Universes as Bigdata:

Superstrings, Calabi-Yau Manifolds & Machine-Learning

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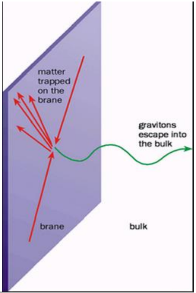
Università di Torino, Nov, 2019

Superstring Theory 9+1 d

Unified theory of quantum gravity

I. 6 Large Dim

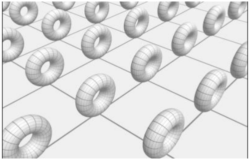
AdS/CFT
Brane World

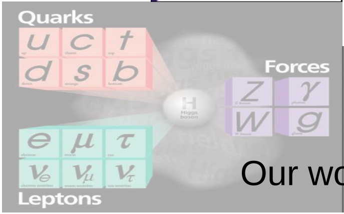


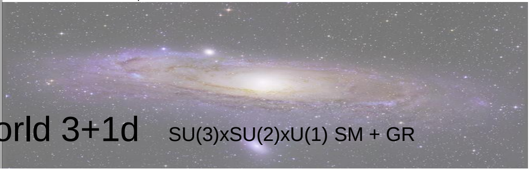
1. *Reduce Dim: 10 = 6+4*
2. *Break SUSY*

II. 6 small dim

Compactification







Our world 3+1d SU(3)xSU(2)xU(1) SM + GR

1984: $10 = 4 + 3 \times 2$

- Heterotic string [Gross-Harvey-Martinec-Rohm]: $E_8 \times E_8$ or $SO(32)$, 1984 - 6
- String Phenomenology [Candelas-Horowitz-Strominger-Witten]: 1986
  - E_8 accommodates SM

$$SU(3) \times SU(2) \times U(1) \subset SU(5) \subset SO(10) \subset E_6 \subset E_8$$

  - 6 extra dimensions is some 6-dimensional manifold X
    1. not just a real 6-manifold but a complex 3-fold X
    2. X is furthermore Kähler ($g_{\alpha\bar{\beta}} = \partial_\alpha \bar{\partial}_{\bar{\beta}} K$) Why SUSY?
    3. X is Ricci flat (vacuum Einstein equations)
    4. Rmk: **there are other classes of solutions** (more later...) but 1,2,3 simplest
- What are such manifolds? **Just so happens that mathematicians were independently thinking of the same problem**

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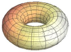
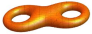
A Classic Problem in Mathematics

- Euler, Gauss, Riemann Σ: $\dim_{\mathbb{R}} = 2, i.e., \dim_{\mathbb{C}} = 1$ (in fact Kähler)
- Trichtomy classification of (compact orientable) surfaces [Riemann surfaces/complex algebraic curves] Σ

		   . . .
$g(\Sigma) = 0$	$g(\Sigma) = 1$	$g(\Sigma) > 1$
$\chi(\Sigma) = 2$	$\chi(\Sigma) = 0$	$\chi(\Sigma) < 0$
Spherical	Ricci-Flat	Hyperbolic
$+$ curvature	0 curvature	$-$ curvature

Euler number $\chi(\Sigma)$, genus $g(\Sigma)$

Classical Results for Riemann Surface Σ

$\chi(\Sigma) = 2 - 2g(\Sigma) =$	$= [c_1(\Sigma)] \cdot [\Sigma] =$	$= \frac{1}{2\pi}\int_\Sigma R =$	$= \sum_{i=0}^{2}(-1)^i h^i(\Sigma)$
Topology	Algebraic Geometry	Differential Geometry	Index Theorem (co-)Homology
Invariants	Characteristic classes	Curvature	Betti Numbers

- First Chern Class $c_1(\Sigma)$
- Rank of (co-)homology group (Betti Number) $h^i(\Sigma)$
- Complexifies (Künneth) $h^i = \sum_{j+k=i} h^{j,k}$, Hodge Number $h^{j,k}$

Calabi-Yau

- $\dim_{\mathbb{C}} > 1$ extremely complicated (high-dim geometry hard: cf. Poincaré Conjecture/Perelman Thm/Thurston-Hamilton Programme)
- Luckily, for our class of **Kähler** complex manifolds:
- CONJECTURE [E. Calabi, 1954, 1957]: M compact Kähler manifold (g, ω) and $([R] = [c_1(M)])_{H^{1,1}(M)}$.

  Then $\exists!(\tilde{g}, \tilde{\omega})$ such that $([\omega] = [\tilde{\omega}])_{H^2(M;\mathbb{R})}$ and $Ricci(\tilde{\omega}) = R$.

Rmk: $c_1(M) = 0 \Leftrightarrow$ Ricci-flat (rmk: Ricci-flat familiar in GR long before strings)

- THEOREM [S-T Yau, 1977-8; Fields 1982] Existence Proof
- Calabi-Yau: Kähler and Ricci-flat (Strominger & Yau were neighbours at IAS)

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Explicit Examples of Calabi-Yau Spaces

An interesting sequence: 1,2, ??? ...

$\dim_{\mathbb{C}} = 1$

Torus $T^2 = S^1 \times S^1$

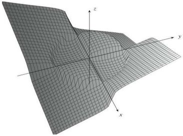
QFT in $10 - 2 = 8d$

$\dim_{\mathbb{C}} = 2$

(1) 4-Torus $T^4 = S^1 \times S^1 \times S^1 \times S^1$

(2) K3 surface

QFT in $10 - 4 = 6d$

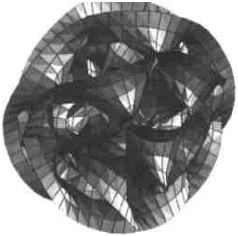


$\dim_{\mathbb{C}} = 3$

Unclassified ???

(Yau's Conjecture: Finite Number)

Desired QFT in $10 - 6 = 4d$



The Inevitability of Algebraic Geometry

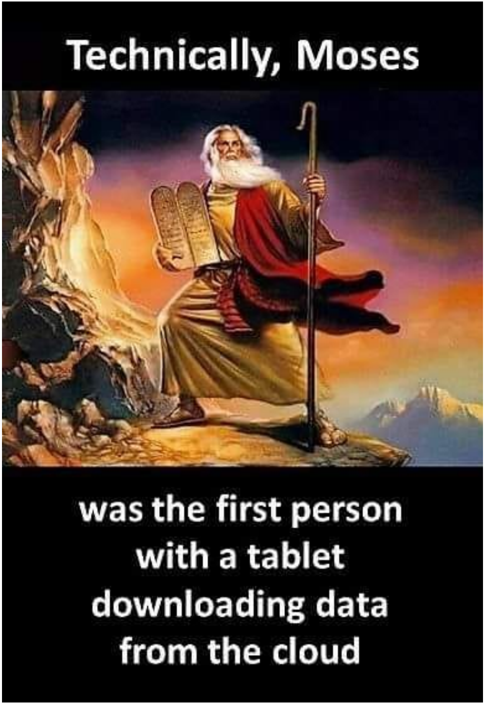
- How to construct CY3? Realize as vanishing locus of polynomials, **Algebraic Geometry** e.g., $\{(p,q) | p^2 + q^2 - 1 = 0\} \subset \mathbb{R}^2$ is a circle (1-real dimension)
- Complexify and Projectivize (Projective algebraic variety)
  - Cubic equation in $\mathbb{CP}^2$: e.g. CY1 $= T^2$ $\{(x,y,z) | x^3 + y^3 + z^3 = 0\} \subset \mathbb{CP}^2$ (elliptic curve); $\dim_{\mathbb{C}} = 2 - 1 = 1$
  - TMH: Homogeneous Eq in $\mathbb{CP}^n$, degree $= n+1$ is Calabi-Yau of $\dim_{\mathbb{C}} = n-1$
- **An Early Physical Challenge to Algebraic Geometry**
  - Particle content in [CHSW]

$$\left.\begin{array}{ll} \text{Generation} & h^1(X,TX) = h^{2,1}_{\bar\partial}(X) \\ \text{Anti-Generation} & h^1(X,TX^*) = h^{1,1}_{\bar\partial}(X) \end{array}\right\} \begin{array}{l} \text{Net-gen: } \chi = 2(h^{1,1} - h^{2,1}) \\ = \text{Euler Number } (X) \end{array}$$

- <u>1986 Question: Are there Calabi-Yau threefolds with $\chi = \pm 6$?</u>

The First Data-sets in Mathematical Physics/Geometry

- [Candelas-A. He-Hübsch-Lutken-Schimmrigk-Berglund] (1986-1990)
  - CICYs (complete intersection CYs) multi-deg polys in products of $\mathbb{CP}^{n_i}$ CICYs
  - Problem: *classify all configuration matrices*; employed the best computers at the time (**CERN supercomputer**); q.v. magnetic tape and dot-matrix printout in Philip's office
  - 7890 matrices, 266 Hodge pairs $(h^{1,1}, h^{2,1})$, 70 Euler $\chi \in [-200, 0]$
- [Candelas-Lynker-Schimmrigk, 1990]
  - Hypersurfaces in Weighted P4
  - 7555 inequivalent 5-vectors w_i, 2780 Hodge pairs, $\chi \in [-960, 960]$
- [Kreuzer-Skarke, mid-1990s - 2000]
  - Hypersurfaces in (Reflexive, Gorenstein Fano) Toric 4-folds
  - 6-month running time on dual Pentium SGI machine
  - at least 473,800,776, with 30,108 distinct Hodge pairs, $\chi \in [-960, 960]$



The age of data science in mathematical physics/string theory not as recent as you might think

of course, experimentals physics had been decades ahead in data-science/machine-learning

After 40 years of research by mathematicians and physicists

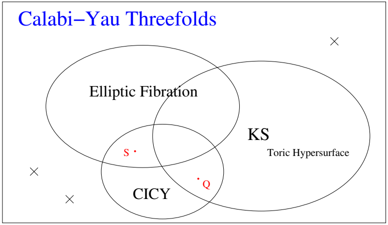
. . . . ..

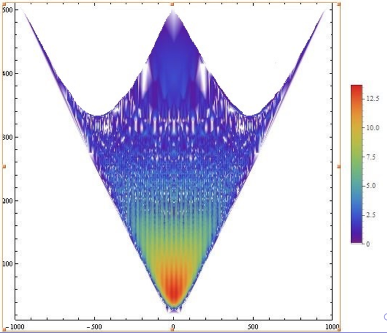
The Compact CY3 Landscape

cf. YHH, *The Calabi-Yau Landscape: from Geometry, to Physics, to Machine-Learning,* 1812.02893, Springer, to appear, 2019/20

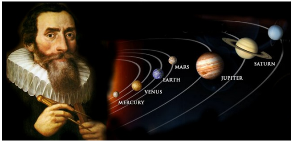
- $\sim 10^{10}$ data-points (and growing, still mined by many international collabs: London/Oxford, Vienna, Northeastern, Jo'burg, Munich, ,...)

  a Georgia O'Keefe Plot for Kreuzer-Skarke





The Geometric Origin of our Universe

- Each CY3 (+ bundles, discrete symmetries) X gives a 4-D universe
  - The geometry (algebraic geometry, topology, differential geometry etc.) of X determines the physical properties of the 4-D world
  - particles and interactions $\sim$ cohomology theory; masses $\sim$ metric; Yukawa $\sim$ Triple intersections/integral of forms over X
- 

  Ubi materia, ibi geometria

  – Johannes Kepler (1571-1630)
- Our Universe:
  - (1) probabilistic/anthropic?
  - (2) Sui generis/selection rule?
  - (3) one of multi-verse ?

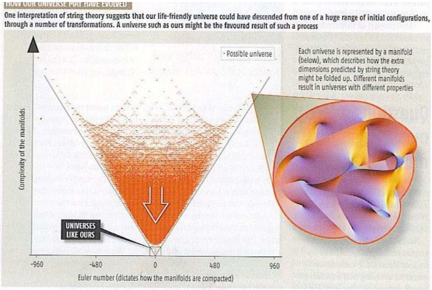
  cf. *Exo-planet/Habitable Zone search*

Triadophilia

Exact (MS)SM Particle Content from String Compactification

- [Braun-YHH-Ovrut-Pantev, Bouchard-Cvetic-Donagi 2005] first exact MSSM
- [Anderson-Gray-YHH-Lukas, 2007-] use alg./comp. algebraic geo & sift
- Anderson-Gray-Lukas-Ovrut-Palti ~ 200 in 10^{10} MSSM Stable Sum of Line Bundles over CICYs (Oxford-Penn-Virginia 2012-)

Constantin-YHH-Lukas '19: 10^{23} exact MSSMs (by extrapolation on above set)?



A Special Corner

[New Scientist, Jan, 5, 2008 feature]

P. Candelas, X. de la Ossa, YHH, and B. Szendroi

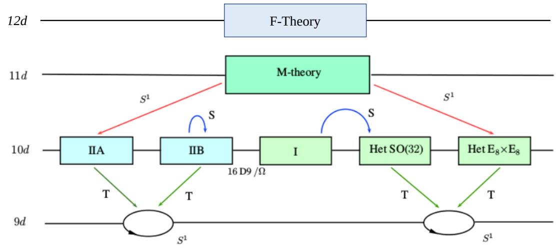
"Triadophilia: A Special Corner of the Landscape" ATMP, 2008

The Landscape Explosion

meanwhile . . . LANDSCAPE grew rapidly with

- D-branes Polchinski 1995
- M-Theory/G_2 Witten, 1995
- F-Theory/4-folds Katz-Morrison-Vafa, 1996
- AdS/CFT Maldacena 1998 Alg Geo of AdS/CFT
- Flux-compactification Kachru-Kallosh-Linde-Trivedi, 2003, . . .

The Vacuum Degeneracy Problem

- 

More solutions related by dualities

Fig. modified from `https://www.physics.uu.se/`

- String theory trades one hard-problem [quantization of gravity] by another [looking for the right compactification] (in many ways a richer and more interesting problem)
- KKLT 2003, Douglas, Denef 2005 - 6 at least 10^{500} possibilities

SUMMARY: Algorithms and Datasets in String Theory

- Growing databases and algorithms (many motivated by string theory): e.g., Singular, Macaulay2, GAP, SAGE, Bertini, grdb, etc; "Periodic table of shapes Project" classify Fanos
- Archetypical Problems
  - Classify configurations (typically integer matrices: polyotope, adjacency, ...)
  - Compute geometrical quantity algorithmically
    - toric $\rightsquigarrow$ combinatorics;
    - quotient singularities $\rightsquigarrow$ rep. finite groups;
    - generically $\rightsquigarrow$ ideals in polynomial rings;
    - Numerical geometry (homotopy continuation);
    - Cohomolgy (spectral sequences, Adjunction, Euler sequences)
- Typical Problem in String Theory/Algebraic Geometry:

$$\begin{array}{ccc} INPUT & & OUTPUT \\ \boxed{\text{integer tensor}} & \longrightarrow & \boxed{\text{integer}} \end{array}$$

Where we stand . . .

The Good: Last 10-15 years: several international groups have bitten the bullet Oxford, London, Vienna, Blacksburg, Boston, Johannesburg, Munich, . . . computed many geometrical/physical quantities and compiled them into various databases **Landscape Data** ($10^{9\sim10}$ entries typically)

The Bad: Generic computation HARD: dual cone algorithm (exponential), triangulation (exponential), Gröbner basis (double-exponential) . . . e.g., how to construct stable bundles over the $\gg 473$ million KS CY3? Sifting through for SM computationally impossible . . .

The ???: **Borrow new techniques from "Big Data" revolution**

A Wild Question

- Typical Problem in String Theory/Algebraic Geometry:

$$\begin{array}{ccc} INPUT & & OUTPUT \\ \boxed{\text{integer tensor}} & \longrightarrow & \boxed{\text{integer}} \end{array}$$

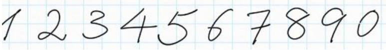
- Q: Can (classes of problems in computational) Algebraic Geometry be "learned" by AI ? , i.e., can we "machine-learn the landscape?"
- [YHH 1706.02714] Deep-Learning the Landscape, PLB 774, 2017: Experimentally, it seems to be the case for many situations
- 2017

  YHH (June), Seong-Krefl (June), Ruehle (June),

  Carifio-Halverson-Krioukov-Nelson (July)

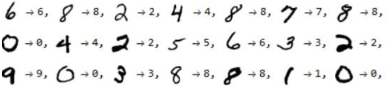
A Prototypical Question

- Hand-writing Recognition, e.g., my 0 to 9 is different from yours:



- How to set up a bijection that takes these to $\{1, 2, \ldots, 9, 0\}$? Find a clever Morse function? Compute persistent homology? Find topological invariants? <u>ALL are inefficient and too sensitive to variation.</u>

- What does your iPhone/tablet do? What does Google do? Machine-Learn
  - Take large sample, take a few hundred thousand (e.g. NIST database)



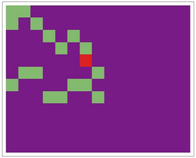
...

ML in 1 min

NN Doesn't Care/Know about Algebraic Geometry

- Hodge Number of a Complete Intersection CY is the association rule, e.g.

$$X = \begin{pmatrix} 1&1&0&0&0&0&0&0 \\ 1&0&1&0&0&0&0&0 \\ 0&0&0&1&0&1&0&0 \\ 0&0&0&0&1&0&1&0 \\ 0&0&0&0&0&0&2&0 \\ 0&1&1&0&0&0&0&1 \\ 1&0&0&0&0&1&1&0 \\ 0&0&0&1&1&0&0&1 \end{pmatrix}, \qquad h^{1,1}(X) = 8 \quad \rightsquigarrow$$



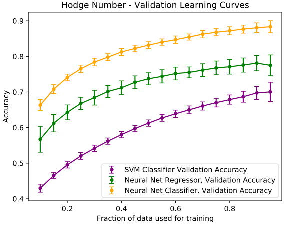
$\longrightarrow 8$

CICY is 12×15 integer matrix with entries $\in [0, 5]$ is simply represented as a 12×15 pixel image of 6 colours Proper Way

- Cross-Validation:
  - Take samples of $X \to h^{1,1}$
  - train a NN, or SVM
  - Validation on *unseen* $X \to h^{1,1}$

Deep-Learning Algebraic Geometry

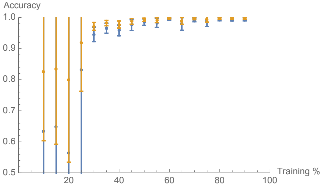
- YHH '17 Bull-YHH-Jejjala-Mishra '18:



Learning Hodge Number $h^{1,1} \in [0, 19]$ so can set up 20-channel NN classifer, regressor, as well as SVM,

bypass exact sequences

- YHH-SJ Lee'19: Distinguishing Elliptic Fibrations in CY3



bypass Oguiso-Kollar-Wilson Theorem/Conjecture

(learning curves for precision and Matthews ϕ)

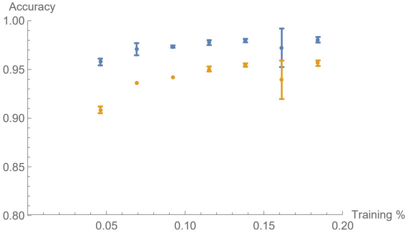
More Success Stories in Algebraic Geometry

- Ruehle '17: genetic algorithm for bundle cohomology
- Brodie-Constantin-Lukas '19: EXACT formulae for line-bundle coho / complex surfaces Interpolation vs Extrapolation $\rightsquigarrow$ Conjecture Formulation
- Ashmore-YHH-Ovrut '19: ML Calabi-Yau metric:
  - No known explicit Ricci-Flat Kähler metric (except T^n) (Yau's '86 proof non-constructive); Donaldson ['01-05] relatively fast method of *numerical* (balanced) such metrics
  - ML improves it to 10-100 times faster with equal/better accuracy (NB. *checking* Ricci-flat is easy)
- RMK: Alg Geo / $\mathbb{C}$ amenable to ML: core computations (Grobner bases, syzygies, long exact sequences, etc) $\sim$ integer (co-)kernel of matrices.

Why stop at string/geometry?

[YHH-MH. Kim 1905.02263] Learning Algebraic Structures

- When is a Latin Square (Sudoku) the Cayley (multiplication) table of a finite group? (rmk: there is a known quadrangle-thm to test this) NN/SVM find to 94.9% ($\phi = 0.90$) at 25-75 cross-validation.
- Can one look at the Cayley table and recognize a finite simple group?



- bypass Sylow and Noether Thm
- rmk: can do it via character-table T, but getting T not trivial
- SVM: space of finite-groups (point-cloud of Cayley tables), ?$\exists$ hypersurface separating simple/non-simple?

Why stop at the mathematics/physics?

[YHH-Jejjala-Nelson] "hep-th" 1807.00735

- Word2Vec: [Mikolov et al., '13] NN which maps words in sentences to a vector space **by context** (much better than word-frequency, quickly adopted by Google); maximize (partition function) over all words with sliding window ($W_{1,2}$ weights of 2 layers, C_α window size, D # windows)

$$Z(W_1, W_2) := \frac{1}{|D|} \sum_{\alpha=1}^{|D|} \log \prod_{c=1}^{C_\alpha} \frac{\exp([\vec{x}_c]^T \cdot W_1 \cdot W_2)}{\sum_{j=1}^{V} \exp([\vec{x}_c]^T \cdot W_1 \cdot W_2)}$$

- We downloaded all $\sim 10^6$ titles of hep-th, hep-ph, gr-qc, math-ph, hep-lat from ArXiv since the beginning (1989) till end of 2017 Word Cloud

  (rmk: Ginzparg has been doing a version of linguistic ML on ArXiv)

  (rmk: abs and full texts in future)

Subfields on ArXiv has own linguistic particulars

- Linear Syntactical Identities

  bosonic + *string-theory* = *open-string*

  holography + *quantum* + *string* + *ads* = *extremal-black-hole*

  string-theory + *calabi-yau* = *m-theory* + *g2*

  space + *black-hole* = *geometry* + *gravity* . . .

- binary classification (Word2Vec + SVM) of formal (hep-th, math-ph, gr-qc) vs phenomenological (hep-ph, hep-lat) : 87.1% accuracy (5-fold classification 65.1% accuracy). ArXiv classifications

- Cf. **Tshitoyan et al.**, "Unsupervised word embeddings capture latent knowledge from materials science literature", **Nature** July, 2019: 3.3. million materials-science abstracts; uncovers structure of periodic table, predicts discoveries of new thermoelectric materials years in advance, and suggests as-yet unknown materials

Summary and Outlook

PHYSICS
- Use AI (Neural Networks, SVMs, Regressor . . .) as
  1. Classifier deep-learn and categorize landscape data
  2. Predictor estimate results beyond computational power

MATHS
- Not solving NP-hard problems, but stochastically bypassing the expensive steps of long sequence-chasing, Gröbner bases, dual cones/combinatorics
- YHH '17: (tried predicting primes with NN); Alessandretti, Baronchelli, YHH, '19: (tried ML on BSD);
- Hierarchy of Difficulty ML struggles with:

  numerical $<$ **algebraic geometry over** $\mathbb{C}$ $<$ **combinatorics/algebra** $<$ **number theory**

GRAZIE

- Boris Zilber [Merton Professor of Logic, Oxford]: "you've managed syntax without semantics. . . "
- Try your favourite problem and see
- 2017:

  First non-human citizen (2017, Saudi Arabia)

  First non-human with UN title (2017)

  First String Data Conference (2017)

Sophia (Hanson Robotics, HK)

An Analogy: $\mathbb{R} + i \sim$ QFT + SUSY

- Do Complex Numbers exist?
- Function Theory and Geometry MUCH BETTER behaved over $\mathbb{C}$ than over $\mathbb{R}$

  Theorem [Fundamental]: $\mathbb{C}$ is algebraic closure of $\mathbb{R}$
- QFT much better behaved with SUSY

  Theorem [Coleman-Mandula/Haag-Lopuszanski-Sohnius]: Nontrivial extension of Poincaré/Gauge Theory-Lie is anticommutator
- Working with QFTs with SUSY is like doing algebra/geometry over $\mathbb{C}$

$$\mathbb{C} \simeq \mathbb{R}[i] \quad \sim \quad (SO(1,3) \rtimes L_{[\ ,\ ]}) \rtimes L_{\{\ ,\ \}} \simeq G_{SUSY}$$

- For us, SUSY $\rightsquigarrow$ Kähler manifolds Back to Compactification

CICYs

$$M = \left[\begin{array}{c|cccc} n_1 & q_1^1 & q_1^2 & \dots & q_1^K \\ n_2 & q_2^1 & q_2^2 & \dots & q_2^K \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ n_m & q_m^1 & q_m^2 & \dots & q_m^K \end{array}\right]_{m \times K}$$

- Complete Intersection Calabi-Yau (CICY) 3-folds
- K eqns of multi-degree $q_j^i \in \mathbb{Z}_{\geq 0}$ embedded in $\mathbb{P}^{n_1} \times \ldots \times \mathbb{P}^{n_m}$
- $c_1(X) = 0 \rightsquigarrow \sum_{j=1}^{K} q_r^j = n_r + 1$
- M^T also CICY

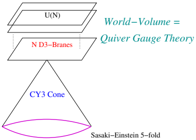
- The Quintic $Q = [4|5]^{1,101}_{-200}$ (or simply $[5]$);
- CICYs Central to string pheno in the 1st decade [Distler, Greene, Ross, et al.]

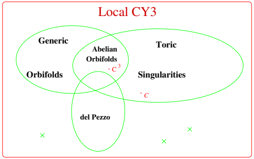
  E_6 GUTS unfavoured; Many exotics: e.g. 6 entire anti-generations

Back to CICYs

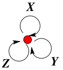
AdS/CFT as a Quiver Rep/Moduli Variety Corr.

a 20-year prog. joint with **A. Hanany**, S. Franco, B. Feng, et al.





D-Brane Gauge Theory
(SCFT encoded as quiver)
$\longleftrightarrow$
Vacuum Space as affine Variety

- $(\mathcal{N} = 4$ SYM$)$ $\Big($  $, W = \mathrm{Tr}([x, y], z)\Big) \longleftrightarrow \mathbb{C}^3 = \mathrm{Cone}(S^5)$ [Maldacena]
- THM [(P) Feng, Franco, Hanany, YHH, Kennaway, Martelli, Mekareeya, Seong, Sparks, Vafa, Vegh, Yamazaki, Zaffaroni . . . (M) R. Böckland, N. Broomhead, A. Craw, A. King, G. Musiker, K. Ueda . . .] (coherent component of) representation variety of a quiver is toric CY3 iff quiver + superpotential graph dual to a bipartite graph on T^2 Back to Landscape

combinatorial data/lattice polytopes $\longleftrightarrow$ gauge thy data as quivers/graphs

A Single Neuron: The Perceptron

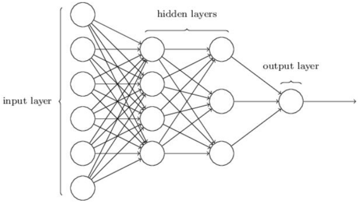
- began in 1957 (!!) in early AI experiments (using CdS photo-cells)
- DEF: Imitates a neuron: activates upon certain inputs, so define
  - Activation Function $f(z_i)$ for input tensor z_i for some multi-index i;
  - consider: $f(w_i z_i + b)$ with w_i weights and b bias/off-set;
  - typically, $f(z)$ is sigmoid, Tanh, etc.
- Given training data: $D = \{(x_i^{(j)}, d^{(j)}\}$ with input x_i and **known output** $d^{(j)}$, minimize

$$SD = \sum_j \left(f(\sum_i w_i x_i^{(j)} + b) - d^{(j)} \right)^2$$

  to find optimal w_i and $b \rightsquigarrow$ "learning", then check against Validation Data
- Essentially (non-linear) regression

The Neural Network: network of neurons $\rightsquigarrow$ the "brain"

- DEF: a connected graph, each node is a perceptron (*Implemented on Mathematica 11.1 + / TensorFlow-Keras on Python*)
  1. adjustable weights/bias;
  2. distinguished nodes: 1 set for input and 1 for output;
  3. iterated training rounds.



Simple case: forward directed only, called multilayer perceptron

- others: e.g., decision trees, support-vector machines (SVM), etc
- Essentially how brain learns complex tasks; **apply to our Landscape Data**

Back to Landscape

Computing Hodge Numbers: Sketch

- Recall Hodge decomposition $H^{p,q}(X) \simeq H^q(X, \wedge^p T^\star X) \rightsquigarrow$

$$H^{1,1}(X) = H^1(X, T_X^\star), \qquad H^{2,1}(X) \simeq H^{1,2} = H^2(X, T_X^\star) \simeq H^1(X, T_X)$$

- Euler Sequence for subvariety $X \subset A$ is short exact:

$$0 \to T_X \to T_M|_X \to N_X \to 0$$

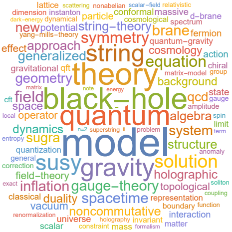
- Induces long exact sequence in cohomology:

$$\begin{array}{cccccccc}
0 & \to & \cancel{H^0(X, T_X)} \overset{0}{\nearrow} & \to & H^0(X, T_A|_X) & \to & H^0(X, N_X) & \to \\
 & \to & \boxed{H^1(X, T_X)} & \overset{d}{\to} & H^1(X, T_A|_X) & \to & H^1(X, N_X) & \to \\
 & \to & H^2(X, T_X) & \to & \ldots & & &
\end{array}$$

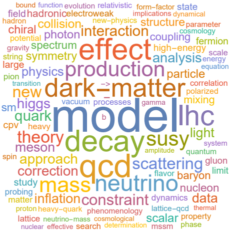
- Need to compute Rk(d), cohomology and $H^i(X, T_A|_X)$ (Cf. Hübsch)

Back to ML

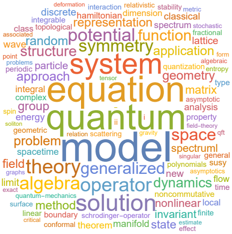
ArXiv Word-Clouds



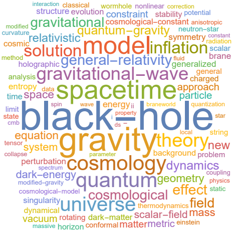
hep-th

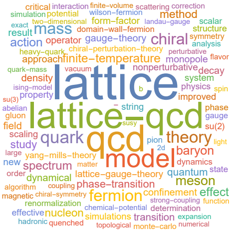


hep-ph



math-ph





Back to Word2Vec

Classifying Titles

Compare, + non-physics sections, non-science (Times), pseudo-science (viXra)

Actual \ Word2Vec + SVM	1	2	3	4	5
1	40.2	6.5	8.7	24.0	20.6
2	7.8	65.8	12.9	9.1	4.4
3	7.5	11.3	72.4	1.5	7.4
4	12.4	4.4	1.0	72.1	10.2
5	10.9	2.2	4.0	7.8	75.1

1 : hep-th
2 : hep-ph
3 : hep-lat
4 : gr-qc
5 : math-ph

Actual \ NN	1	2	3	4	5	6	7	8	9	10
viXra-hep	11.5	47.4	6.8	13.	11.	4.5	0.2	0.3	2.2	3.1
viXra-qgst	13.3	14.5	1.5	54.	8.4	1.8	0.1	1.1	2.8	3.

6: cond-mat, 7: q-fin, 8: stat, 9: q-bio, 10: Times of India Back to Main