetry conditions stated by the principle (symmetry constraint).

 \rightarrow CP has thus an important **methodological function**:

• it furnishes a kind of selection rule (given an initial situation with a specified symmetry, only certain phenomena are allowed to happen);

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- it furnishes a kind of selection rule (given an initial situation with a specified symmetry, only certain phenomena are allowed to happen);
- it offers a kind of *falsification* criterion for physical theories (a violation of Curie's principle may indicate that something is wrong in the physical description).

Issue: Status and validity of CP

- What sort of principle? (*A priori*? *A posteriori*? *Methodological* rule? *Pragmatic receipt*? *Truism*? *Hazard*?)
- Reducible to another principle? (*Leibniz's principle of sufficient reason? Causality principle? Determinism & Invariance?*)

 $\textbf{FOCUS} \rightarrow \textbf{the philosophical discussion on CP stimulated by the growing interest in the role of symmetry and symmetry breaking in physics.$

For example: the validity of the principle has been typically questioned in relation to *spontaneous symmetry breaking*.

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Curie's own view

Support for CP \rightarrow in showing its utility in the practice of physics

(not in providing an argument for its validity).

Curie's focus:

• the *heuristic* aspect of CP

 \rightarrow what can be obtained, in the prediction of physical phenomena, by means of the relationships between the symmetries of a medium (cause) and the phenomena (effects) occurring in it.

Utility of CP:

- → to "prevent us from searching for unrealizable phenomena" ("firm but negative conclusions");
- → to provide "precious **directions** for the discovery of new phenomena" ("positive conclusions, but not offering the same certainty of their results").

Issue \rightarrow *status* and *significance* of symmetries and symmetry principles in physics?

Note \rightarrow exploring the role and meaning of physical symmetries is deeply intertwined with basic questions regarding:

- physical reality \rightarrow ontological aspects of s.
- physical knowledge \rightarrow epistemological aspects of s.
- guiding strategies of physical research → methodological aspects of s.

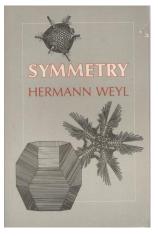
Classic references:

H. Weyl, *Symmetry* (1952) and **E. P. Wigner**, *Symmetries and Reflections* (1967)

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Hermann Weyl (1885-1955)





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Eugene P. Wigner (1902-1995)



E. P. WIGNER Symmetries and Reflections 1967



Invariance in Physical Theory (1949
 Symmetry and Conservation Laws (1964)
 The Role of Invariance Principles in Natural Philosophy (1963)
 Events, Laws of Nature, and Invariance Principles (1964)
 Relativistic Invariance and Quantum Phenomena (1957)

5. Functions of physical symmetries

Main functions of symmetries in physics:

• Classificatory → classification of *physical objects* on the grounds of *symmetry properties*.

Examples:

- \bullet Crystals \rightarrow classific. by means of the sym. properties of their morphology and structure;
- Elementary particles \rightarrow classific. by means of the *irreducible* representations of the theory's fundamental symmetry groups (**Wigner** 1939: seminal paper on the *unitary representations of the inhomogeneous Lorentz group*).

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Note: if a classification includes all the *necessary properties* for characterizing a type of entity (for ex: all the *quantum numbers* for a particle type), symmetries may assume a **definitory** function (defining types of entities on the basis of their transformation properties)

 \Rightarrow used for a *structural approach* to the entities of modern physics (ex: *group-theoretical account of objects*).

Classificatory

- Normative → symmetries as constraints (restrictions on the form of the dynamical equations [ex: Einstein's use of general covariance for his gravitational equations]; selections rules; Curie's principle; ..)
- Unificatory → symmetry groups as a powerful tool for theoretical unification: possibility of unifying different interaction theories (gauge theories) by means of the unification of the corresponding symmetry groups (*Grand Unification Theories* [GUTs], *Theory of Everything*, ..)

Explanatory

Heuristic

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Functions of physical symmetries (3)

 Explanatory → many physical phenomena can be 'explained' as (more or less direct) consequences of (a) symmetry principles or (b) symmetry arguments.

(a): the explanatory role of symmetries arises from their place in the *hierarchy of the structure of physical theory*.

 \rightarrow For ex.: an explanatory role for symmetries with respect to **conservation laws** might be claimed on the basis of Noether's 1918 *connection between symmetries and conservation laws* (along with the more fundamental status of symmetries in the hierarchy).

• Wigner (1967) \rightarrow "This .. the progression from events to laws of nature, and from laws of nature to symmetry or invariance principles, is what I meant by hierarchy of our knowledge of the world around us".

 \Rightarrow Symmetries may be used to explain (i) the *form of the laws*, and (ii) the *occurrence (or non-occurrence) of certain events* (in a manner analogous to the way in which the laws explain why certain events occur and not others).

Functions of physical symmetries (4)

- Heuristic \rightarrow methodological aspects of s. in guiding to
- the formulation of **new physical laws**;
- the prediction of the occurrence of **new phenomena** (for ex., *via* Curie's principle);
- \bullet the prediction of the existence of $\textbf{new particles} \rightarrow$
 - via the classificatory role, on the grounds of vacant places in symmetry classification schemes \rightarrow as in the famous case of the 1962 prediction of the **particle** Ω^- (in the context of the hadronic classification scheme known as the "Eightfold Way").
 - via the unificatory role → the paradigmatic example is the prediction of the W and Z particles (experimentally found in 1983) in the context of the Weinberg-Salam gauge theory proposed in 1967 for the unification of the weak and electromagnetic interactions.
 - via a symmetry-based theoretical "mechanism" → the 1964 prediction of the "Higgs", experimentally found in 2012 at LHC.

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Natural question: Why such a role of physical symmetries?

1. Ontological views \rightarrow symmetries as a *substantial part* of the physical world: the symmetries of theories represent properties **existing in nature**, characterizing the **structure of the physical world**.

Motivations:

- the ontological status of symmetries provides a *reason for their success* in physics (see the successful predictions of new particles);
- the *geometrical interpretation* of spatiotemporal symmetries → as symmetries of spacetime itself (the *geometrical structure* of the physical world).

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2. Epistemological views \rightarrow symmetries are seen as being primarily epistemological (\rightarrow *conditions, modalities* and *limits* in physical inquiry)

Motivations:

(a) Wigner $(1967) \rightarrow$ the spatiotemporal invariance principles play the role of a **prerequisite** for the very possibility of discovering the laws of nature: "*if the correlations between events changed from day to day, and would be different for different points of space, it would be impossible to discover them*".

 \rightarrow Two readings:

- Wigner: conception related to a fact of *ignorance* (if we could directly know all the laws of nature, we would not need to use symmetry principles in our search for them);
- symmetry principles as *transcendental principles* in the Kantian sense
 (→ conditions for the possibility of knowledge)
- (b) Symmetry and equivalence (*irrelevance*, *redundancy*)

(b) Symmetry and equivalence \rightarrow irrelevance, redundancy the presence of equivalent elements (corresponding to a symmetry) suggests the presence of irrelevant or redundant features in the physical description.

Examples:

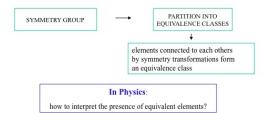
- global spacetime symmetries → the equivalence interpreted in the sense of the irrelevance of "absolute" features. For ex.: translational symmetry (equivalence of space points) → irrelevance of an absolute position to the physical description; ...;
- local (gauge) symmetries → the irrelevant elements correspond to the presence of "surplus structure" in the theory.

 \rightarrow Two readings:

- symmetries as associated with *unavoidable redundancy* in our descriptions of the world (see **Dirac** (1930))
- symmetries as indicating a *limitation* of our epistemic access → there are certain properties of objects, such as their absolute position, that are *not observable* (T. D. Lee, 1981)

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SYMMETRY, EQUIVALENCE, IRRELEVANCE



EQUIVALENCE → IRRELEVANCE

the presence of a symmetry (equivalent elements) corresponds to the presence of irrelevant features in the theory

"[Nature's] fundamental laws control a substratum of which we cannot form a mental picture without introducing irrelevancies. The formulation of these laws requires the use of the mathematics of transformations." (P.A.M. Dirac, 1930)

IRRELEVANCE → NONOBSERVABILITY

irrelevant theoretical features make no observable difference

"The root of all symmetry prinicples lies in the assumption that it is impossible to observe certain basic quantities," (T.D. Lee, 1981).

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4. Connection between symmetry and objectivity \rightarrow on either an *ontological* or an *epistemological* account.

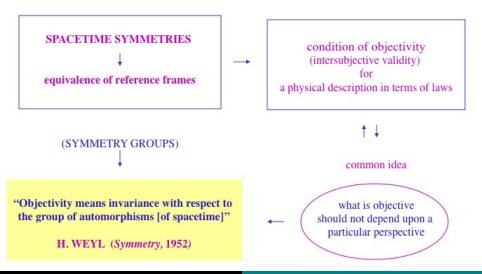
Starting point \rightarrow once again by spacetime symmetries: the laws by means of which we describe the evolution of physical systems have an *objective validity* because they are the same for all observers.

 \rightarrow The old and natural idea that *what is objective should not depend upon the particular perspective* under which it is taken into consideration is thus reformulated in the following *group-theoretical terms*:

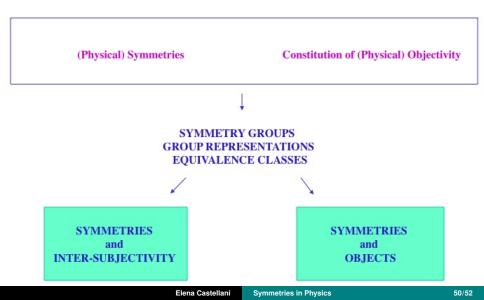
objective is what is invariant with respect to the transformation group of reference frames.

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Symmetry and Intersubjectivity



Symmetry and Objectivity



Summing up

- Symmetries in physics offer various interpretational options.
- The position that one takes will depend in part on one's preferred approach to other issues in philosophy of science → scientific realism, the laws of nature, the relationship between mathematics and physics, the nature of theoretical entities, ...
- It will also depend on whether one views symmetries as ultimately *fundamental* or *derivative* (be that in a methodological sense or in an ontological sense).
- ⇒ How to understand the status and significance of physical symmetries clearly presents a challenge to both physicists and philosophers.

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- K. Brading-E.C.-N. Teh, Symmetriy and Symmetry Breaking, SEP, 2017.

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[–] K. Brading-E.C. (eds.), *Symmetries in Physics: Philosophical Reflections*, CUP, 2003.

Thank you!

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