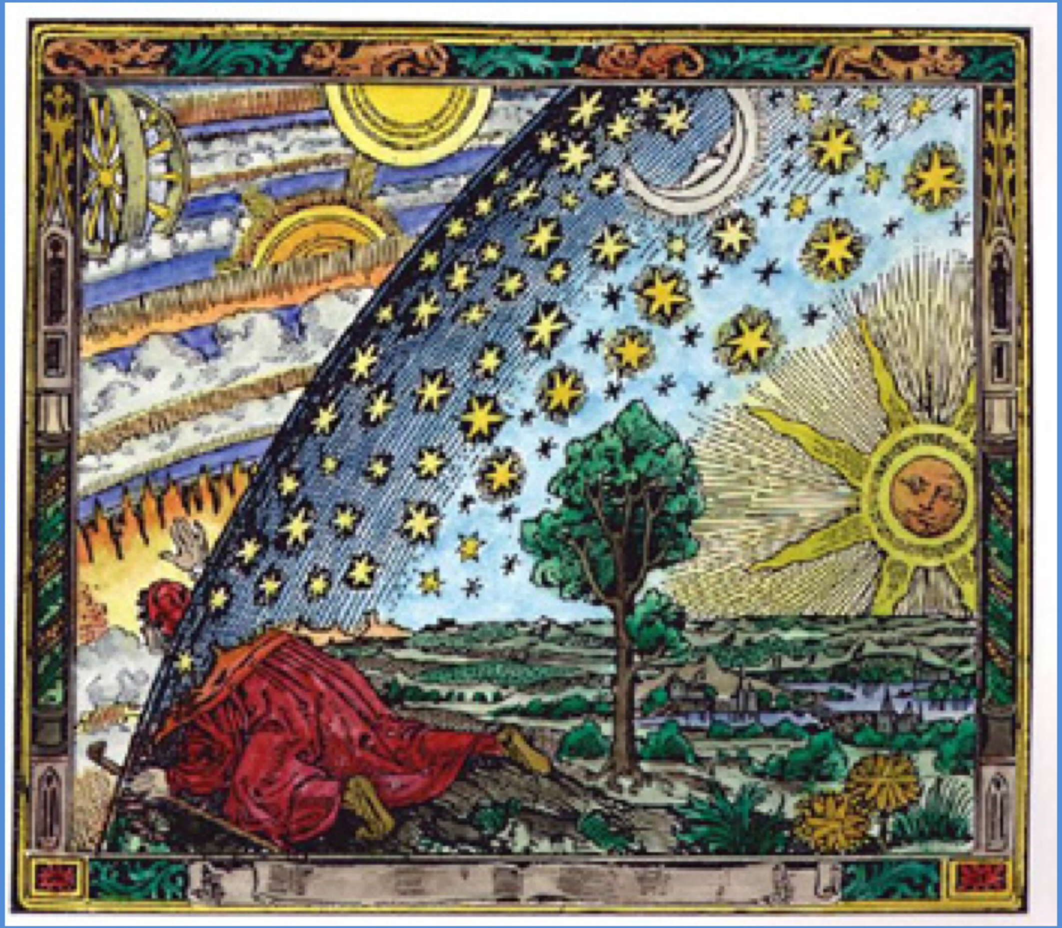
Irreversibility Revisited; my way

Lamberto Rondoni

Politecnico di Torino
INFN
CECAM



Dipartimento di Fisica
Università di Torino

14/06/2019

A medieval monk tells he found the spot where heaven and hearth meet
(C. Flammarion)

Outline

- Physical and psychological irreversibility (layman way)
- Atomism: irreversible world from reversible dynamics
  (my way to most logically consistent approach)

  Boltzmann entropy and the Boltzmann equation

  Loschmidt and Zermelo paradoxes

  Cosmology and initial conditions: predictions?
- Closer to us: novel ergodic notions (my way)

How come future looks to us so very differenty from past?
We see present caused by past & causing future, never the opposite.
Why are we so sure that there is an asymmetry in time?
If a dice rests on a table, we do not know how it got there; knowledge of its state is too coarse: maybe we neglect details of its microscopic phases, necessary to know its past. With same information, we predict that it will stay there, as the forces acting on it sum to 0

We can't travel backward in time, so no interest in acting on past, while we want to have a better future.

Reichenbach, thought that such an asymmetry is due to the fact that we search for ***relations*** among things. A footprint on the sand is never interpreted as a physically possible ***spontaneous event***; we assume that someone walked on the sand, as we deem unreasonable that such a phenomenon may spontaneously occur.

Why do we ask such questions?
Do we imply that the distinction between past and future is just ***psychological?***

Why do we ask such questions?
Do we imply that the distinction between past and future is just ***psychological?***

Einstein's friend M. Besso passed away a few months before Einstein, who then wrote a moving letter to his friend's widow and son:

Michele has preceded me a little in leaving this strange world.
This is not important.
For us, who are convinced physicists, the distinction between past, present and future is only an illusion, however persistent (quoted by Prigogine).

Why do we ask such questions?
Do we imply that the distinction between past and future is just ***psychological?***

Einstein’s friend M. Besso passed away a few months before Einstein, who then wrote a moving letter to his friend’s widow and son:

Michele has preceded me a little in leaving this strange world.
This is not important.
For us, who are convinced physicists, the distinction between past, present and future is only an illusion, however persistent (quoted by Prigogine).

Popper dismissed this view. Is before and after Hiroshima an illusion?

Feynman on «vague» theories

https://www.youtube.com/watch?v=hz2SENYI1rE

Physics describes phenomena that can be measured. Therefore, the measurement tools have an impact on the resulting picture of reality. Measurements provide quantitative data, that can be cast in mahematical form (one way of looking at Nature).

Classical view: measurement tools do not perturb the system of interest, and the result of measurements represents intrinsic properties of the system. This is not justified when the energies involved in the measurement are comparable to those of the phenomenon under consideration.

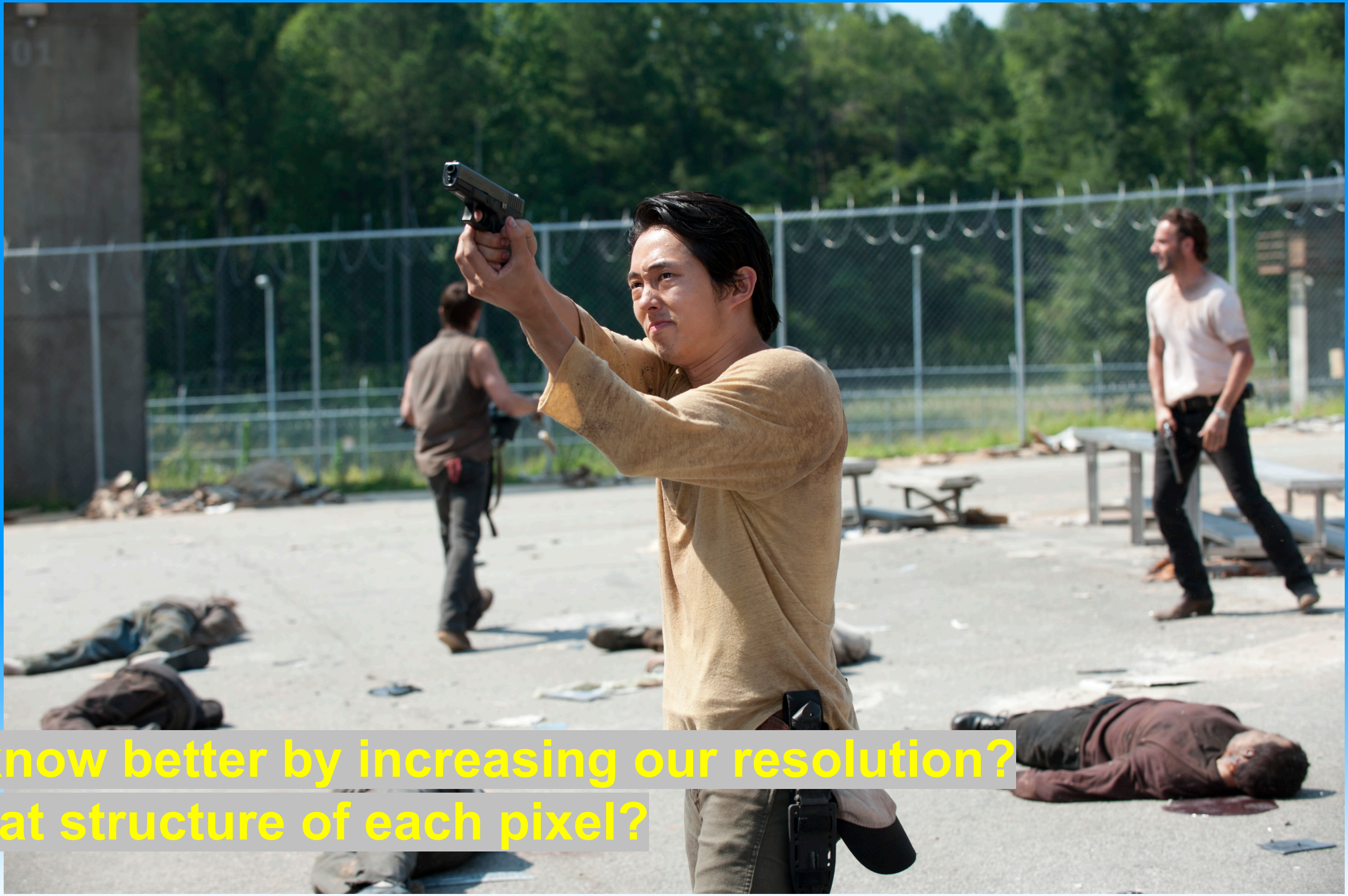
Different measurement tools probe different levels of reality; they amount to different perspectives requiring different descriptions, even for a same given object.

Measurement tools and scales are chosen by ***someone*** in order to characterize one phenomenon and not another.

If we want to observe more closely a cloud, we see fog.

If we watch the cloud we don't see the fog and viceversa.

An objective representation as a photograph cannot answer all questions: is this a massacre, a movie, a hoax?



Would we know better by increasing our resolution?
By looking at structure of each pixel?

TAKE HOME MESSAGE

A physical theory, because based on data on subjectively chosen space and time scales, has limited applicability.
This reflects our limitations in looking at Nature:
if we see a brick, we don't see the cathedral, and viceversa.

No subjectivity: on given levels of observation we must all agree.
Different perspectives on the same phenomenon cannot contradict each other, even when they look contradictory:
the cathedral beauty is not contradicted by the brick ugliness.

Loss of information about irrelevant quantities seems necessary for understanding; would we understand better a movie, by looking at the pixels of each single frame? the cathedral, by investigating the chemical compositions of bricks? Yes? No?

TAKE HOME MESSAGE

A physical theory, because based on data on subjectively chosen space and time scales, has limited applicability.
This reflects our limitations in looking at Nature:
if we see a brick, we don't see the cathedral, and viceversa.

No subjectivity: on given levels of observation we must all agree.
Different perspectives on the same phenomenon cannot contradict each other, even when they look contradictory:
the cathedral beauty is not contradicted by the brick ugliness.

Loss of information about irrelevant quantities seems necessary for understanding; would we understand better a movie, by looking at the pixels of each single frame? the cathedral, by investigating the chemical compositions of bricks? Yes? No?
Who decides what is relevant and what is not?

On our scale, thermodynamics, successfully grasps the idea of irreversibility, formalizing the observation that heat invariably flows from hot to cold bodies, not vice versa: the Second Law states that the entropy of isolated systems does not decrease.

Work, that may result in a higher order, in the formation of patterns, requires energy that is eventually dissipated in the form of heat; hence it is irreversibly lost.

Eddington popularized the link between entropy growth and the ***ARROW of TIME*****: the arrow distinguishing past from future.**

What if we change perspective?

Let us try the *(Hamiltonian) Atomistic Hypothesis:*

If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generations of creatures, what statement would contain the most Information in the fewest words? I believe it is the atomic hypothesis (or the atomic *fact*, or whatever you wish to call it) that all things are made of atoms—little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another. In that one sentence, you will see, there is an enormous amount of information about the world, if just a little imagination and thinking are applied...

Hamiltonian particle systems

$\mathcal{M}$ = space space of "microscopic" phases of system,

$\Gamma = (\mathbf{q}, \mathbf{p}) \in \mathcal{M} \subset I\!R^{6N}$, plus evolution rule:

$$\dot{\mathbf{q}} = \frac{\partial H}{\partial \mathbf{p}} \ , \quad \dot{\mathbf{p}} = -\frac{\partial H}{\partial \mathbf{q}}$$

$S^t : \mathcal{M} \to \mathcal{M}$ evolution operator (Γ at time 0, $S^t\Gamma$ at time t).
Phase functions $\mathcal{O} : \mathcal{M} \mapsto I\!R$.

Rules of the game: system of such atoms represented at time t by one phase Γ; measurement takes a time T, we say it yields average over values $\mathcal{O}(S^t\Gamma)$ along trajectory, for a time T.

Which properties of which objects are described by this approach?

Successful for several macroscopic/astrophysical objects, now used for assemblies of $N \ggg 1$ *atoms*!

Assume inter-particle forces *F* depend only on particles positions, Newton's equations for *N* particles system read:

$$m_i \frac{d^2 x_i}{dt^2} = F_i(x_1, \ldots, x_N), \quad i = 1, \ldots, N$$

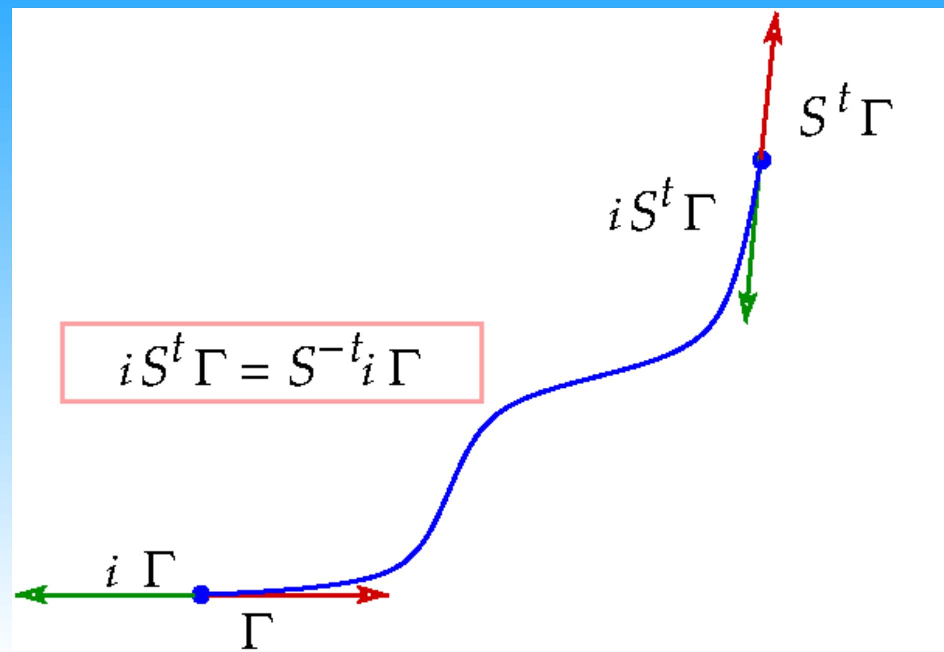
$\Gamma = (x, v)$ initial microscopic condition,

S^t evolution operator,

Time reversal invariance (TRI) implies that there exists time reversal operator i such that:

$iS^t i = S^{-t}$ or $iS^t\Gamma = S^{-t} i\Gamma$

Configurations traced back with opposite velocities (e.g. spin-echo)



Classical and quantum mechanics are TRI !!
(even in presence of a magnetic field!)

How on earth can a world obeying such laws be irreversible? Indeed there are contrasting opinions.

1. **shall we assume neutral kaons decay is responsible for time-symmetry breaking in macroscopic systems in *standard* temperature, pressure etc. conditions? (microscopic irreversibility)**

2. **shall we assume that irreversibility does not depend on internal structure of molecules, hence to restrict to translation degrees of freedom, which are adequately described by TRI mechanical laws (microscopic reversibility)?**

One may take route 1, *postulating* microscopic irreversibility (Prigogine). We prefer route 2 (Lebowitz, Penrose… Boltzmann).

How should that work?

Perhaps same reason for which total mess is so easy to achieve, while it takes (tough) **work** (***energy***) to tidy up our room.

This is the case, however, because we deal with **many** objects

Answer starts stating that a system with low entropy lies in a small phase space volume, and it naturally evolves toward a state of higher entropy, lying in a larger volume.

This leads to a question: what is the origin of the low entropy initial states? If they occupy very small volumes, why should a system start there?

One possible answer: we create states of low entropy (*e.g.* using Sun energy we refrigerate a bottle of water).

How can we? We must be in even lower entropy states… low entropy states on Earth are thus gradually traced back to a distant past Universe exceedingly low entropy state (hugely small phase space volume): **PAST HYPOTHESIS**

The ***Boltzmann entropy*** affords one approach to irreversibility consistent with all that… with some *proviso*

For isolated thermodynamic system in a *microstate* *X* belonging to the class *M(X)* of all microstates with same value of a given *macroscopic* observable (e.g. the density of particles), let the ***entropy*** be defined by:

$$S_B(M) = k_B \log|M|$$

$|M|$ = fraction of microstates *X* in class of macrostate *M(X)*

$k_B = 1.38064 \cdot 10^{-23}\ JK^{-1}$ = *remarkable* Boltzmann constant.

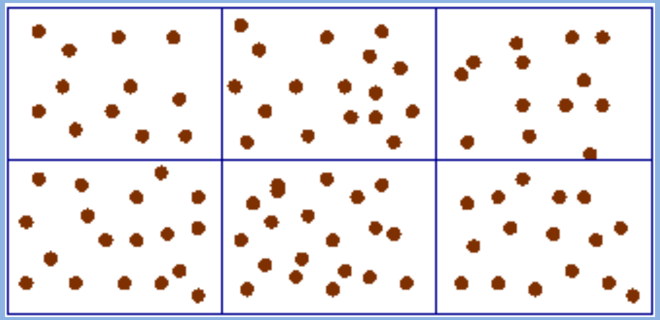
$|M|$ typically grows in time, so that S_B does. Maximum at equilibrium.

Note: *thermodynamic entropy* growth is not average property of an ensemble of macroscopic bodies, but of **EACH** macroscopic object.

Analogously, growth of the *Boltzmann entropy* is not just an average growth.

Boltzmann justified the growth of S_B for dilute gases made of atoms repelling each other at short distances.

One relevant observable is the mass density distribution



$$f\left(x_i, p_j, t\right)\Delta x \Delta p = n_{ij}$$

$$\int f\left(x, p, t\right) dx\, dp = N$$

$$\left(\frac{\partial}{\partial t} + \frac{p}{m} \cdot \nabla_x\right) f\left(x, p, t\right) = \left(\frac{df}{dt}\right)_{coll}$$

Without collision term, the Boltzmann equation looks like the Liouville equation in phase space, but:

1. This is in *6* dimensions, not *6N* dimensions
2. Particles, not points, occupy space; statistics needs $N >> n_{ij} >> 1$

Boltzmann's crucial assumption: ***"molecular chaos"***:
momenta of particles interacting around *x*, are independent:

$$\left(\frac{df}{dt}\right)_{coll} =$$

$$\int dp_1' dp' dp_1 F(x, p, p', p_1, p_1') \cdot \left[f(x, p', t) f(x, p_1', t) - f(x, p, t) f(x, p_1, t) \right]$$

p', p_1' = momenta of two particles before collision
p, p_1 = momenta of two particles after collision

In contrast to Newton's equation for microstates, Boltzmann eq. for a macrostate, is not invariant under time reversal.

$t \to -t$ and $p \to -p$ yields: $\left(\frac{df}{dt}\right)_{coll} \mapsto -\left(\frac{df}{dt}\right)_{coll}$

Introduce Boltzmann's H-functional:

$$\mathcal{H}(t) = k_B \int dx dp\, f(x,p,t) \log f(x,p,t)$$

which equals $-S_B(t) = -k_B \log\left(\left| f(t) \right|\right)$ in the case of a rarefied gas not too far from equilibrium.

Because of the irreversibility of the Boltzmann equation, its solutions obey the

H-theorem

$$\boxed{-\frac{d\mathcal{H}}{dt}(t) = \frac{dS_B}{dt}(t) \geq 0}$$

"=" if and only if **f** = Maxwell–Boltzmann equilibrium distribution.

Theorem reflects irreversible (time-asymmetric) character of Boltzmann equation; based on molecular chaos hypothesis: it holds for dynamics such that distribution is smoothed forward in time!

The “reversibility objection” or “Loschmidt paradox”:

H-theorem cannot be a consequence of reversible microscopic mechanics: if H decreases in time, a reversal of velocities of all atoms yields an initial condition for an increase of H.

The “recurrence objection” or “Zermelo paradox”

Based on Poincaré recurrence theorem: given any tolerance, a mechanical system with bounded phase space takes a finite time T_R to return to its initial condition within that tolerance. If H initially decreases, it must increase again within the time T_R.

Indeed, these objections are well posed, but:

Boltzmann himself noted that T_R for a macroscopic system is, however, extremely long: for instance, $10^{10^{19}}$ years for 1 cm^3 of air in normal conditions. Universe only 10^{10} years.

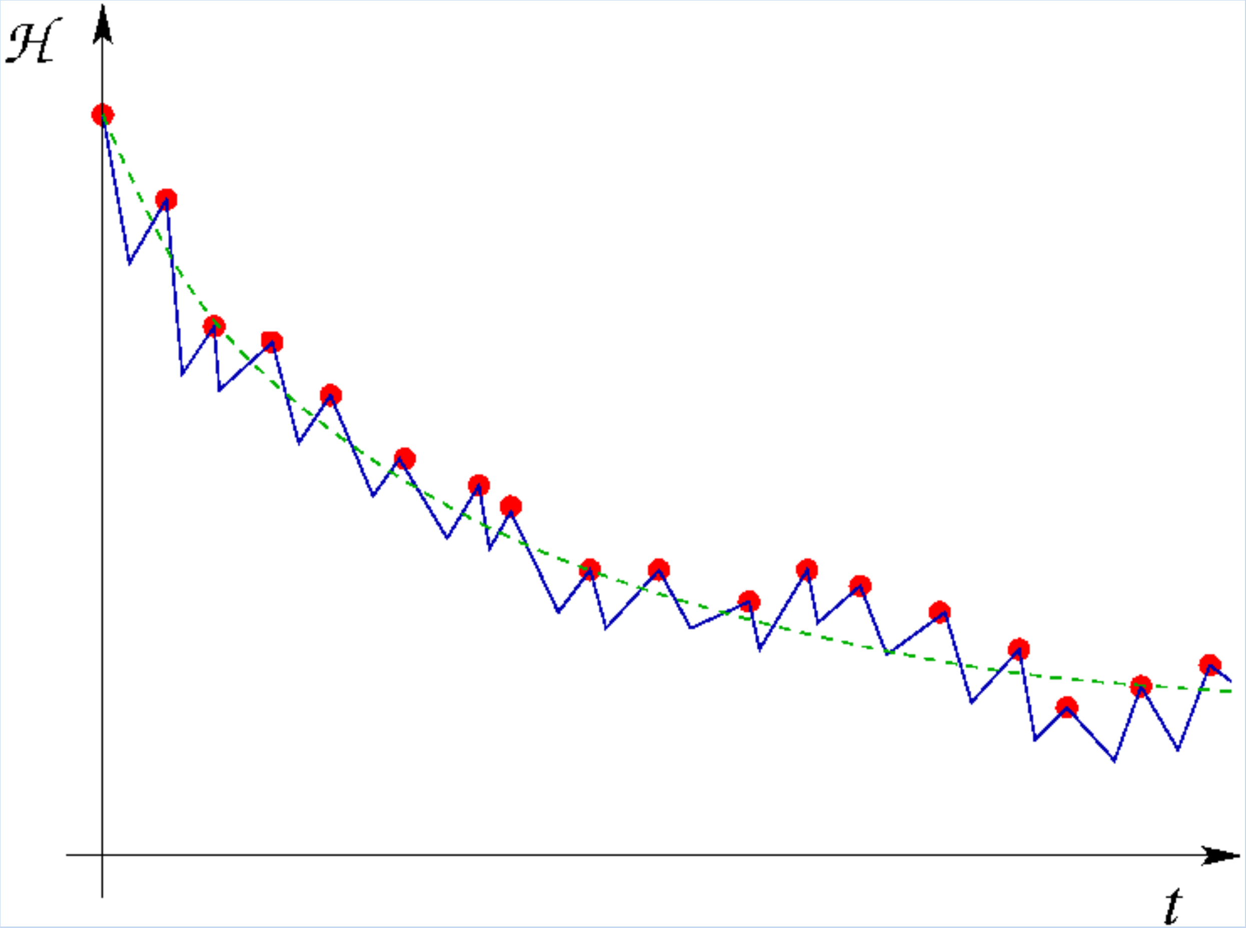
In practice, the answer to these questions is this figure for the *H-functional*: peaks are states in which molecular chaos holds.

As N grows, peaks become denser.

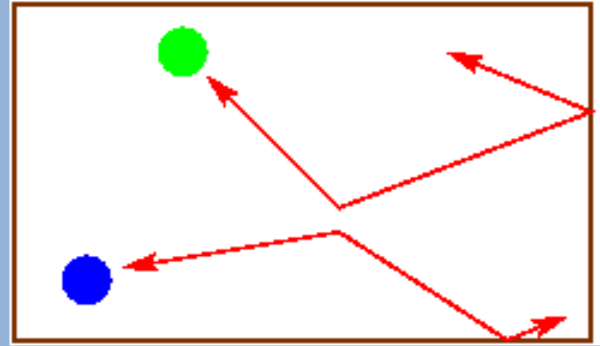
In the large N limit, *H-functional* is monotonic and follows smooth curve.

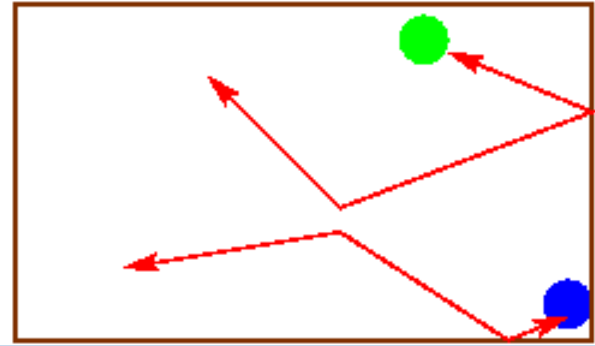
For finite N, there will be recurrence at finite times;

but this suffices to explain the irreversibility of a dilute gas in an isolated box, within our time scales

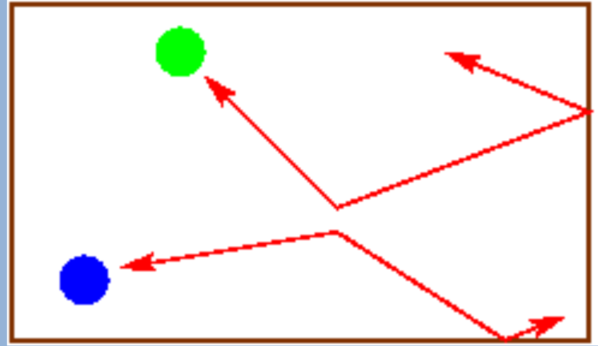


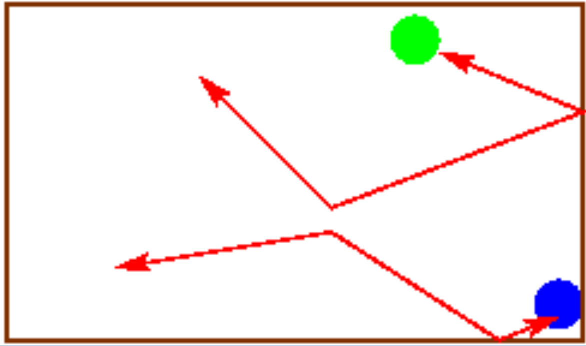
Question: which picture has been taken first?





Question: which picture has been taken first?

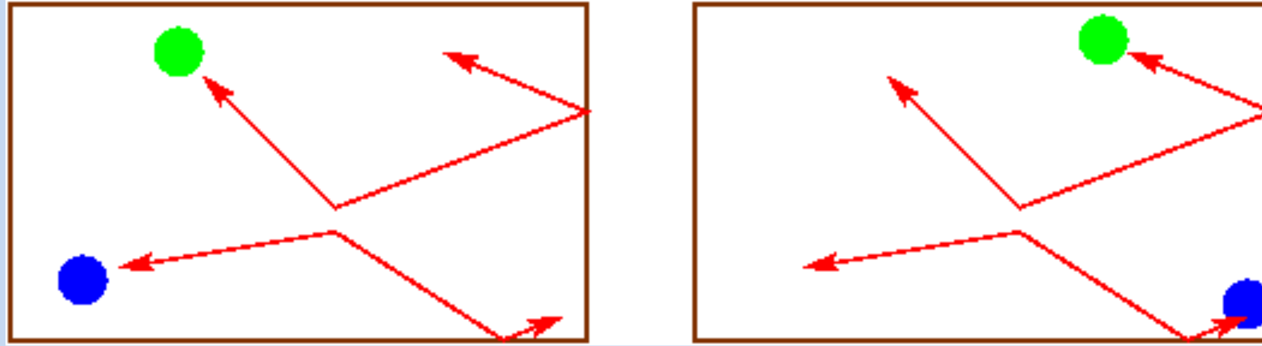




Reversibility of dynamics makes it impossible to answer, whether we look at the pictures or at the movie, because we don’t know whether the movie is being played forward or is rewinding.

The two processes are equally plausible.

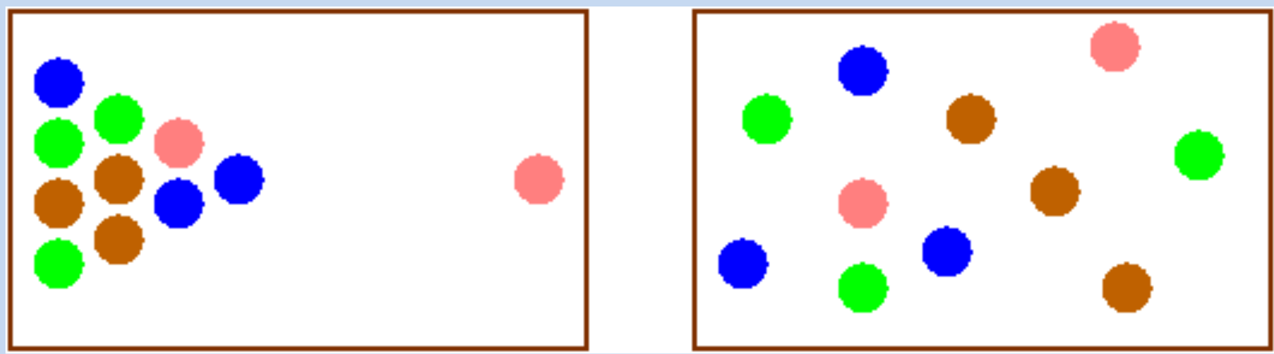
Question: which picture has been taken first?



Reversibility of dynamics makes it impossible to answer, whether we look at the pictures or at the movie, because we don't know whether the movie is being played forward or is rewinding.

The two processes are equally plausible.

But let us consider a larger number of balls.

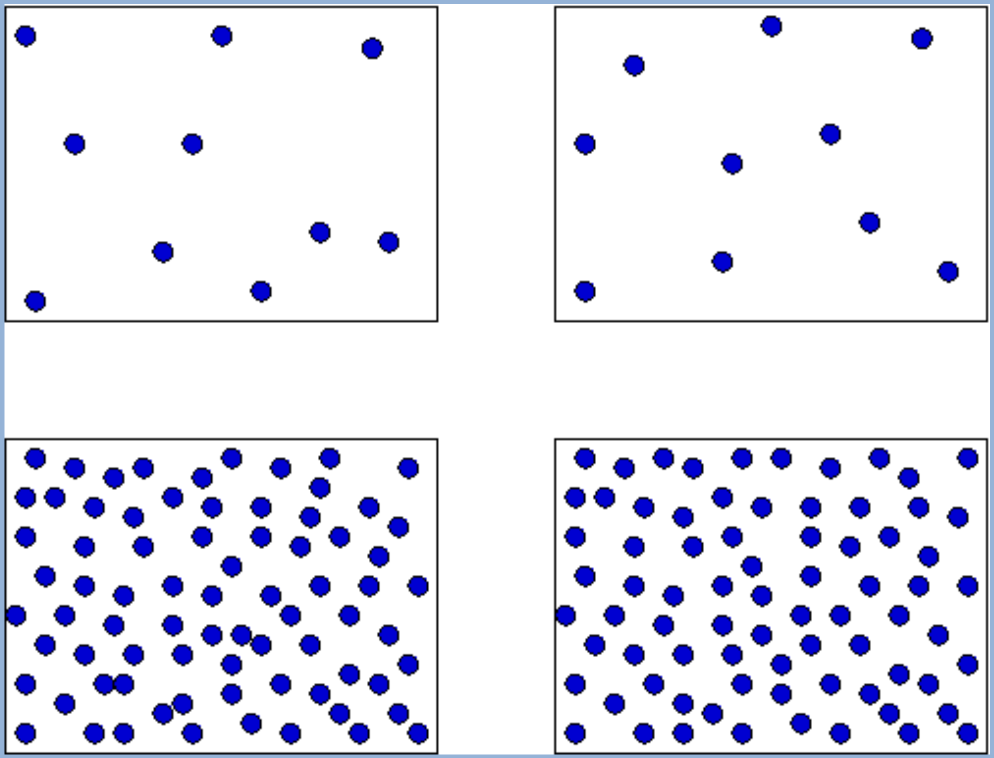


If we know that this is a spontaneous process, we have no doubts:

It suffices to look at the pictures; the movie is not necessary.

Impossible! This, however, is the same, whether we aim at a specific ordered or at a specific disordered (uniform) configuration.

Why is disorder preferred?



The fact is: we do not distinguish disordered configurations and the higher the number of particles the harder to distinguish them.

If we have very many particles, most disordered configurations cannot be distiguished: they are practically the same state!

The disordered (uniform) configurations are much more numerous.

Classical picture: “relaxation to by far most numerous” state, with negligible fluctuations.

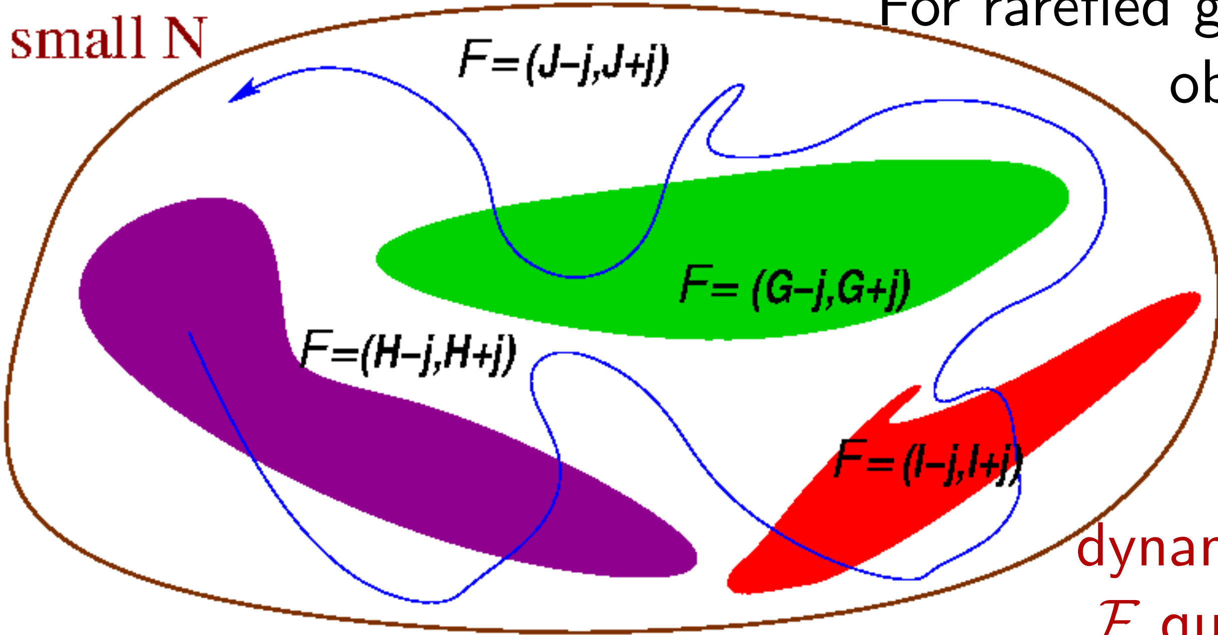
Numerosity! This seems to be the key to irreversibility

In certain models one may count the number of states and find out that, while the number of macrosstates states grows polynomially with the size N of the system, the number of microstates grows **exponentially** with N.

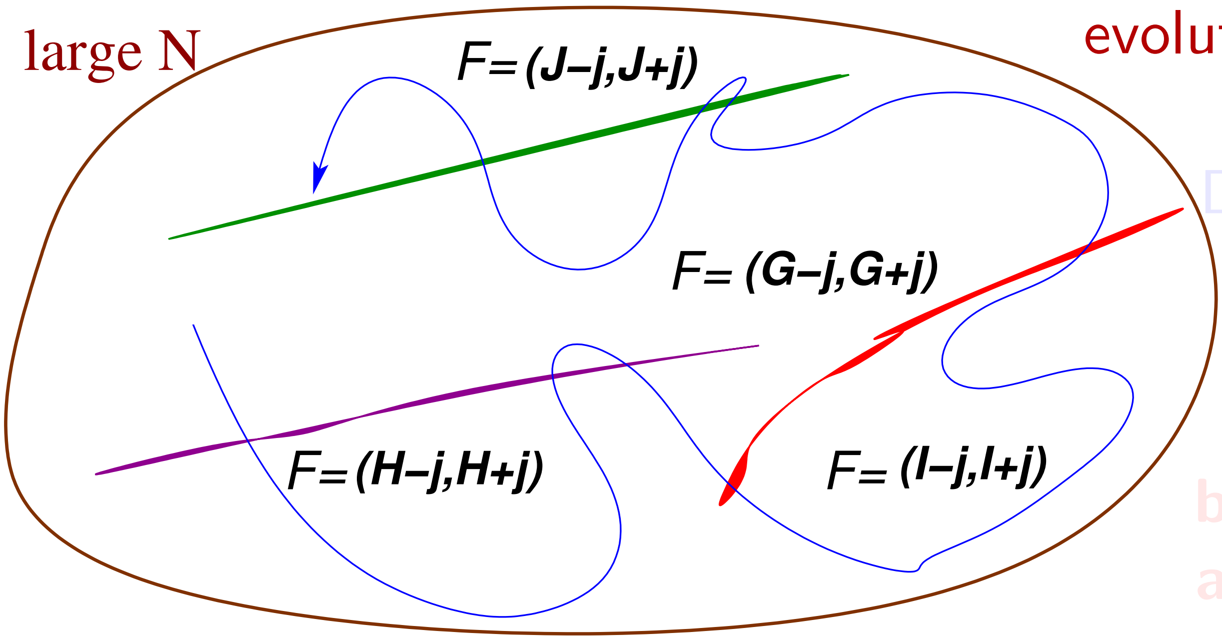
Therefore (little exercise in large deviations theory):

for large N the biggest class contains the ***vast majority*** of states.

small N



large N

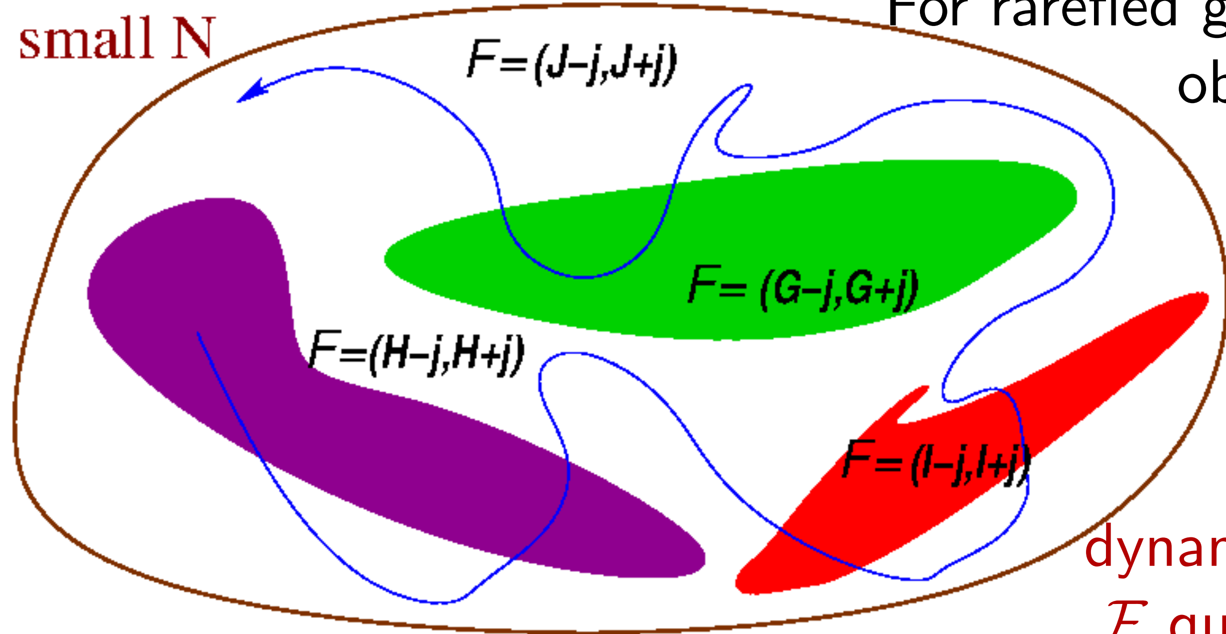


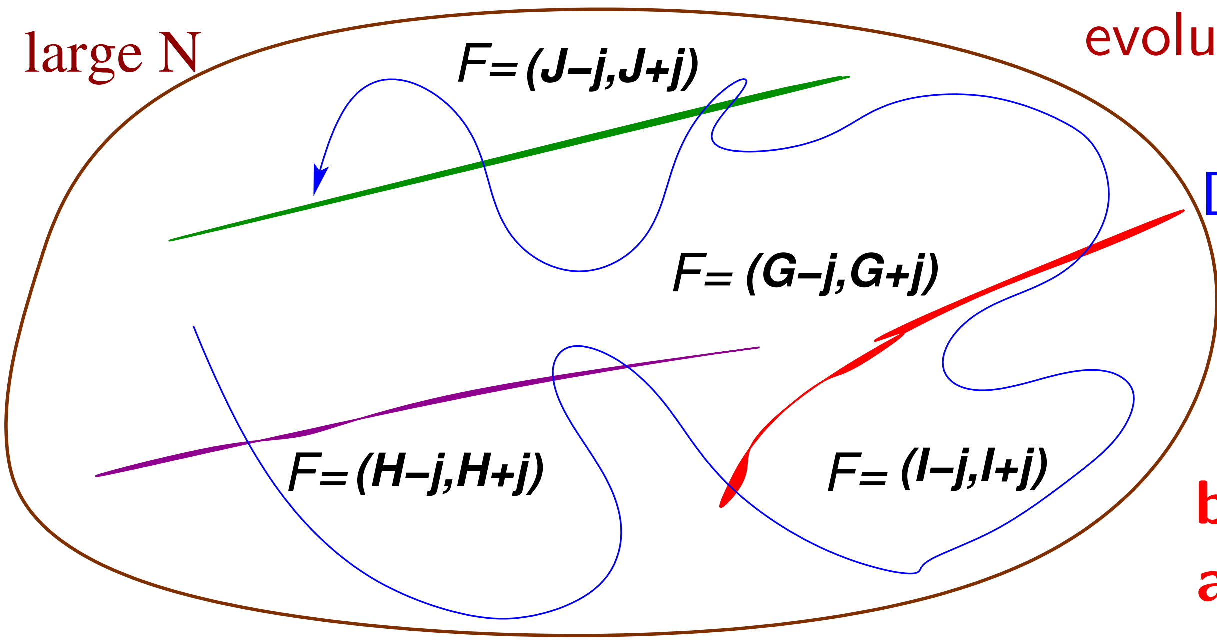
For rarefied gases, $N \ggg 1$, and special observables, give same color to region M with same value of $\mathcal{F}$ within chosen tolerance.

If M much larger than for other macrostates and dynamics prefer no microstates, $\mathcal{F}$ quickly settles about $\mathcal{F}(M)$: evolution almost all time within "equilibrium" state M.

Dynamics **barely relevant**; mere **counting** suffices for **relaxation/irreversibility** **entropy growth**, **but**. . .

both FORWARD **and** BACKWARD in time!!



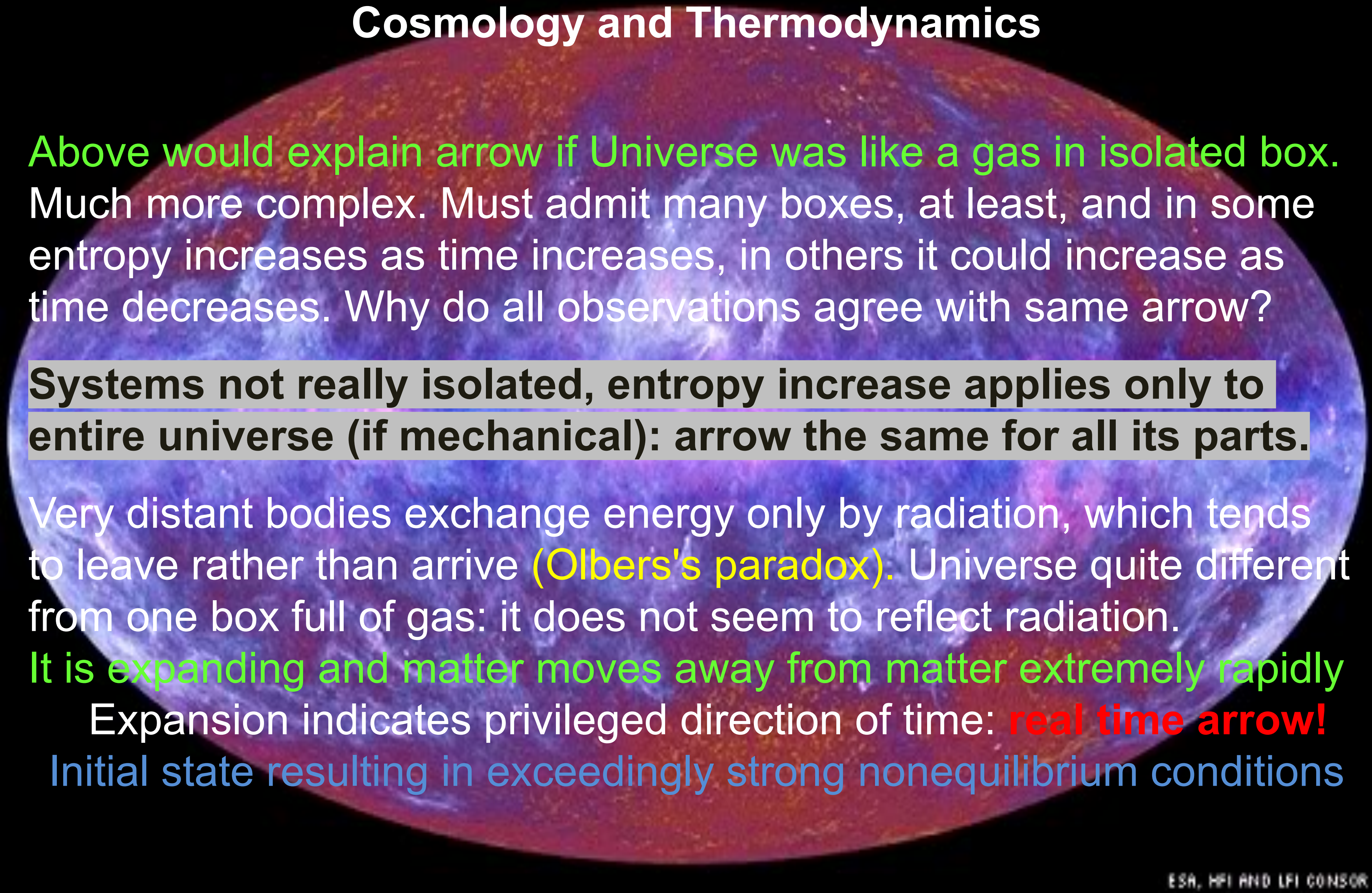


For rarefied gases, $N \ggg 1$, and special observables, give same color to region M with same value of $\mathcal{F}$ within chosen tolerance.

If M much larger than for other macrostates and dynamics prefer no microstates, $\mathcal{F}$ quickly settles about $\mathcal{F}(M)$: evolution almost all time within “equilibrium” state M.

Dynamics **barely relevant**; mere **counting** suffices for **relaxation/irreversibility** **entropy growth**, **but**. . .
both FORWARD
and BACKWARD in time!!

Cosmology and Thermodynamics



Above would explain arrow if Universe was like a gas in isolated box. Much more complex. Must admit many boxes, at least, and in some entropy increases as time increases, in others it could increase as time decreases. Why do all observations agree with same arrow?

Systems not really isolated, entropy increase applies only to entire universe (if mechanical): arrow the same for all its parts.

Very distant bodies exchange energy only by radiation, which tends to leave rather than arrive (Olbers's paradox). Universe quite different from one box full of gas: it does not seem to reflect radiation.

It is expanding and matter moves away from matter extremely rapidly

Expansion indicates privileged direction of time: **real time arrow!**

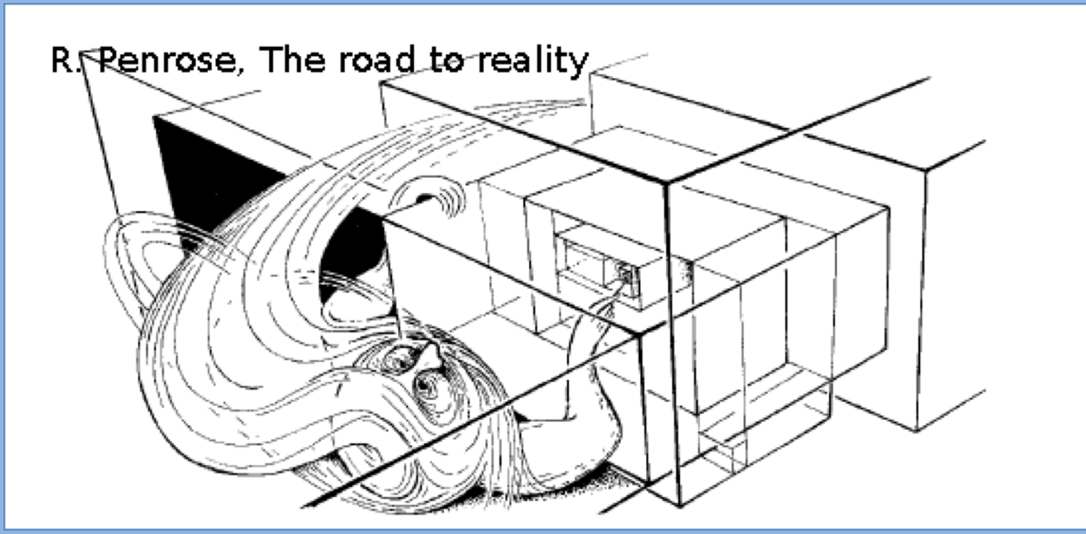
Initial state resulting in exceedingly strong nonequilibrium conditions

If the disorder of our Universe always grows, it must have started from very low value, i.e. in a very small fraction of all microstates. Penrose estimates it to be the ridiculous number: ***one part in*** $10^{10^{123}}$ ***!***

That would allow disorder growth for ultra-astronomic times.

Newton: laws of nature do not suffice, they need initial conditions:
"blind fate cannot make planets move in a single and same fashion in concentric orbits. This uniformity must be due to a choice."

Initial conditions in a very small volume are not ruled out: they simply do not follow any known law of physics. But they are so special; so, why not an i.c. making disorder increase backwards in time?



C. Callender: no need to explain i.c.: **take them as a law themselves!**

The End

The End?

Maybe not…

What does this theory actually predict?

Is irreversibility as so far considered too «vague» a problem?
How can we validate or refute one explanation or another?

Not all systems are rarefied gases (Prigogine),

Apparently “**sufficient** although **minimal**”

In reality, far from minimal: it requires regression to initial conditions of the universe...
Such a **special initial condition:** does it make any (physical) sense?

Can a theory span so many space-time scales?

Shall we really blame the Big Bang for friction between my hands?
Shall we blame on the initial condition of the universe that a bottle of water taken out of a fridge reaches room temperature in 30 minutes?

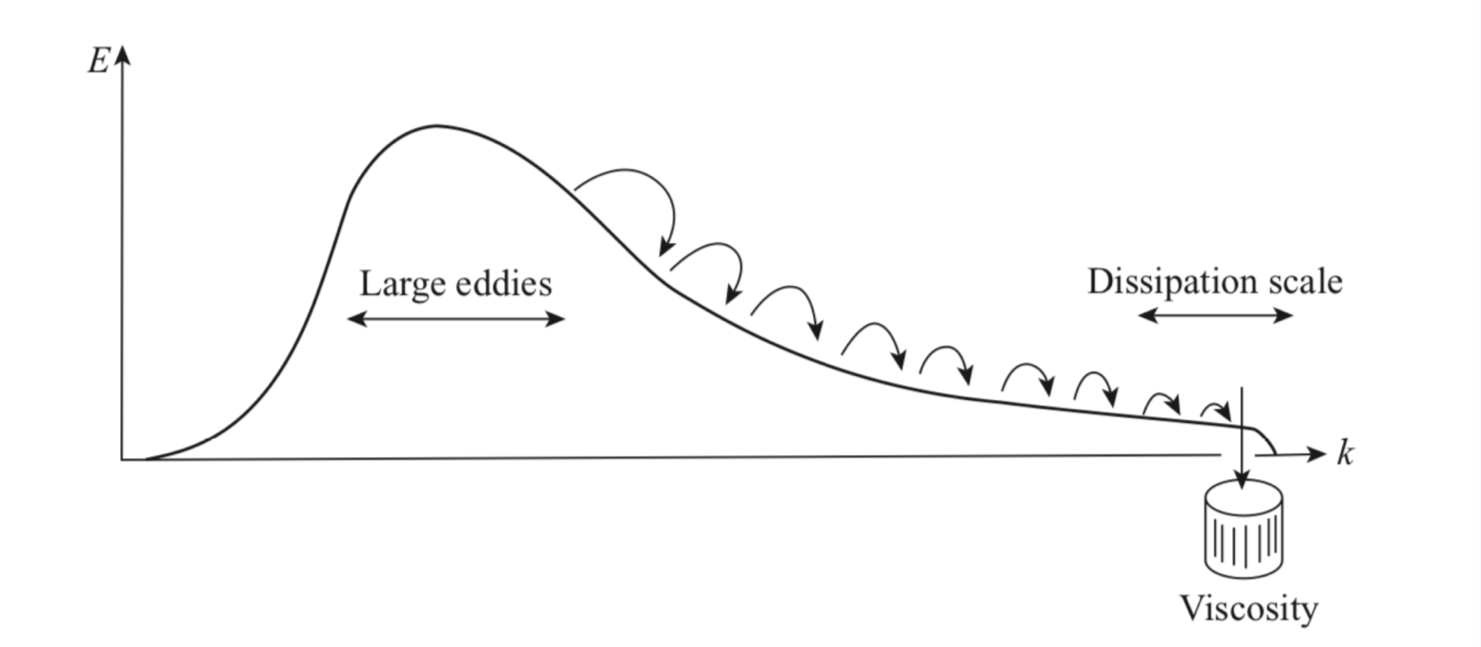
If we do, aren’t we back to square 1, relying on high energy physics?

Let us restrict our field of investigation.

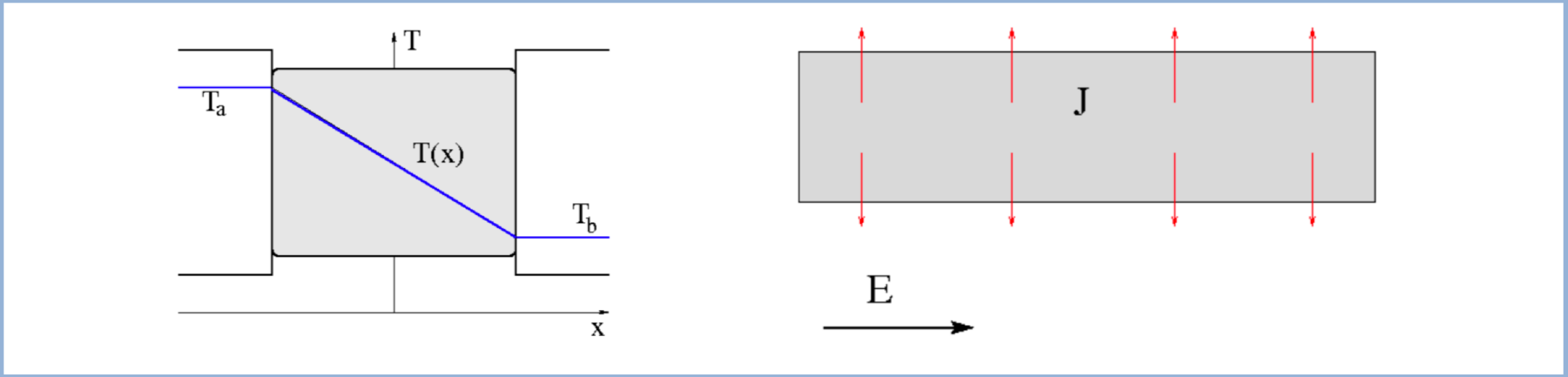
For instance, the theory of turbulence and energy cascades, even though NS equation is intrinsically irreversible

$$\frac{\partial \mathbf{u}}{\partial t} - \nu\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \mathbf{f},$$

$$\nabla\cdot\mathbf{u} = 0,$$



OPEN SYSTEMS



TRI *dissipative* models of nonequilibrium molecular dynamics describe rehology of fluids. Evans-Cohen-Morriss (1993) got FR, symmetry of energy dissipation rate, interpreted in terms of 2nd Law

$$\frac{\mathrm{Prob.}\left(\overline{E}_\tau \approx A\right)}{\mathrm{Prob.}\left(\overline{E}_\tau \approx -A\right)} = \exp\left[A\tau\right]$$

Later we identified **necessary** conditions, rather than sufficient conditions, for relaxation to a stationary state. New ergodic notions are currently being investigated in this approach.

The relavant fact is that necessary conditions unveil unavoidable physics: if some phenomenon takes place,
the necessary mechanisms are at work.

Will that approach provide a less vague theory?

In 1905, Boltzmann thought that one day his atomistic hypothesis could perhaps be disproved and matter be better described by a continuum.
He regretted that one should die before the question could be settled.
He did not know that the in very same years 1905 - 1908 all doubts would have been dispelled. So he stated:

"How immoderate we mortals are! Delight in watching the fluctuations of the contest is our true lot."

Discussion

1. Standard hamiltonian picture based on power of counting
2. convincing and rigorous for rarefied gases, but needs past hypothesis on i.c. of Universe
3. referring irreversibility observed on our daily life to i.c. of Universe appears *vague*: hard to test and spread over huge range of scales
4. restricting scope, for dissipative but TRI models, one finds new ergodic conditions, necessary to obtain irreversible response.

Bibliography

J. L. Lebowitz, Boltzmann's entropy and times' arrow, Physics Today, 46, 9, 32 (1993)

R. Penrose, The Road to Reality: A Complete Guide to the Laws of the Universe (2004)

G. Gallavotti, Nonequilibrium and Irreversibility, Springer (2014)

S. Chibbaro, LR, A Vulpiani, Reductionism, Emergence and Levels of Reality; The Importance of Being Borderline, Springer (2014)

S. Dal cengio, LR, Broken versus Non-Broken Time Reversal Symmetry: Irreversibility and Response. Symmetry 2016, 8, 73

D.J. Evans, S.R. Williams, D.J. Searles, LR, *On typicality in nonequilibrium steady states,* Journal of Statistical Physics ***164*** (4) 842-857 (2016).