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# Factorization and High-Precision Jet Physics

André H. Hoang

University of Vienna

*fdk*  $\Pi$  Doktoratskolleg  
Particles and Interactions



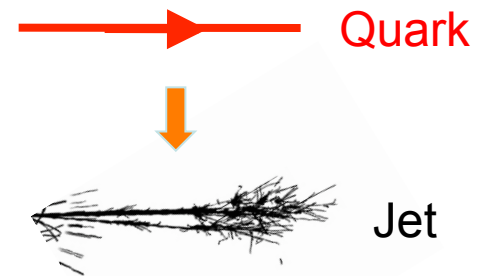
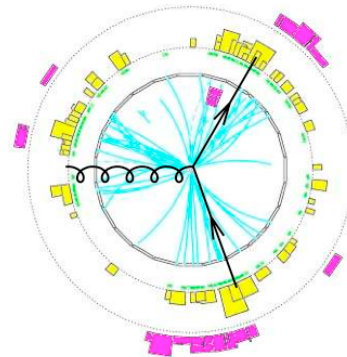
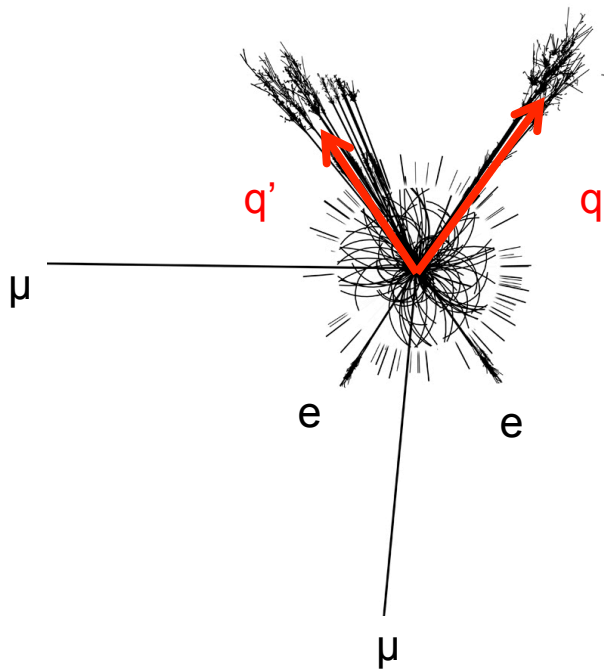
**FWF**  
Der Wissenschaftsfonds.

# Jets

→ Jet: cluster of energetic hadrons leaving tracks and energy deposits in the detectors.



→ Most common object arising in high energy collisions and heavy particle decays



ATLAS  
EXPERIMENT  
<http://atlas.ch>

# Jets

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## Characteristics of jets:

- Represent very rich dynamical objects
- Can behave like unambiguous “particles” or quantum objects, that are defined by the measurement prescription, depending on what question we ask.
- Contain perturbative physics at different energy scales as well as non-perturbative effects. Portion depends on which observables we consider.

**Aims:** Precise (conceptual and) quantitative understanding of jet properties in the framework of QCD.

- Disentangle details of physics of the underlying hard reactions (QCD, Higgs, decays of new physics particles, ...)
- Test our understanding of QCD and our tools to describe it quantitatively

# Main Theoretical Tool

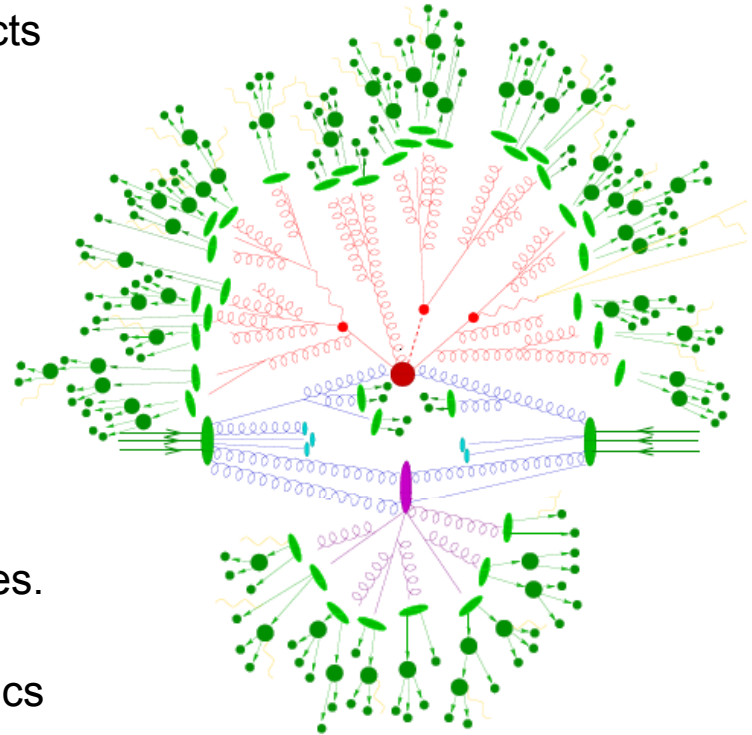
## Monte-Carlo event generators:

→ Separation/factorization of dynamical effects from different energy scales.

- Hard interactions
- Parton evolution to higher multiplicities
- Hadronization
- Secondary interactions

→ The workhorse for all experimental analyses.

- Full description of all aspects of collider physics observables down to all properties of the individual final state hadrons
- Extremely versatile



# Main Theoretical Tool

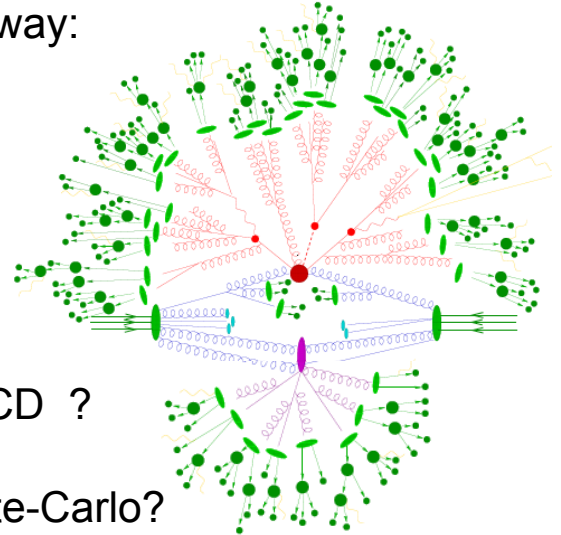
→ Brickwall problems that cannot be addressed in that way:

- Parton showers do not have more than LL precision
- Strong model component (hadronization, UE model,...)
- Limited theoretical precision for many subtle aspects
- What is the theory precision of tuning?
- Monte-Carlo: more **model** OR more first principles QCD ?

What is the meaning of the QCD parameters in the Monte-Carlo?

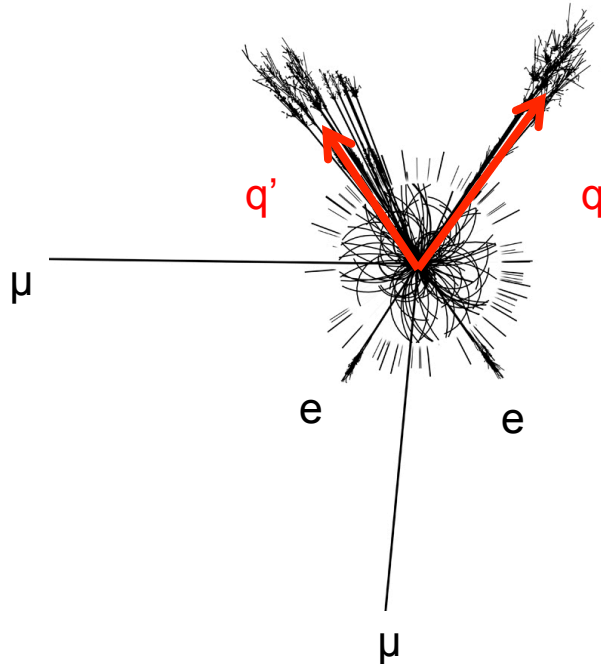
$$\alpha_s, m_{\text{top}}, \dots$$

→ We also have to go different ways, and describe jets with first principles QCD.



# Theory for Jets from Mode Separation

$$E_{\text{jet}} \gg m_{\text{jet}} \gg E_{\text{soft particles}}$$



- Jet as multi-scale quantum system
- Separate quantum modes that live in separated areas of phase space
- Different quark and gluon fields for each separated sector in phase space
- Lagrangian formulation

*Effective Field Theory Approach*

*Soft-Collinear-Effective Theory (SCET)*

- 15 years ago: EFT approach invented to describe jets in B decays, for which EFTs are the only known theory approach
- Until 5 years ago: EFT approach only reproduced many collider physics results already known before from the classic pQCD approach to jets.
- Today: EFT approach addresses problems not addressed before ...

# Outline

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- Soft-Collinear Effective Theory (SCET)
- Anatomy of the SCET method
- Strong coupling from event shapes
- Top quark Monte-Carlo mass parameter

# Basic idea of mode separation

→ First developed for single jet problems in B-physics.

Bauer, Fleming, Pirjol, Stewart  
2000-2001

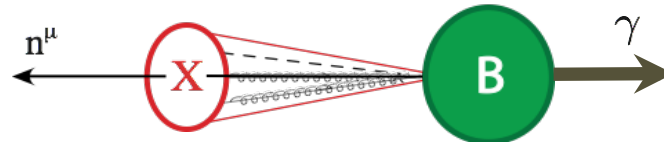
$$B \rightarrow X_s \gamma$$

$$E_\gamma \rightarrow E_\gamma^{\max}$$

jet invariant mass

$$m_X^2 \ll Q^2$$

$$Q = m_b$$



We talk about a jet if:  $m_X^2 \lesssim Q\Lambda_{\text{QCD}}$

Light-cone coordinates:

$$n^\mu = (1, 0, 0, -1)$$

$$\bar{n}^\mu = (1, 0, 0, 1)$$

$$p^\mu = p^+ \frac{\bar{n}^\mu}{2} + p^- \frac{n^\mu}{2} + p_\perp$$

$$= (p^+, p^-, p_\perp)$$

$$p^+ = n \cdot p = p_0 + p_3$$

$$p^- = \bar{n} \cdot p = p_0 - p_3$$



# Basic idea of mode separation

→ First developed for single jet problems in B-physics.

Bauer, Fleming, Pirjol, Stewart  
2000-2001

$$B \rightarrow X_s \gamma$$

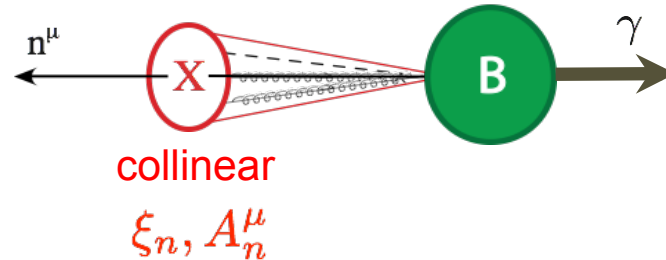
$$E_\gamma \rightarrow E_\gamma^{\max}$$

jet invariant mass

$$m_X^2 \ll Q^2$$

ultrasoft  
 $q_{us}, A_{us}^\mu, h_v$

$$Q = m_b$$



## SCET 1

→ Separation of modes:  $Q^2 \gg m_X^2 \gg \frac{m_X^4}{Q^2} \gtrsim \Lambda_{\text{QCD}}^2$

$$m_X^2 \sim Q \Lambda$$

$$\lambda = \sqrt{\frac{\Lambda}{Q}}$$

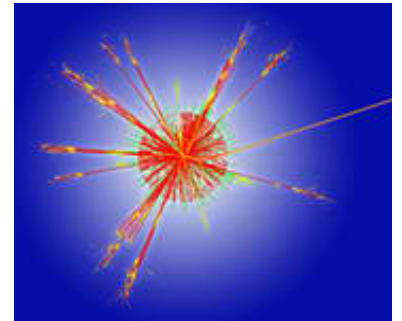
modes	$p^\mu = (+, -, \perp)$	$p^2$	fields
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2 \lambda^2$	$\xi_n, A_n^\mu$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2 \lambda^4$	$q_{us}, A_{us}^\mu$

# Jets from Mode Separation

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## Soft-Collinear Effective Theory:

- Doing jet physics using the concept of mode and scale separation at the Lagrangian and operator level
  - Feynman rules
  - systematic power counting
- Lagrangian level access to jet physics problems.
- IR-log resummation (soft+collinear) through UV-renormalization.
- Approach to access power corrections and subleading twist terms, double counting issues at operator level.
- Leads to results theoretically equivalent to classic pQCD wherever dedicated results have been derived in both approaches.



Differences in the way how results are implemented in applications (subleading).

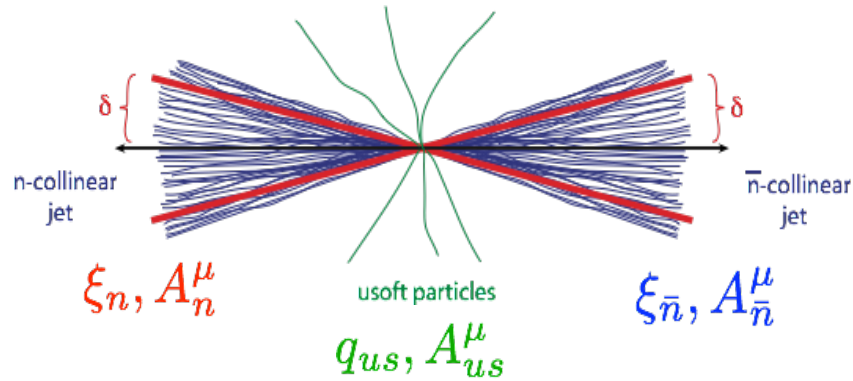
Some problems appear harder / easier in either approach.

# Effective Lagrangian

Consider simple example:

$$e^+ e^- \rightarrow 2 \text{jets}$$

(massless quarks)

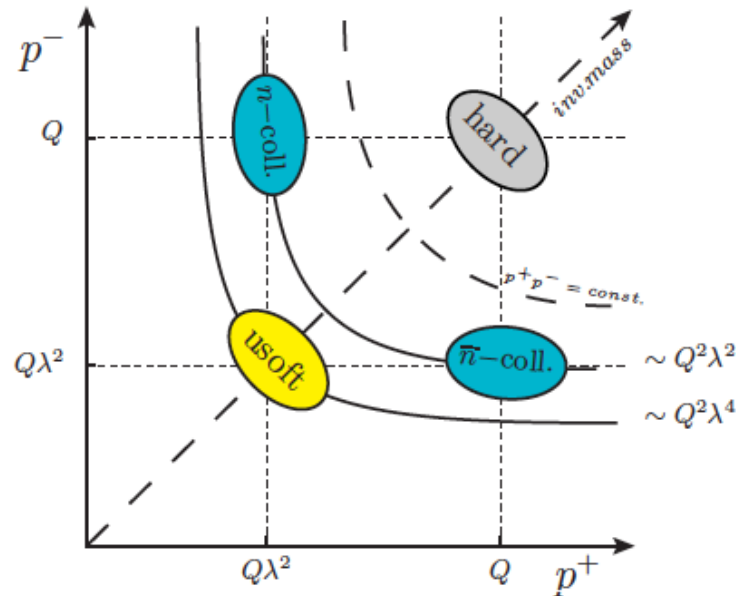


$$Q = E_{\text{cm}}$$

## SCET

$$m_X^2 \sim Q \Lambda$$

$$\lambda = \sqrt{\frac{\Lambda}{Q}}$$



The physical measurement fixed the relevant setup of the quantum modes !

# Effective Lagrangian

## Collinear Lagrangian:

$$\mathcal{L}_{\text{QCD}} = \bar{\psi} i \not{D} \psi$$

$$\psi = \xi_{n_i} + \chi_{\bar{n}_i}$$

$$\chi_{n_i} = \frac{\bar{n}_i n_i}{4} \psi \quad \text{small}$$

$$\xi_{n_i} = \frac{n_i \bar{n}_i}{4} \psi \quad \text{large}$$

$$\mathcal{L}_{c,n} = \bar{\xi}_n \left( i n \cdot D_{us} + i \not{D}_{c,\perp} \frac{1}{i \bar{n} \cdot D_c} i \not{D}_{c,\perp} \right) \frac{\bar{n}}{2} \xi_n$$

$$i D_{us}^\mu = i \partial^\mu + g A_{us}^\mu$$

“Integrate out small component”

“Foldy-Wouthuysen-Tani transformation”

## Effective Lagrangian: (leading in $\lambda$ )

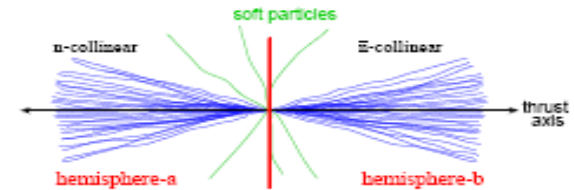
similar to QCD Lagrangian

$$\mathcal{L}_{\text{SCET}} = \sum_{\text{jets } i} \mathcal{L}_{c,n_i}(\xi_{n_i}, A_{n_i}^\mu) + \mathcal{L}_s(q_{us}, A_{us}^\mu)$$

# Effective Lagrangian

Effective jet currents: (dijet production in  $e^+e^-$ )

$$\mathcal{J}^\mu(\omega, \bar{\omega}) = \bar{\chi}_{n,\omega}(0) \Gamma^\mu \chi_{\bar{n},\bar{\omega}}(0)$$

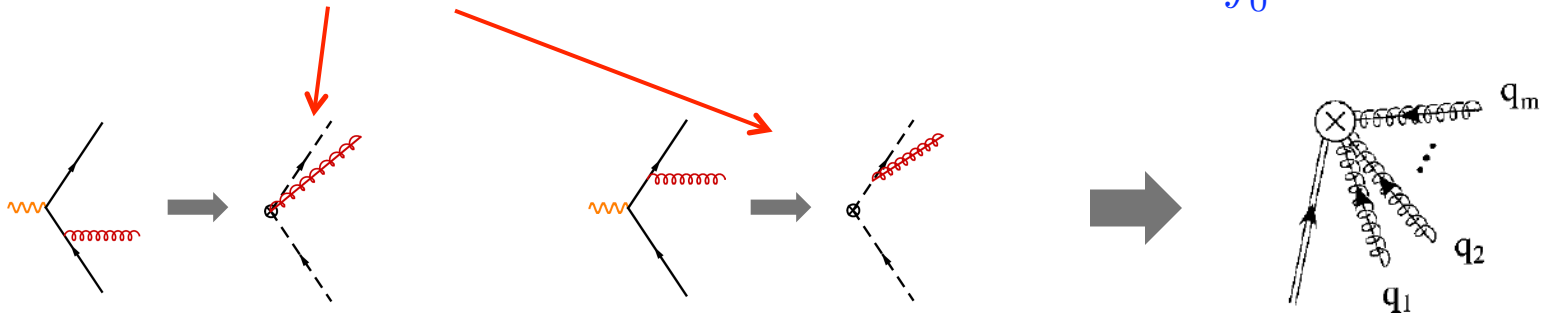


jet field

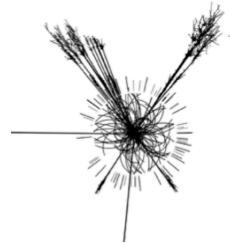
$$\chi_{n,\omega}(0) = (W^\dagger \xi_n)(0)$$

n-collinear Wilson line

$$W_n(0) = P \exp \left( ig \int_0^\infty ds \bar{n} \cdot A_n(s\bar{n}) \right)$$



- ➔ Jet fields are gauge invariant under collinear gauge transformations. Complete gauge invariance in connection with all soft processes.
- ➔ Explains the existence of jets + soft radiation between jets!



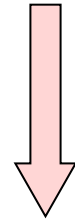
# Factorization at Operator Level

## Factorization:

$$\mathcal{L}_{c,n} = \bar{\xi}_n i n \cdot D_{us} \frac{\bar{n}}{2} \xi_n$$

ultrasoft Wilson line

Soft field redefinition:  
soft-collinear decoupling



$$\xi_n \rightarrow Y_n \xi_n,$$

$$A_n^\mu \rightarrow Y_n A_n^\mu Y_n^\dagger$$

$$Y_n(x) = \bar{P} \exp \left( -ig \int_0^\infty ds n \cdot A_{us}(ns + x) \right)$$

$$\mathcal{L}_{c,n} = \bar{\xi}_n i n \cdot \partial_{us} \frac{\bar{n}}{2} \xi_n$$

$$|X\rangle \longrightarrow |X_n X_{\bar{n}} X_{us}\rangle = |X_n\rangle \otimes |X_{\bar{n}}\rangle \otimes |X_{us}\rangle$$

$$\mathcal{J}^\mu(\omega, \bar{\omega}) \rightarrow \bar{\chi}_{n,\omega}(0) Y_n^\dagger Y_{\bar{n}} \Gamma^\mu \chi_{\bar{n},\bar{\omega}}(0)$$

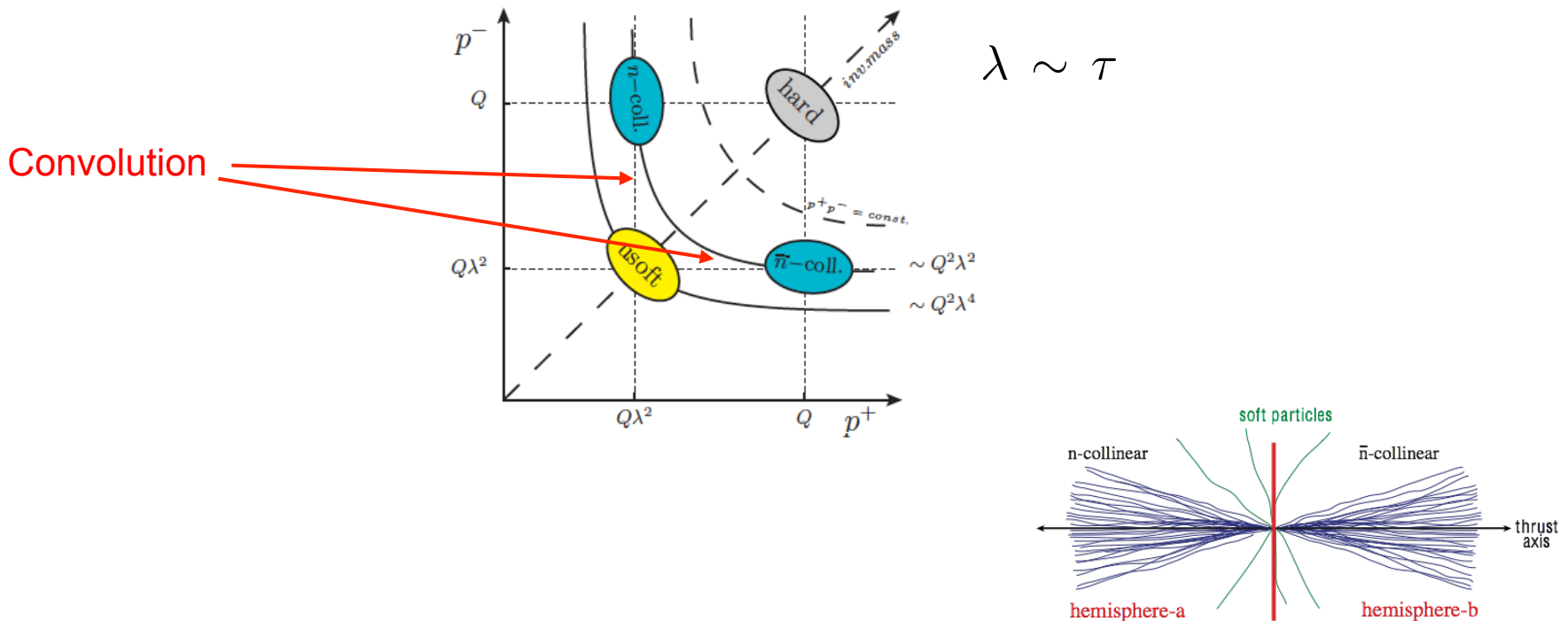
soft-collinear decoupling  
at the operator level

# Anatomy of SCET Predictions

## Singular Cross section (SCET)

Korchemsky, Sterman; Bauer et al.  
Fleming, Mantry, Stewart, AHH  
Schwartz

$$\left(\frac{d\sigma}{d\tau}\right)_{\text{part}}^{\text{sing}} \sim \sigma_0 H(Q, \mu_Q) U_H(Q, \mu_Q, \mu_s) \int d\ell d\ell' U_J(Q\tau - \ell - \ell', \mu_Q, \mu_s) J_T(Q\ell', \mu_j) S_T(\ell - \Delta, \mu_s)$$



# Anatomy of SCET Predictions

## Matrix element terms (fixed-order)

$$\left(\frac{d\sigma}{d\tau}\right)_{\text{part}}^{\text{sing}} \sim \sigma_0 H(Q, \mu_Q) U_H(Q, \mu_Q, \mu_s) \int dldl' U_J(Q\tau - l - l', \mu_Q, \mu_s) J_T(Ql', \mu_j) S_T(l - \Delta, \mu_s)$$

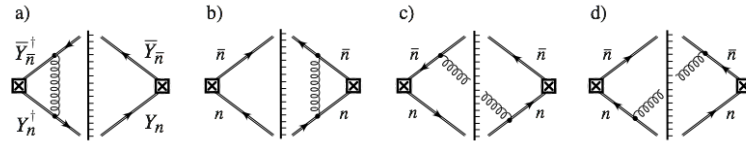
Hard function

Each factor gauge invariant !

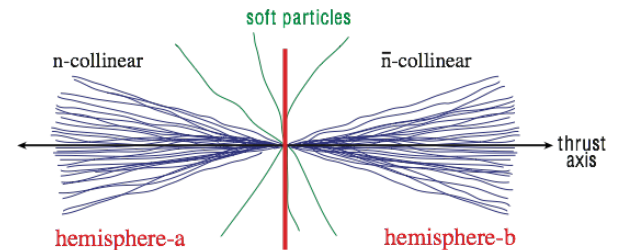
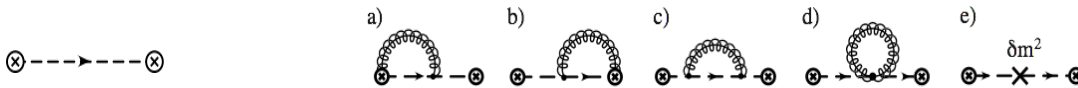
Soft function

Jet function

$$S_{\text{hemi}}(\ell^+, \ell^-, \mu) = \frac{1}{N_c} \sum_{X_s} \delta(\ell^+ - k_s^{+a}) \delta(\ell^- - k_s^{-b}) \langle 0 | \bar{Y}_{\bar{n}} Y_n(0) | X_s \rangle \langle X_s | Y_n^\dagger \bar{Y}_{\bar{n}}^\dagger(0) | 0 \rangle$$



$$J_n(Qr_n^+, \mu) = \frac{-1}{8\pi N_c Q} \text{Disc} \int d^4x e^{ir_n \cdot x} \langle 0 | T \bar{\chi}_{n,Q}(0) \hat{n} \chi_n(x) | 0 \rangle$$





# Anatomy of SCET Predictions

## Summation of large logarithms (RG-summation, SCET 1)

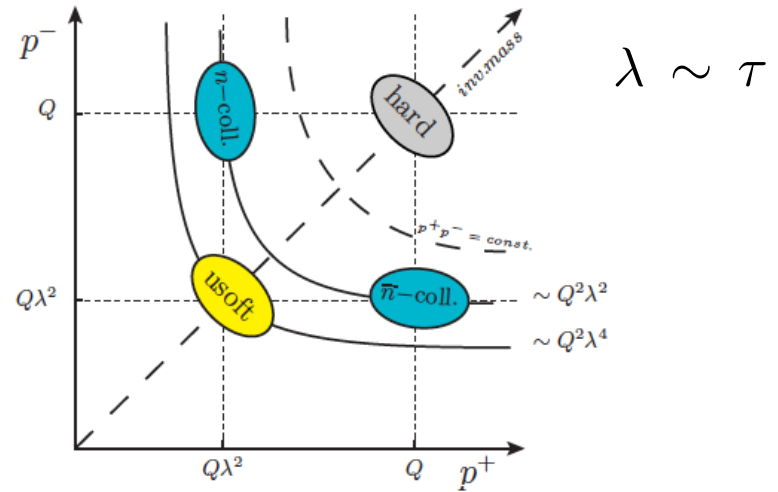
$$\left(\frac{d\sigma}{d\tau}\right)_{\text{part}}^{\text{sing}} \sim \sigma_0 H(Q, \mu_Q) U_H(Q, \mu_Q, \mu_s) \int d\ell d\ell' U_J(Q\tau - \ell - \ell', \mu_Q, \mu_s) J_T(Q\ell', \mu_j) S_T(\ell - \Delta, \mu_s)$$

### 2-jet production current

$$\mu \frac{d}{d\mu} H_Q(Q, \mu) = \gamma_{H_Q}(Q, \mu) H_Q(Q, \mu)$$

$$\gamma_{H_Q}(Q, \mu) = \Gamma_{H_Q}[\alpha_s] \ln\left(\frac{\mu^2}{Q^2}\right) + \gamma_{H_Q}[\alpha_s]$$

NNLL summations possible!



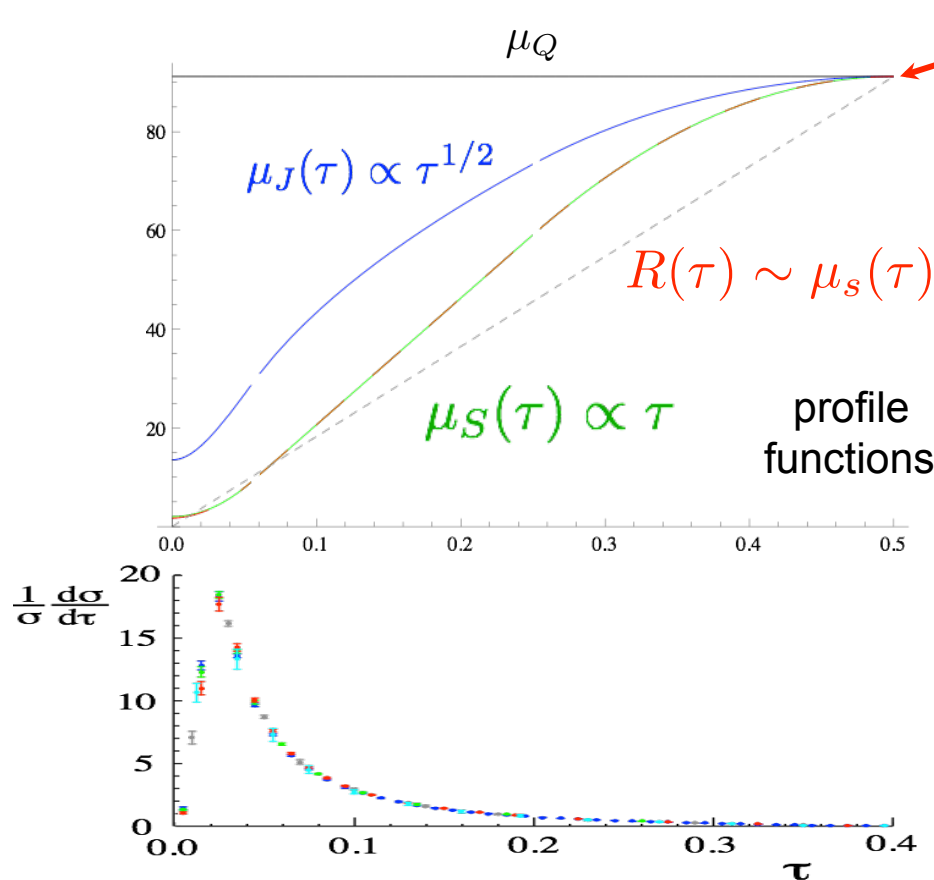
### Jet function evolution

$$\mu \frac{d}{d\mu} J(y, \mu) = \gamma_J(y, \mu) J(y, \mu) = \left[ 2\Gamma^{\text{cusp}}(\alpha_s) \ln(iy\mu^2 e^{\gamma_E}) + \gamma_J(\alpha_s) \right] J(y, \mu)$$

# Anatomy of SCET Predictions

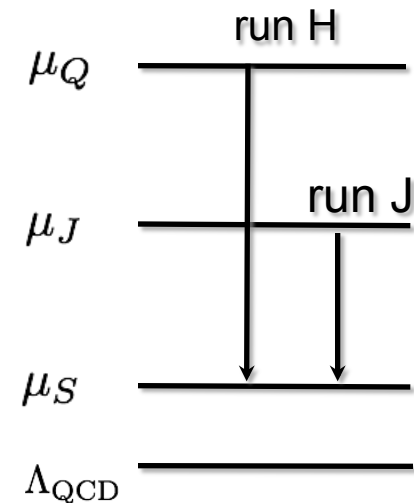
## Summation of large logarithms (RG-summation, SCET 1)

$$\left(\frac{d\sigma}{d\tau}\right)_{\text{part}}^{\text{sing}} \sim \sigma_0 H(Q, \mu_Q) U_H(Q, \mu_Q, \mu_s) \int d\ell d\ell' U_J(Q\tau - \ell - \ell', \mu_Q, \mu_s) J_T(Q\ell', \mu_j) S_T(\ell - \Delta, \mu_s)$$



profile functions

scales become equal for multijet region



# Anatomy of SCET Predictions

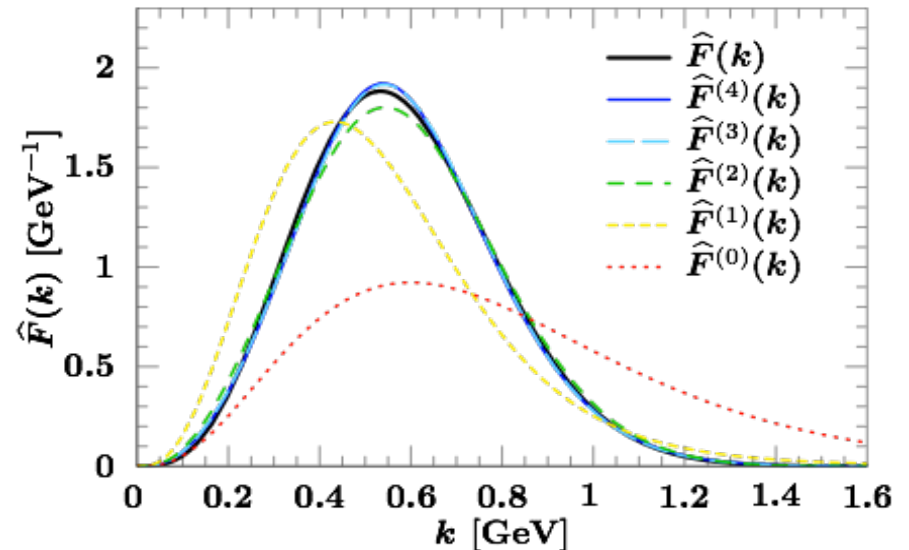
## Combination for hadron level prediction

$$\left(\frac{d\sigma}{d\tau}\right) = \int d\ell \left[ \left(\frac{d\sigma}{d\tau}\right)_{\text{part}}^{\text{sing}} \left(\tau - \frac{\ell}{Q}\right) + \left(\frac{d\sigma}{d\tau}\right)_{\text{part}}^{\text{nonsing}} \left(\tau - \frac{\ell}{Q}\right) \right] S^{\text{mod}}(\ell)$$

Fixed-order minus terms  
already resummed

Soft matrix element  
model function

- + Non-singular contributions
- + Renormalon subtraction





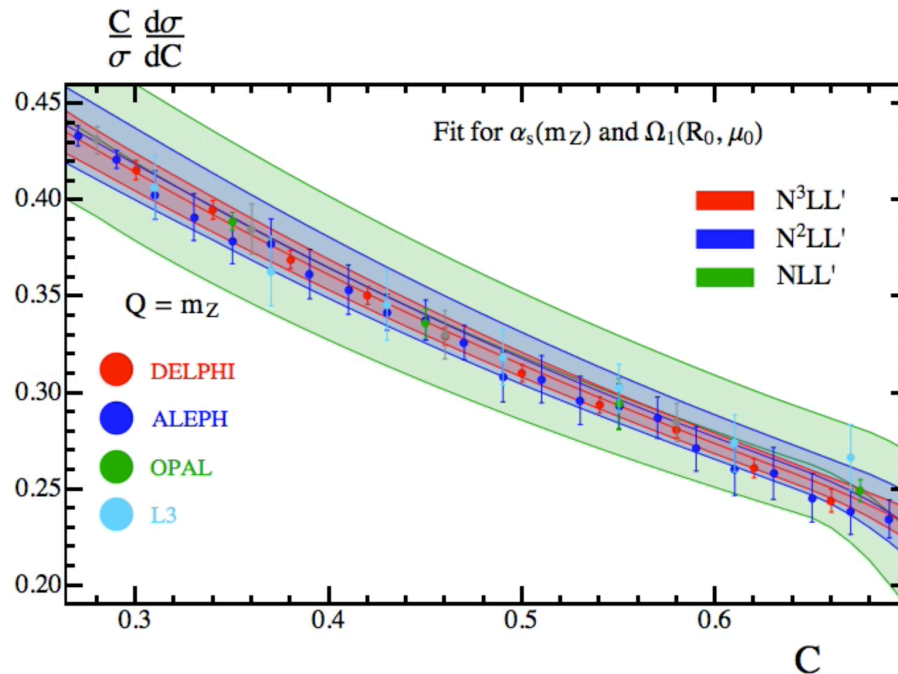
# Application of SCET 1

- Can be applied to global jet shape variables, not sensitive to transverse momenta: e.g.  $e^+e^-$  eventshapes

Thrust 
$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

C-parameter 
$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$$

- Analyses at NNLL +  $O(\alpha_s^2)$  fixed order using tail data (all available  $Q > 25$  GeV)



Becher, Schwartz (partonic resummation)

Full analysis incl. nonpert. effects:

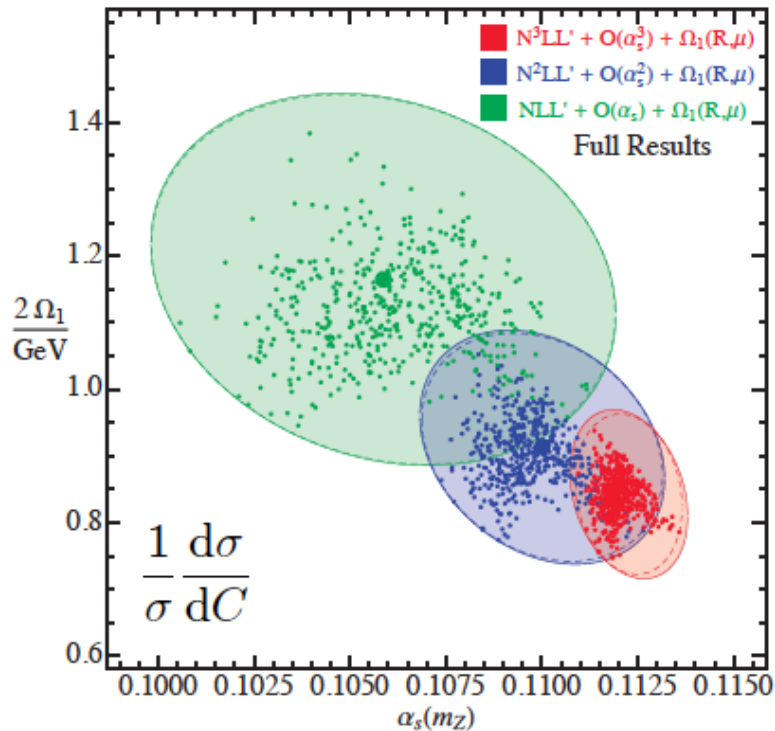
Abbate, Fickinger, AHH, Mateu, Stewart (thrust)

AHH, Kolodrubetz, Mateu, Stewart (C-para)

# Strong Coupling from $e^+e^-$ Event Shapes

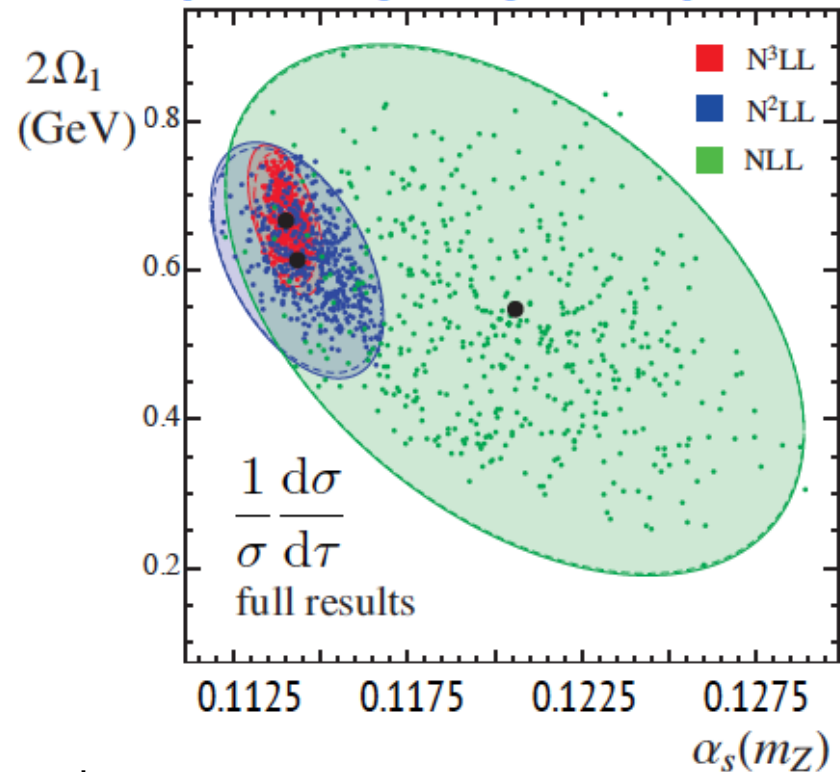
C-parameter Tail  $\alpha_s$ - $\Omega_1$  Global Fit

[Hoang, Kolodrubetz, VM Stewart]



Thrust Tail  $\alpha_s$ - $\Omega_1$  Global Fit

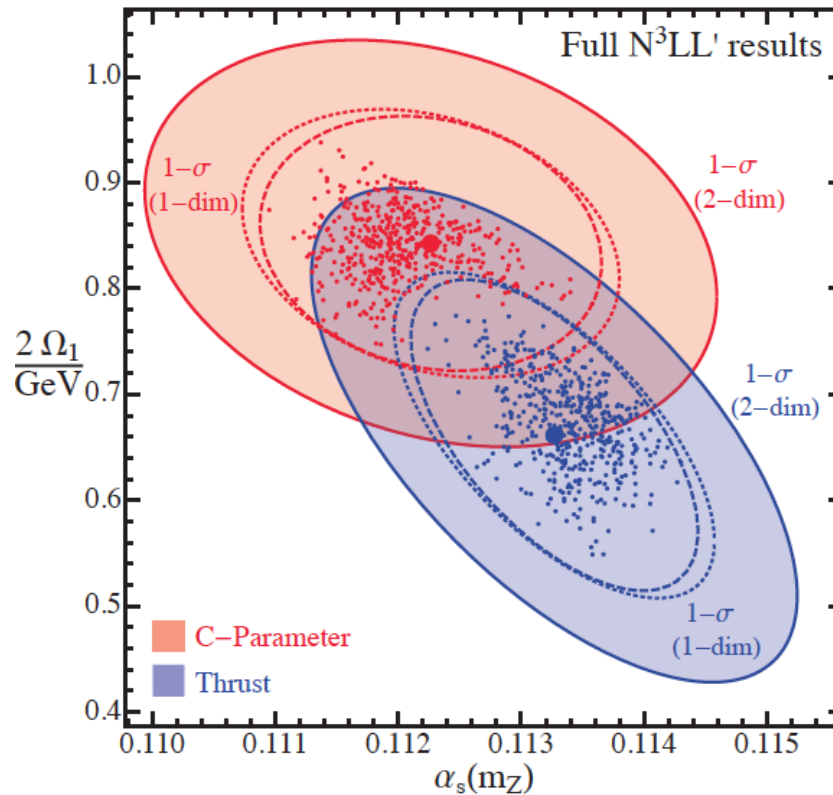
[Abbate, Fickinger, Hoang, VM Stewart]



- ➔ Different behavior of fits with increase order
- ➔ Good convergence
- ➔ Very good agreement at  $N^3LL + O(\alpha_s^3)$  with renormalon subtraction.
- ➔ Error dominated by theory uncertainty (particularly pQCD)

# Strong Coupling from $e^+e^-$ Event Shapes

C-parameter versus Thrust Tail Global Fit



C-parameter:

$$\alpha_s(M_Z) = 0.1123 \pm 0.0015$$

AHH, Kolodrubetz, Mateu, Stewart;  
PRD 91 (2015) 9, 094018

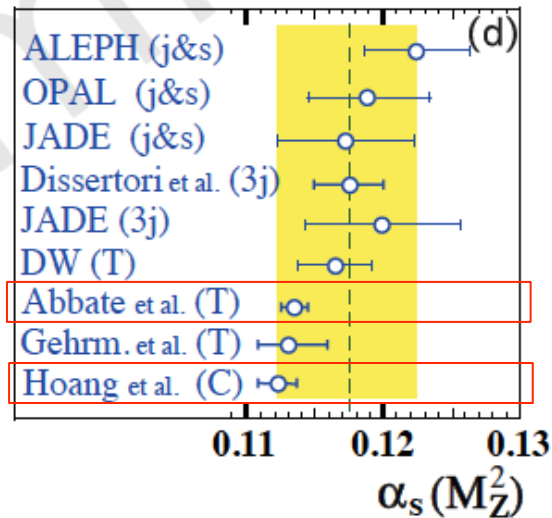
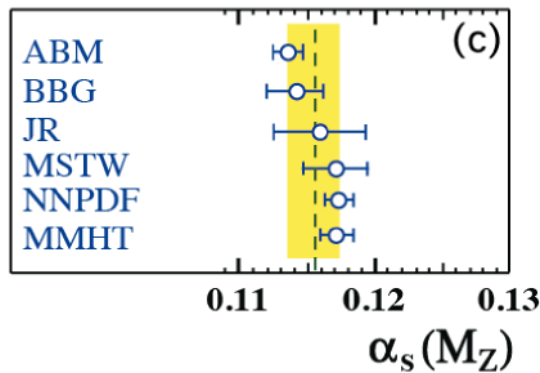
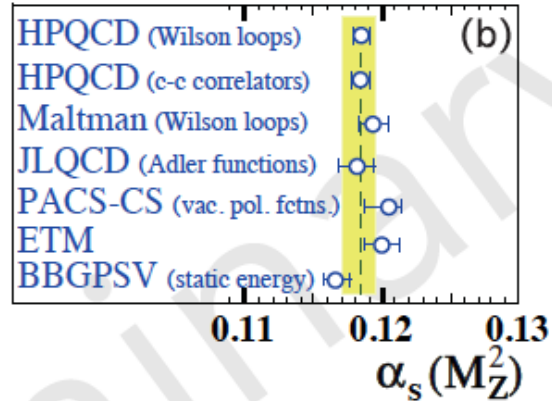
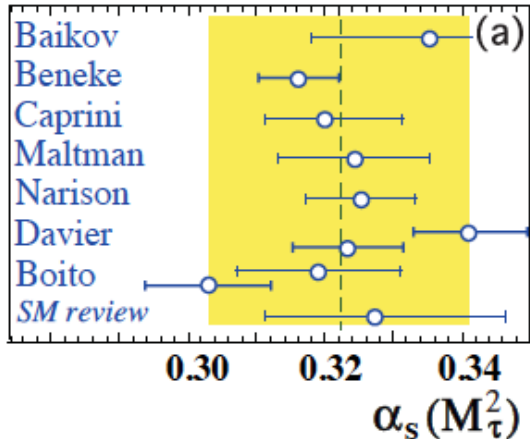
Thrust:

$$\alpha_s(M_Z) = 0.1135 \pm 0.0010$$

Abbate, Fickinger, AHH, Mateu,  
Stewart; PRD 83 (2011) 074021

→ Thrust and C-parameter results are fully compatible concerning fit of non-perturbative matrix element  $\Omega_1$ .

# Strong Coupling 2015 World Average

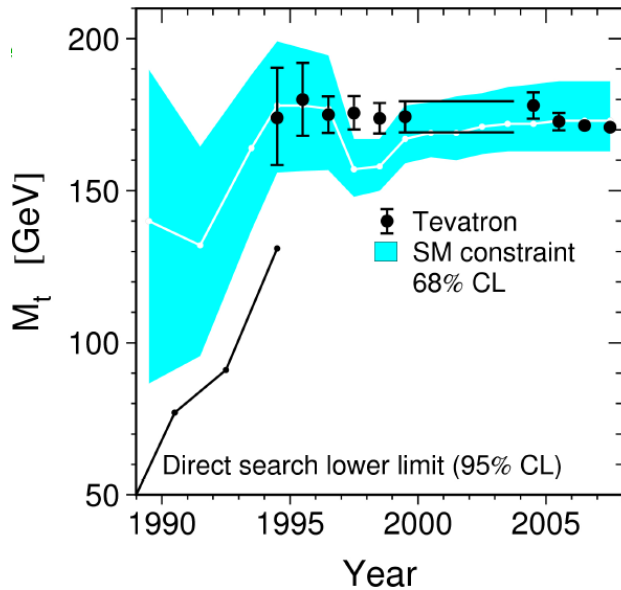


2016 Worldaverage:

$$\alpha_s(M_Z) = 0.1181 \pm 0.0013$$



# A small history on top mass reconstruction

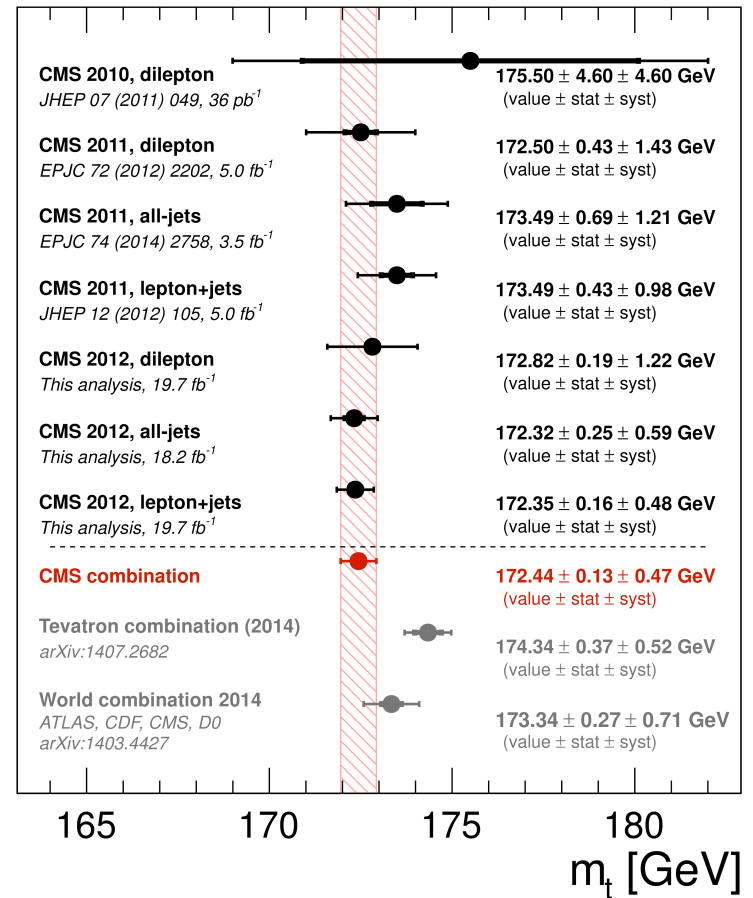


- Many individual measurements with uncertainties below 1 GeV.

$$m_t^{\text{MC}} = 174.34 \pm 0.64 \quad (\text{Tevatron final, 2014})$$

$$m_t^{\text{MC}} = 172.44 \pm 0.49 \quad (\text{CMS Run-1 final, 2015})$$

$$m_t^{\text{MC}} = 172.84 \pm 0.70 \quad (\text{ATLAS Run-1 final, 2016})$$



# Main Top Mass Measurements Methods

## LHC+Tevatron

### Direct Reconstruction:

#### Kinematic Fit

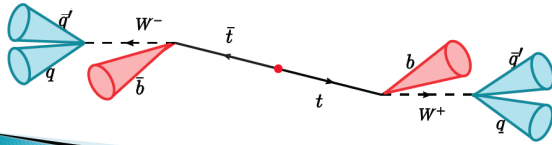
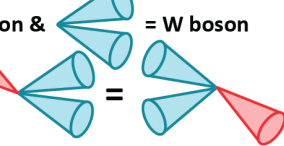
Selected objects:

- 4 untagged jets
- 2 b-tagged jets



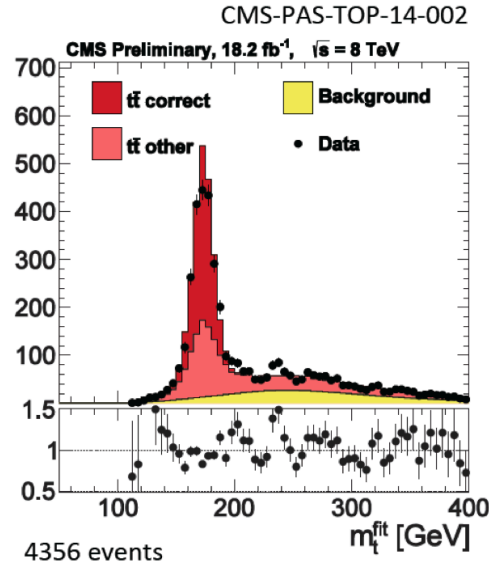
Constraints:

- $2 \times m_{jj} = m_W$
- $m_{top} = m_{j_{bb,1}} = m_{j_{bb,2}} = m_{antitop}$

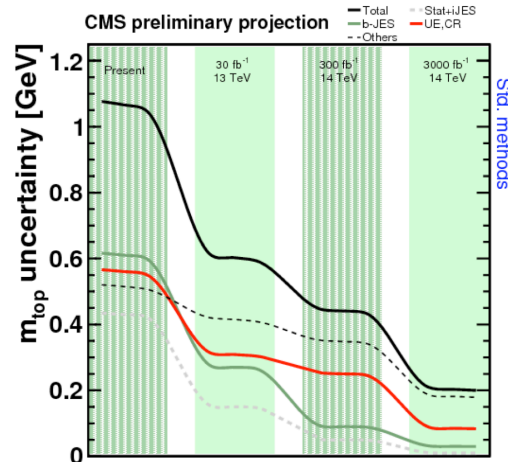


11 Eike Schlieckau - Universität Hamburg September 30th 2014

kinematic mass determination



Determination of the best-fit value of the Monte-Carlo top quark mass parameter

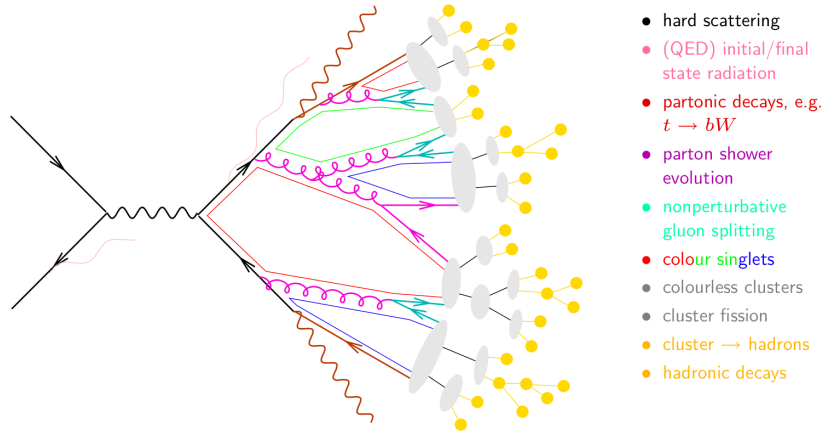


- ⊕ High top mass sensitivity
- ⊖ Precision of MC ?
- ⊖ Meaning of  $m_t^{MC}$  ?

$\Delta m_t \sim 0.5 \text{ GeV}$

←  $\Delta m_t \sim 200 \text{ MeV (projection)}$

# Monte-Carlo Event Generators



- 1) Matrix elements (LO/NLO)
- 2) Parton shower (LL)
- 3) Hadronization model

- Full simulation of all processes (all experimental aspects accessible)
- QCD-inspired: partly first principles QCD  $\Leftrightarrow$  partly model
- Description power of data better than intrinsic theory accuracy.
- Top quark in parton shower: treated like a real particle ( $m_t^{\text{MC}} \approx m_t^{\text{pole}} + ?$ ).
- Top quark in matrix elements:  $m_t^{\text{MC}} = m_t^{\text{pole}}$

But pole mass ambiguous by  $O(\Lambda_{\text{QCD}})$  due to confinement.  
Short mass definition more suitable.

Uncertainty (a): But how precise is modelling?  $\rightarrow$  Part of exp. Analyses

Uncertainty (b): What is the meaning of MC QCD parameters?  $\rightarrow$  This work

# MC Top Quark Mass (for reconstruction)

AHH, Stewart 2008 AHH, 2014

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(R = 1 \text{ GeV}) + \Delta_{t,\text{MC}}(R = 1 \text{ GeV})$$

$$\Delta_{t,\text{MC}}(1 \text{ GeV}) \sim \mathcal{O}(1 \text{ GeV})$$

- small size of  $\Delta_{t,\text{MC}}$
- Renormalon-free
- little parametric dependence on other parameters

## MSR Mass Definition

MS Scheme: ( $\mu > \bar{m}(\bar{m})$ )

$$\bar{m}(\bar{m}) - m^{\text{pole}} = -\bar{m}(\bar{m}) [0.42441 \alpha_s(\bar{m}) + 0.8345 \alpha_s^2(\bar{m}) + 2.368 \alpha_s^3(\bar{m}) + \dots]$$

MSR Scheme: ( $R < \bar{m}(\bar{m})$ )



$$m_{\text{MSR}}(R) - m^{\text{pole}} = -R [0.42441 \alpha_s(R) + 0.8345 \alpha_s^2(R) + 2.368 \alpha_s^3(R) + \dots]$$

$$m_{\text{MSR}}(m_{\text{MSR}}) = \bar{m}(\bar{m})$$

⇒  $m_{\text{MSR}}(R)$  Short-distance mass that smoothly interpolates all R scales  
≈ “pole mass subtraction for scales larger than R”

# Calibration of the MC Top Mass

## Method:

- ✓ 1) Strongly mass-sensitive hadron level observable (as closely as possible related to reconstructed invariant mass distribution !)
- ✓ 2) Accurate hadron level QCD predictions at  $\geq$  NLL/NLO with full control over the quark mass scheme dependence.
- ✓ 3) QCD masses as function of  $m_t^{\text{MC}}$  from fits of observable.
- 4) Cross check observable independence / universality

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(R = 1 \text{ GeV}) + \Delta_{t,\text{MC}}(R = 1 \text{ GeV})$$

$$\Delta_{t,\text{MC}}(1 \text{ GeV}) = \bar{\Delta} + \delta\Delta_{\text{MC}} + \delta\Delta_{\text{pQCD}} + \delta\Delta_{\text{param}}$$

Experimental systematics

### Monte Carlo dependence:

- different tunings
- parton showers
- color reconnection
- Intrinsic error, ...

### QCD errors:

- perturbative error
- scale uncertainties
- electroweak effects

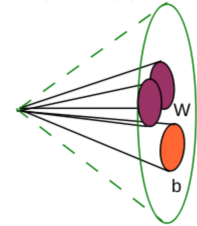
### Parametric errors:

- strong coupling  $\alpha_s$
- Non-perturbative parameters

Treated in our analysis

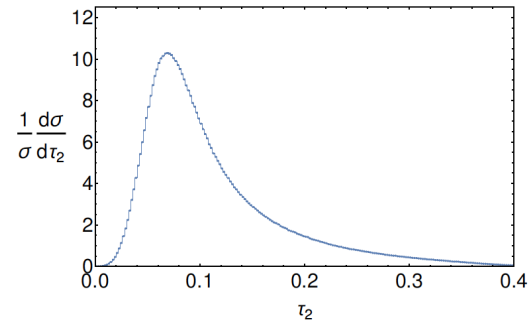
# Thrust Distribution

Observable: 2-jettiness in  $e^+e^-$  for  $Q = 2p_T \gg m_t$  (boosted tops)



$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{Q}$$

$$\tau_2 \xrightarrow{\text{peak}} \approx \frac{M_1^2 + M_2^2}{Q^2}$$

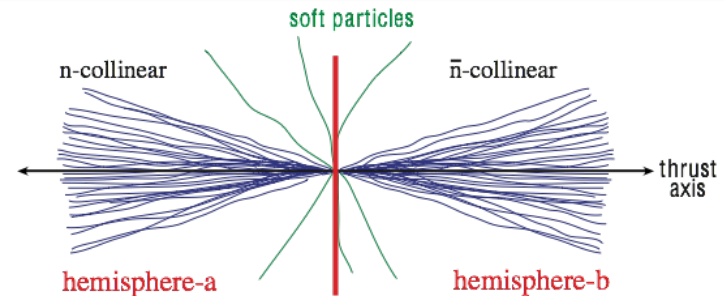


Invariant mass distribution in the resonance region of wide hemisphere jets !

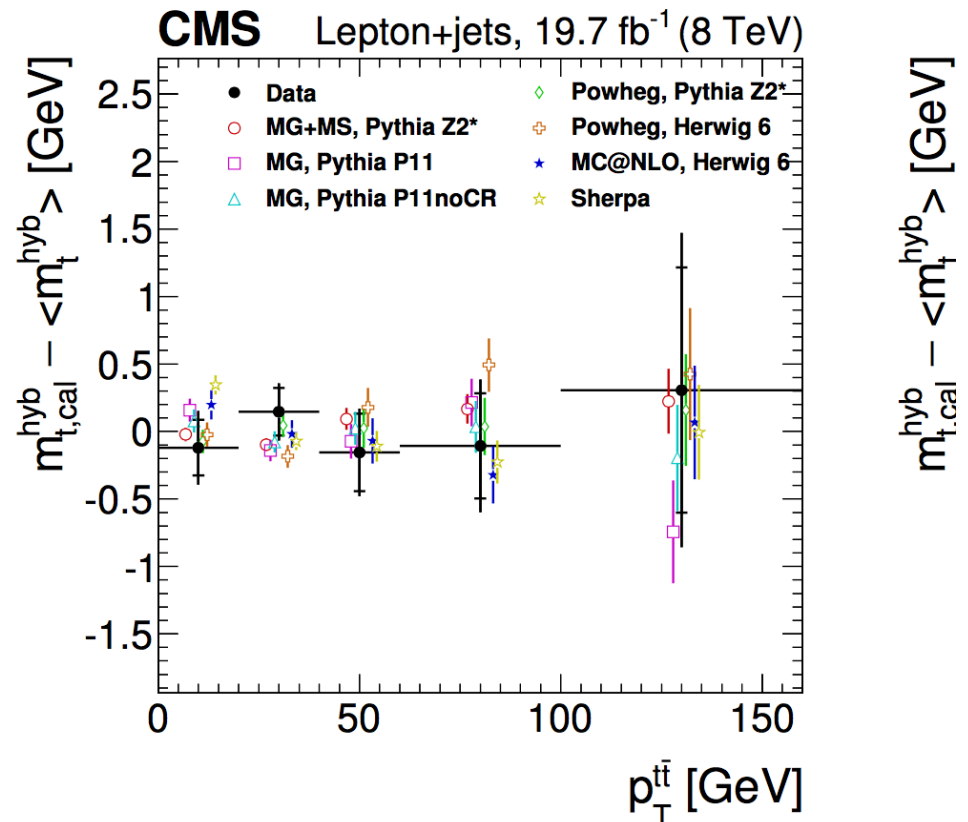
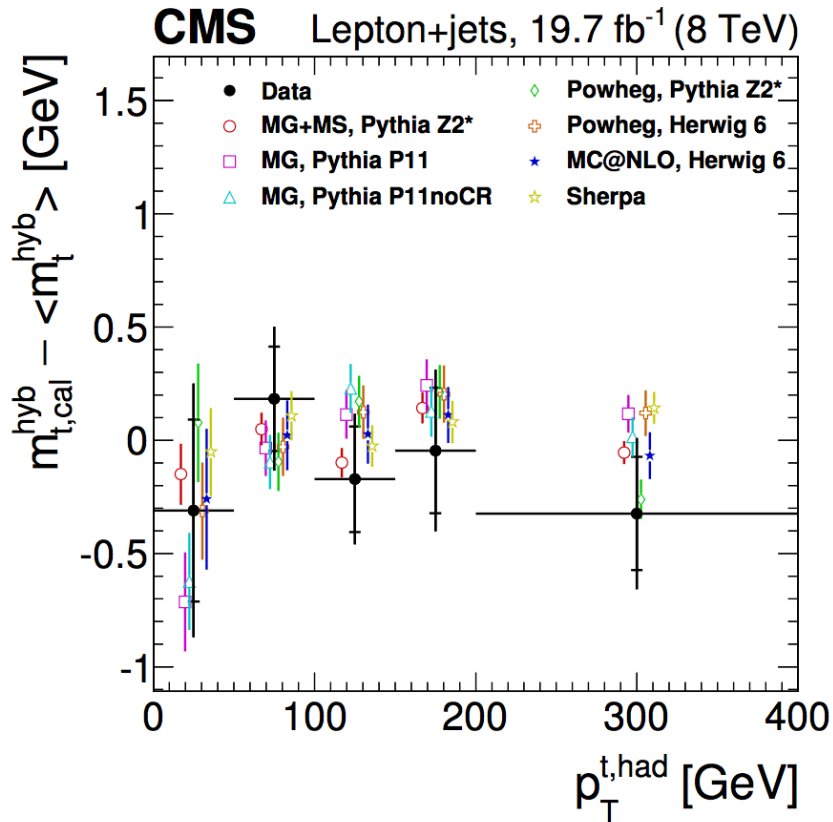
$$\frac{d\sigma}{d\tau} = Q^2 \sigma_0 H_0(Q, \mu) \int dl J_0(Ql, \mu) S_0(Q\tau - l, \mu)$$

Excellent mass sensitivity:

$$\tau_2^{\text{peak}} = 1 - \sqrt{1 - \frac{4m_t^2}{Q^2}} \quad (\text{tree level})$$



# $p_T$ Dependence of CMS Top Mass



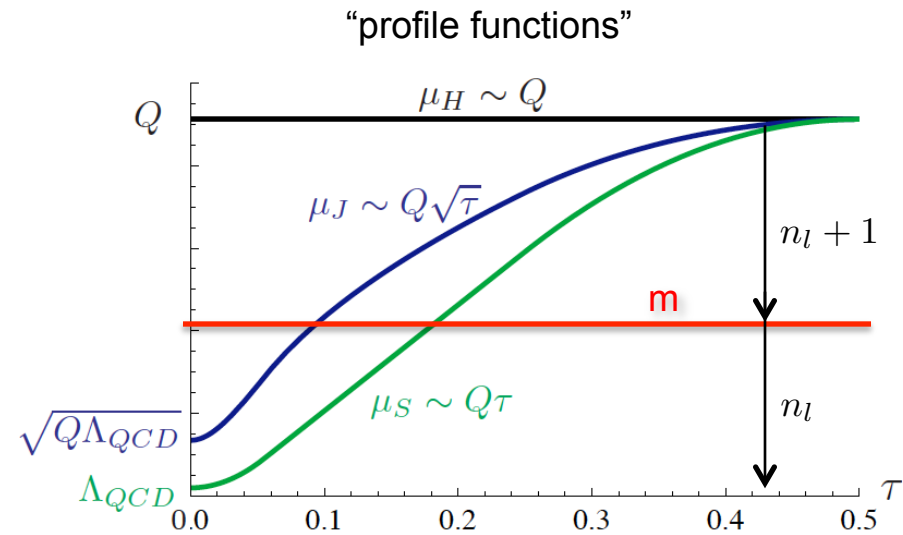
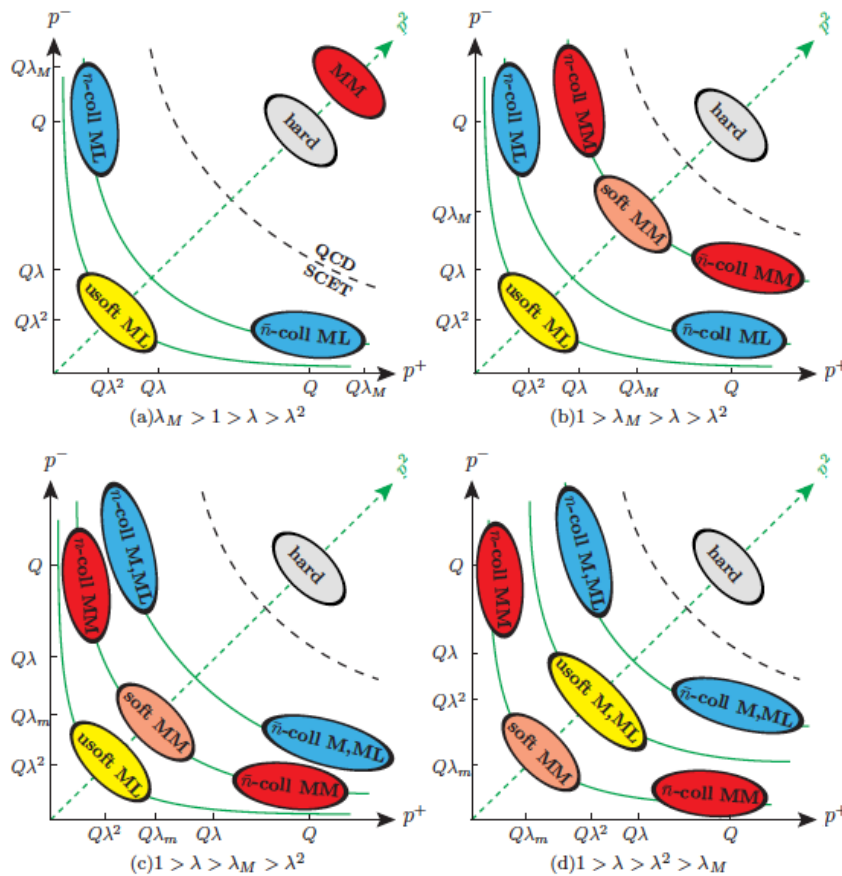
- Top mass from reconstruction of boosted tops consistent with low  $p_T$  results.
- More precise studies possible with more statistics from Run-2.
- Meaning of  $m_t^{\text{MC}}$  for boosted tops and slow top quarks consistent.

# Further Developments (small selection)

→ Extension of massless SCET-1 to massive quarks: Pietrulewicz, AHH, Jemos, Mateu

Variable Flavor Number scheme for final state jets (can be combined with PDF)

For arbitrary masses and full log resummation in any kinematic regime.



“profile functions”

mode	$p^\mu = (+, -, \perp)$	$p^2$
$n$ -coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	$m^2$
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	$m^2$



# 2-Jettiness for Top Production (QCD)

$$\frac{d\sigma}{d\tau_2} = f(\underbrace{m_t^{\text{MSR}}(R)}_{\text{any scheme possible}}, \underbrace{\alpha_s(M_Z), \Omega_1, \Omega_2, \dots}_{\text{Non-perturbative}}, \underbrace{\mu_h, \mu_j, \mu_s, \mu_m, R, \Gamma_t}_{\text{renorm. scales finite lifetime}})$$

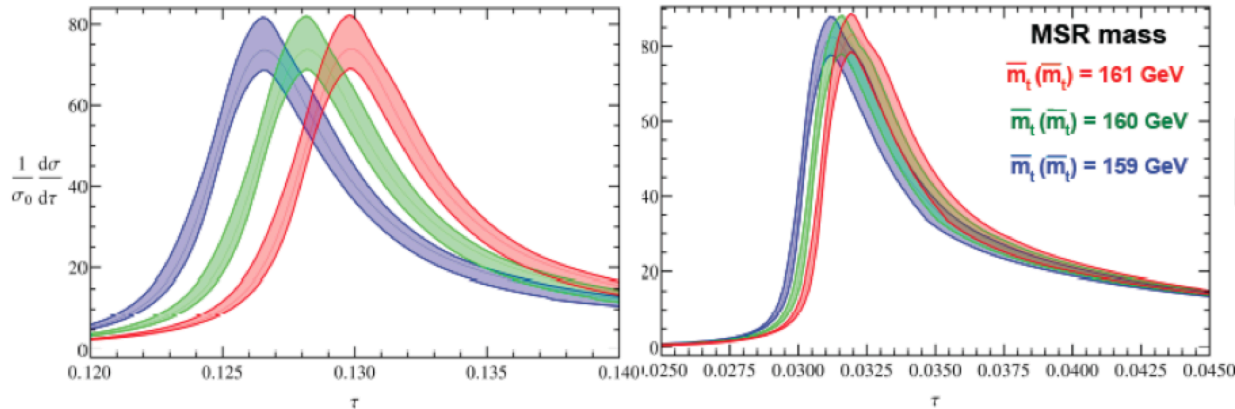
any scheme possible

Non-perturbative

renorm. scales finite lifetime

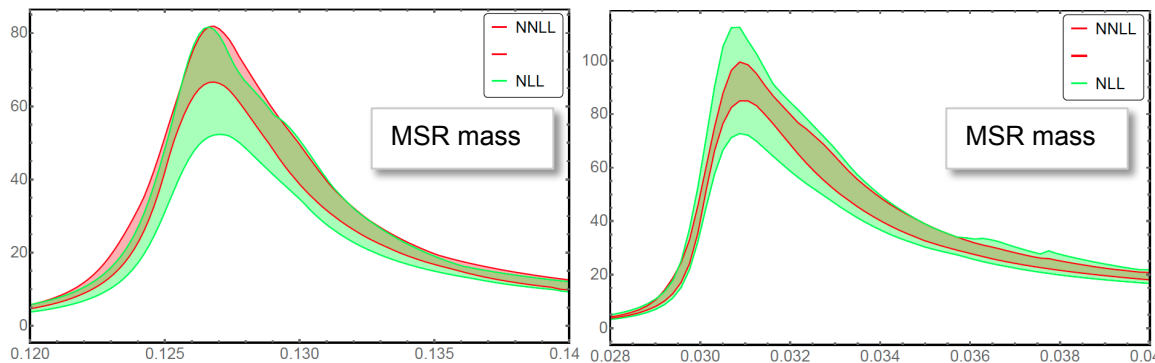
Q=700 GeV ( $p_T = 350$  GeV)

Q=1400 GeV ( $p_T = 700$  GeV)



Q=700 GeV

Q=1400 GeV



- Higher mass sensitivity for lower Q ( $p_T$ )
- Finite lifetime effects included
- Dependence on non-perturbative parameters
- Convergence:  $\Omega_{1,2,\dots}$
- Good convergence
- Reduction of scale uncertainty (NLL to NNLL)
- Control over whole distribution

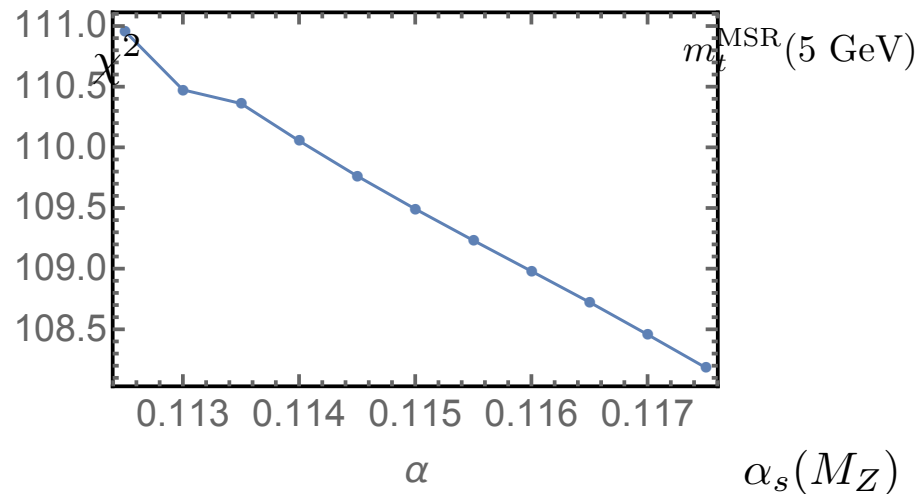
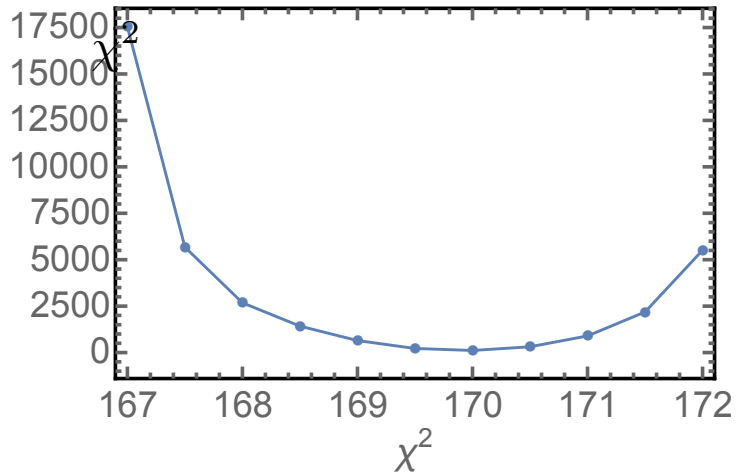
# Fit Procedure Details

Butenschoen, Dehnadi, AHH, Mateu, Preisser, Stewart; PRL to appear

- $\frac{d\sigma}{d\tau} = f(m_t^{\text{MSR}}, \alpha_s(m_Z), \Omega_1, \Omega_2, \dots, \mu_H, \mu_J, \mu_S, \mu_M, R, \Gamma_t)$  (NNLL + NLO)  
any scheme    non-perturbative    renorm. scales    finite lifetime
- Generating PYTHIA Samples: (PYTHIA 8.205)  
at different energies:  $Q = 600, 700, 800, \dots, 1400$  GeV
  - ▶ masses:  $m_t^{\text{MC}} = 170, 171, 172, 173, 174, 175$  GeV
  - ▶ width:  $\Gamma_t = 1.4$  GeV
  - ▶ Statistics:  $10^7$  events for each set of parameters
  - ▶ Tune 7 (Monash)
- Feed MC data into **Fitting Procedure**: all ingredients are there  
Fit parameters:  $m_t^{\text{MSR}}, \alpha_s(m_Z), \Omega_1, \Omega_2, \dots$ 
  - ▶ standard fit based on  $\chi^2$  minimization
  - ▶ analysis with 500 sets of profiles ( $\tau_2$  dependent renorm. scales) for the each MC sample
  - ▶ different Q-sets: 7 sets with energies between 600 - 1400 GeV
  - ▶ different n-sets: 3 choices for fitranges - (xx/yy)% of maximum peak height } 21 fit setups

# Peak Fits Parameter Sensitivity

Default renormalization scales;  $\Gamma_t = 1.4$  GeV, tune 7,  $\Omega_{1,\text{smear}} = 2.5$  GeV,  $m_t^{\text{Pythia}} = 171$  GeV,  $Q = \{700, 1000, 1400\}$  GeV, peak fit (60/80)%



→  $\chi^2_{\text{min}} \sim O(100)$

- Very strong sensitivity to  $m_t$
- Low sensitivity to strong coupling
- Take PDF strong coupling as input:  $\alpha_s(M_Z) = 0.1181(13)$  (error irrelevant for  $m_t^{\text{MSR}}$ ,  $m_t^{\text{pole}}$ )
- $\chi^2_{\text{min}}$  and  $\delta m_t^{\text{stat}}$  do not have any physical meaning
- PDF rescaling method:  $(\chi^2_{\text{min}})^{\text{rescale}} = 1$  can be used to define an incompatibility uncertainty

# Fit Result: Pythia 8.205 vs. Theory

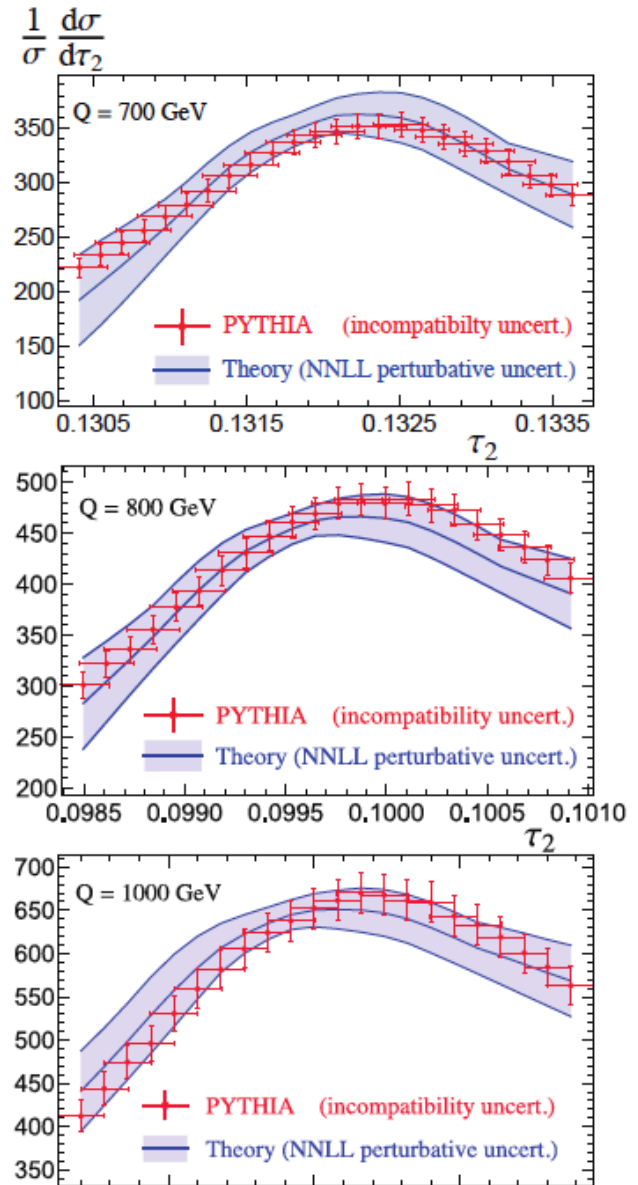
$\Gamma_t = 1.4$  GeV, tune 7,

$m_t^{\text{MC}} = 173$  GeV

$\Omega_1 = 0.44$  GeV,

$m_t^{\text{MSR}(1\text{GeV})} = 172.81$  GeV

- Good agreement of PYTHIA with NNLL/NLO theory predictions
- **Perturbative uncertainties** of theory predictions based on scale uncertainties (profiles)
- **MC uncertainties:**
  - Vertical: rescaled statistical error (PDF rescaling method)  $\rightarrow$  independent on statistics
  - Horizontal: fit coverage from 21 fit setups (incompatibility uncertainty)



# Convergence & Stability: MSR vs. Pole Mass

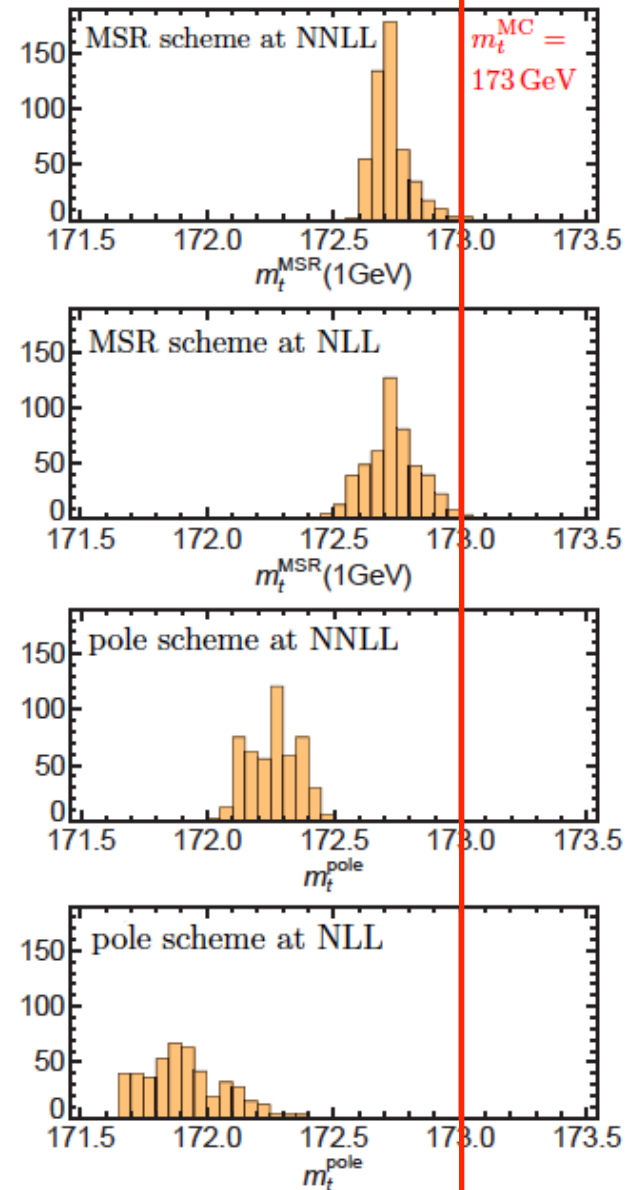
500 profiles;  $\alpha_s = .118$ ;  $\Gamma_t = 1.4$  GeV; tune 7;  
 $Q = 700, 1000, 1400$  GeV; peak(60/80)%

Input:  $m_t^{\text{MC}} = 173$  GeV

fit to find  $m_t^{\text{MSR}}(1\text{GeV})$  or  $m_t^{\text{pole}}$

- Good convergence & stability for **MSR mass**
- Mass  $m_t^{\text{MSR}}(1\text{GeV})$  mass definition closest to the MC top mass  $m_t^{\text{MC}}$ .
- **Pole mass** shows worse convergence.
- Poles mass not compatible with MC mass within errors
- 1100/700 MeV difference at NLL/NNLL

Similar analyses from the 20 other Q-set and n-range setups.



# Final Result for $m_t^{\text{MSR}}(1 \text{ GeV})$

- All investigated MC top mass values show consistent picture

- MC top quark mass is indeed closely related to MSR mass

within uncertainties:

$$m_t^{\text{MC}} \simeq m_t^{\text{MSR}}(1 \text{ GeV})$$

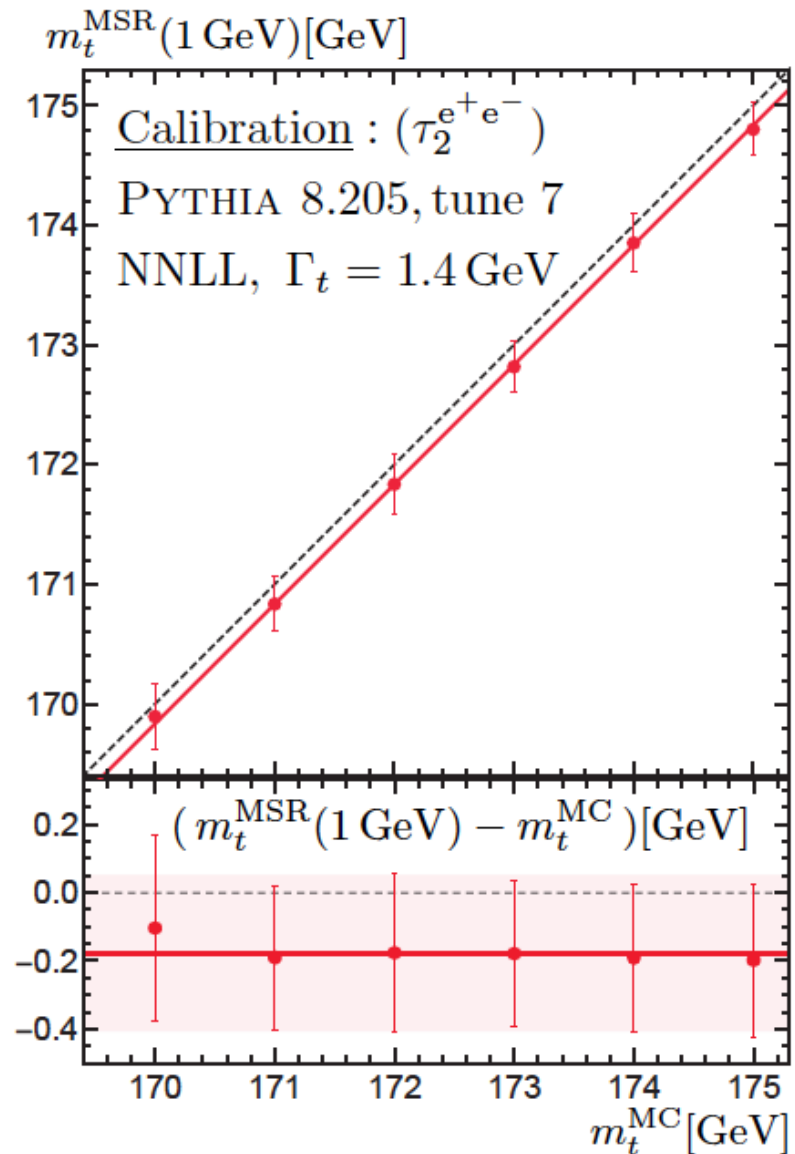
$$m_t^{\text{MC}} = 173 \text{ GeV} \quad (\tau_2^{e^+e^-})$$

mass	order	central	perturb.	incompatibility	total
$m_{t,1 \text{ GeV}}^{\text{MSR}}$	NLL	172.80	0.26	0.14	0.29
$m_{t,1 \text{ GeV}}^{\text{MSR}}$	N <sup>2</sup> LL	172.82	0.19	0.11	0.22
$m_t^{\text{pole}}$	NLL	172.10	0.34	0.16	0.38
$m_t^{\text{pole}}$	N <sup>2</sup> LL	172.43	0.18	0.22	0.28

↓  
Spread of results  
from 21 fit setups

$$\Omega_1^{\text{PY}} = 0.41 \pm 0.07 \pm 0.02 \text{ GeV at NLL}$$

$$\Omega_1^{\text{PY}} = 0.42 \pm 0.07 \pm 0.03 \text{ GeV at N}^2\text{LL}$$



# Conclusions

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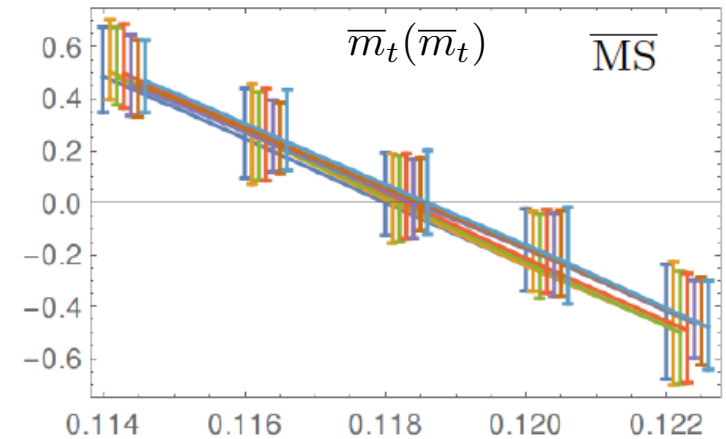
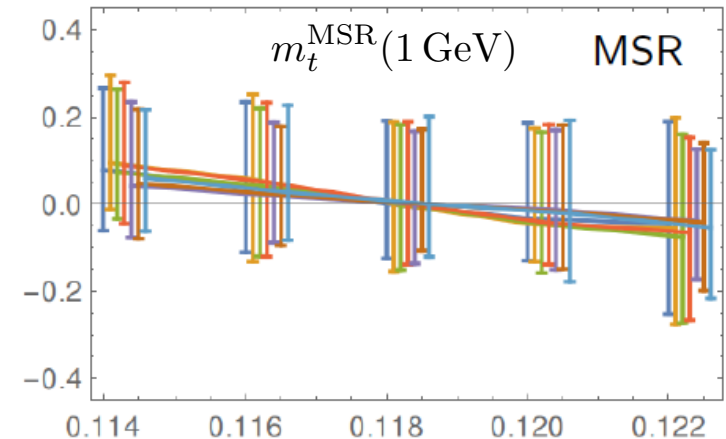
- SCET allows for high precision computations for jet physics
- SCET allows for very complicated mode setups to solve previously unsolved problems
  
- Soft-Collinear Effective Theory: aimed at making internal dynamics of jets accessible to pQCD and factorization in a systematically improvable matter
  
- Event shape distributions
- Monte-Carlo top quark mass calibration

# MSR/MS Parametric Dependence on $\alpha_s$

500 profiles;  $\Gamma_t = 1.4, -1$  GeV; tune 7;  
diff. Q-sets; peak(60/80)%

$$m_t^{\text{PYTHIA}} = 173 \text{ GeV}$$

- $\alpha_s$  dependence:  
 $m^{\text{scheme}}[\alpha_s] - m^{\text{scheme}}[.118]$
- small dependence of MSR mass on  $\alpha_s$   
 $\sim 50 \text{ MeV error } (\delta\alpha_s = .002)$
- large sensitivity of  $\overline{\text{MS}}$  mass on  $\alpha_s$
- not an error:  
calculated from MSR





# MSR Mass Tune Dependence

500 profiles;  $\Gamma_t = 1.4, -1$  GeV; tune 1, 3, 7;  
diff. Q-sets; peak(60/80)%

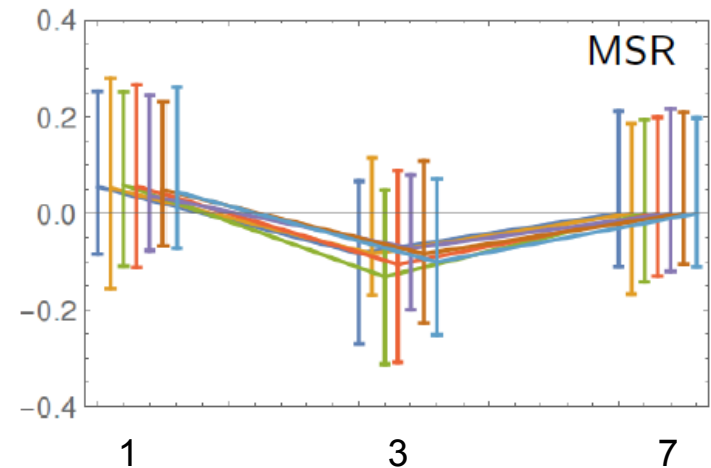
$$m_t^{\text{PYTHIA}} = 173 \text{ GeV}$$

- tune dependence:

$$m^{\text{MSR}}[\text{tune}] - m^{\text{MSR}}[7]$$

- clear sensitivity to tune
- $m^{\text{MC}}$  will depend on tune
- tune dependence is not a calibration uncertainty:

(different tune  $\Rightarrow$  different MC  $\Rightarrow m_t^{\text{MC}}$ )

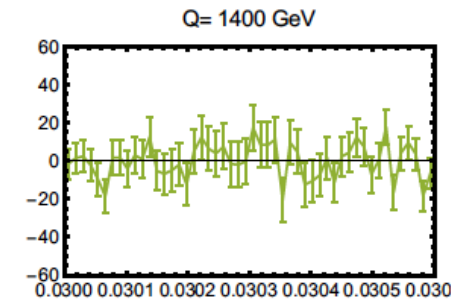
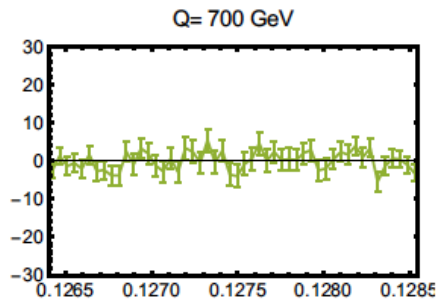
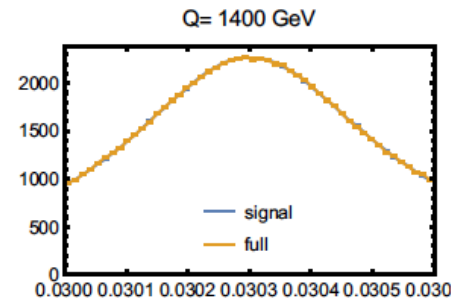
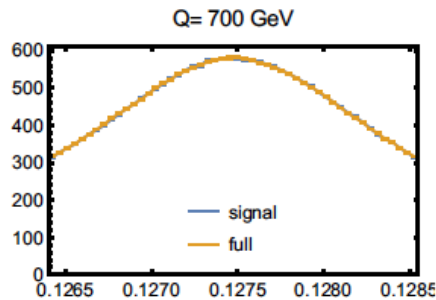
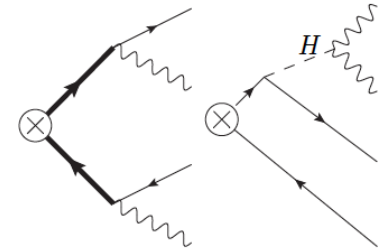


# Signal $t\bar{t}b\bar{b}$ vs full $ee \rightarrow WWbb$

MadGraph 5 study:

- Non-resonant contributions are irrelevant for  $\tau_2$  distribution

- ▶ PYTHIA (or similar MCs) will give a good description of the production process at LO
- ▶ hemisphere invariant mass  $\sim$  top invariant mass (no pollution from background)

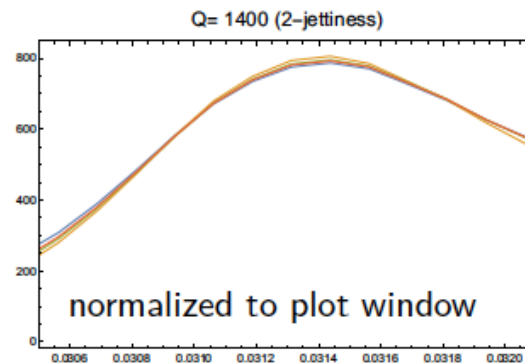
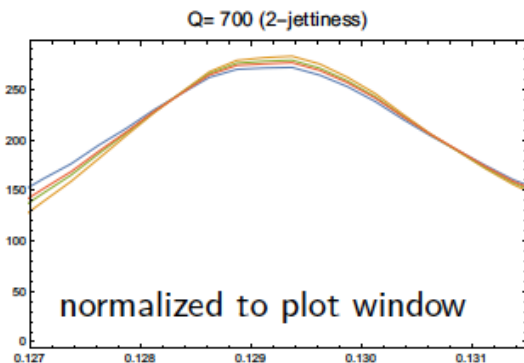
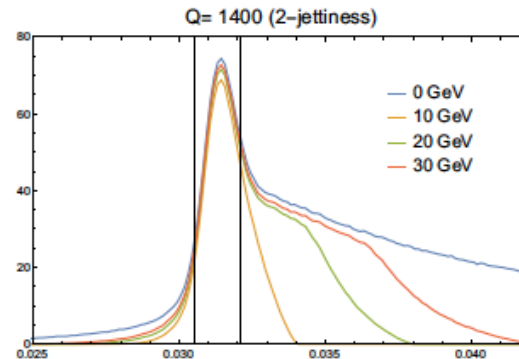
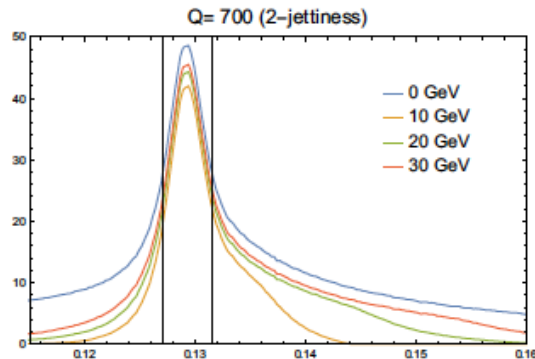


# Pythia Study: Hemisphere Mass Cuts

- In our theory description we treat the top decay as inclusive w.r.t. hemisphere
  - ▶ violated by decay products which cross to the other hemisphere
  - ▶ no differential impact in resonance region (irrelevant when normalized to signal region)

Cuts on hemisphere invariant mass above and below:

$$M_i^{\text{cut}} = m_t^{\text{MC}} \pm \Delta^{\text{cut}}$$



# Pole Mass from MSR Mass

---

$$\alpha_s(M_Z) = 0.118$$
$$n_f = 5$$

$$m_t^{\text{pole}} - m_t^{\text{MSR}}(1 \text{ GeV}) = \begin{array}{cccc} \mathcal{O}(\alpha_s) & \mathcal{O}(\alpha_s^2) & \mathcal{O}(\alpha_s^3) & \mathcal{O}(\alpha_s^4) \\ 0.173 & + 0.138 & + 0.159 & + 0.23 \text{ GeV} \leftarrow \text{calculated} \\ + 0.53 & + 1.43 & + 4.54 & + 16.6 \text{ GeV} \leftarrow \text{extrapolated} \\ + 68.6 & + 317.7 & + 1629 & + 9158 \text{ GeV} \end{array}$$

- Size of terms consistent with scale error estimate of calibration.
- No stable determination of pole mass.

# Top Mass Reconstruction Error Budget

Lepton+jets channel	$m_t$ fit type			
	2D $\delta m_t^{2D}$ (GeV)	$\delta$ JSF	1D $\delta m_t^{1D}$ (GeV)	hybrid $\delta m_t^{\text{hyb}}$ (GeV)
<b>Experimental uncertainties</b>				
Method calibration	0.04	0.001	0.04	0.04
<b>Jet energy corrections</b>				
– JEC: Intercalibration	<0.01	<0.001	+0.02	+0.01
– JEC: In situ calibration	–0.01	+0.003	+0.24	+0.12
– JEC: Uncorrelated non-pileup	+0.09	–0.004	–0.26	–0.10
– JEC: Uncorrelated pileup	+0.06	–0.002	–0.11	–0.04
Lepton energy scale	+0.01	<0.001	+0.01	+0.01
$E_T^{\text{miss}}$ scale	+0.04	<0.001	+0.03	+0.04
Jet energy resolution	–0.11	+0.002	+0.05	–0.03
b tagging	+0.06	< 0.001	+0.04	+0.06
Pileup	–0.12	+0.002	+0.05	–0.04
Backgrounds	+0.05	< 0.001	+0.01	+0.03
<b>Modeling of hadronization</b>				
<b>JEC: Flavor-dependent</b>				
– light quarks (u d s)	+0.11	–0.002	–0.02	+0.05
– charm	+0.03	<0.001	–0.01	+0.01
– bottom	–0.32	<0.001	–0.31	–0.32
– gluon	–0.22	+0.003	+0.05	–0.08
<b>b jet modeling</b>				
– b fragmentation	+0.06	–0.001	–0.06	<0.01
– Semileptonic b hadron decays	–0.16	<0.001	–0.15	–0.16
<b>Modeling of perturbative QCD</b>				
PDF	0.09	0.001	0.06	0.04
Ren. and fact. scales	+0.17 ± 0.08	–0.004 ± 0.001	–0.24 ± 0.06	–0.09 ± 0.07
ME-PS matching threshold	+0.11 ± 0.09	–0.002 ± 0.001	–0.07 ± 0.06	+0.03 ± 0.07
ME generator	–0.07 ± 0.11	–0.001 ± 0.001	–0.16 ± 0.07	–0.12 ± 0.08
Top quark $p_T$	+0.16	–0.003	–0.11	+0.02
<b>Modeling of soft QCD</b>				
Underlying event	+0.15 ± 0.15	–0.002 ± 0.001	+0.07 ± 0.09	+0.08 ± 0.11
Color reconnection modeling	+0.11 ± 0.13	–0.002 ± 0.001	–0.09 ± 0.08	+0.01 ± 0.09
Total systematic	0.59	0.007	0.62	0.48
Statistical	0.20	0.002	0.12	0.16
Total	0.62	0.007	0.63	0.51

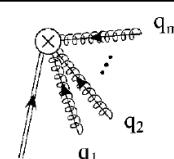
$m_t^{\text{MC}} = 172.44 \pm 0.49$   
 (CMS Run-1 final, 2015)  
 arXiv:1509.04044

← NLO ME corrections

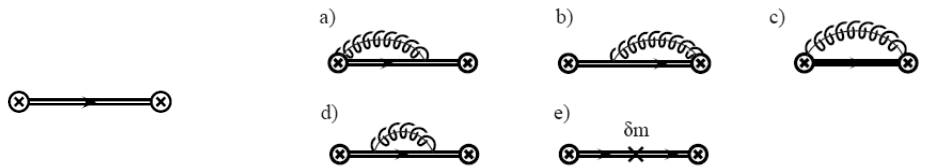
# b(oosted)HQET Factorization

Jet function:  $B_+(2v_+ \cdot k) = \frac{-1}{8\pi N_c m} \text{Disc} \int d^4x e^{ik \cdot x} \langle 0 | T \{ \bar{h}_{v_+}(0) W_n(0) W_n^\dagger(x) h_{v_+}(x) \} | 0 \rangle$

- perturbative, any mass scheme
- depends on  $m_t, \Gamma_t$
- Breit-Wigner at tree level
- Gauge-invariant off-shell top quark dynamics



$$W = \sum_{m=0}^{\infty} \sum_{\text{perms}} \frac{(-g)^m}{m!} \frac{\bar{n} \cdot A_{n,q_1}^{a_1} \cdots \bar{n} \cdot A_{n,q_m}^{a_m}}{\bar{n} \cdot q_1 \bar{n} \cdot (q_1 + q_2) \cdots \bar{n} \cdot (\sum_{i=1}^m q_i)} T^{a_m} \cdots T^{a_1}$$



$$\hat{s} = \frac{M^2 - m_t^2}{m_t}$$

$$\mathcal{B}_{\pm}(\hat{s}, 0, \mu, \delta m) = -\frac{1}{\pi m} \frac{1}{\hat{s} + i0} \left\{ 1 + \frac{\alpha_s C_F}{4\pi} \left[ 4 \ln^2 \left( \frac{\mu}{-\hat{s} - i0} \right) + 4 \ln \left( \frac{\mu}{-\hat{s} - i0} \right) + 4 + \frac{5\pi^2}{6} \right] \right\} - \frac{1}{\pi m} \frac{2\delta m}{(\hat{s} + i0)^2}$$

Fleming, AHH, Mantry, Stewart 2007

# b(oosted)HQET Factorization

Is the pole mass determining the top single particle pole?

**NO !**

$$\hat{s} = \frac{M_t^2 - m_t^2}{m_t}$$

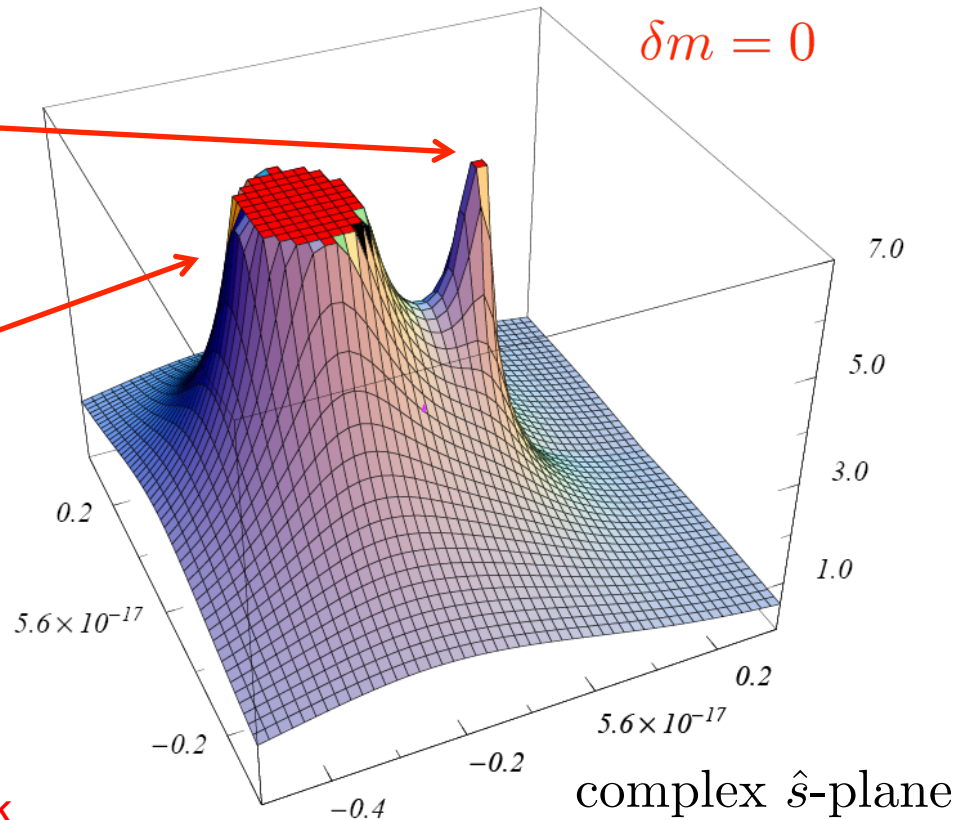
$$|\mathcal{B}_{\pm}(\hat{s}, \Gamma_t, \mu)|^2$$

$$\delta m = 0$$

pole mass peak  
Invisible for  $\Gamma_t > 0.5 \text{ GeV}$

observable peak

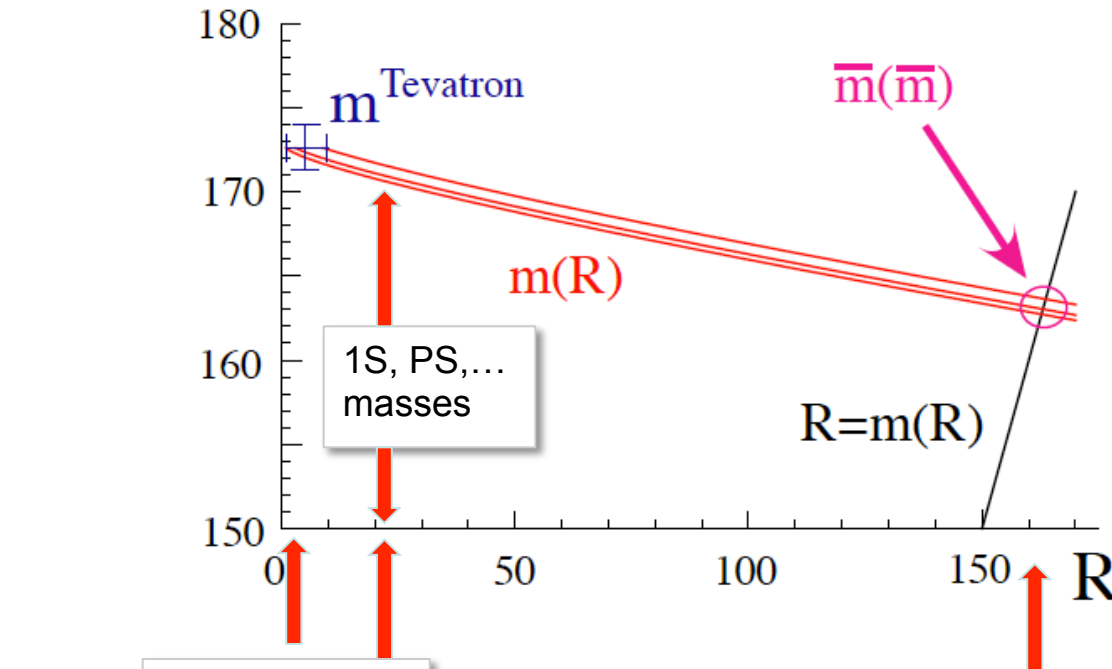
- pole mass and observable peak separated by renormalon
- pole mass peak residue decreases with order
- MSR mass close to observable peak



# MSR Mass Definition

AH, Stewart: arXiv:0808.0222

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(3_{-2}^{+6} \text{ GeV}) = m_t^{\text{MSR}}(3 \text{ GeV})_{-0.3}^{+0.6}$$

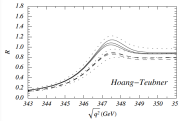
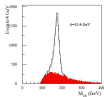


Good choice for R:  
Of order of the typical scale of the observable used to measure the top mass.

Peak of invariant mass distribution, endpoints

Total cross section, e.w.precision obs., Unification, MSbar mass

Top-antitop threshold at the ILC





# Masses Loop-Theorists Like to use

## Total cross section (LHC/Tev):

$$m_t^{\text{MSR}}(R = m_t) = \bar{m}_t(\bar{m}_t)$$

$$M_t = M_t^{(O)} + M_t(0)\alpha_s + \dots$$

- more inclusive
- sensitive to top production mechanism (pdf, hard scale)
- indirect top mass sensitivity
- large scale radiative corrections

## Threshold cross section (ILC):

$$m_t^{\text{MSR}}(R \sim 20 \text{ GeV}), m_t^{1S}, m_t^{\text{PS}}(R)$$

$$M_t = M_t^{(O)} + \langle p_{\text{Bohr}} \rangle \alpha_s + \dots$$

$$\langle p_{\text{Bohr}} \rangle = 20 \text{ GeV}$$

## Inv. mass reconstruction (ILC/LHC):

$$m_t^{\text{MSR}}(R \sim \Gamma_t), m_t^{\text{jet}}(R)$$

$$M_t = M_t^{(O)} + \Gamma_t \alpha_s + \dots$$

$$\Gamma_t = 1.3 \text{ GeV}$$

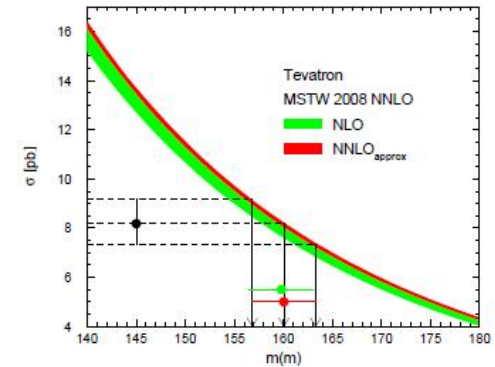
- more exclusive
- sensitive to top final state interactions (low scale)
- direct top mass sensitivity
- small scale radiative corrections



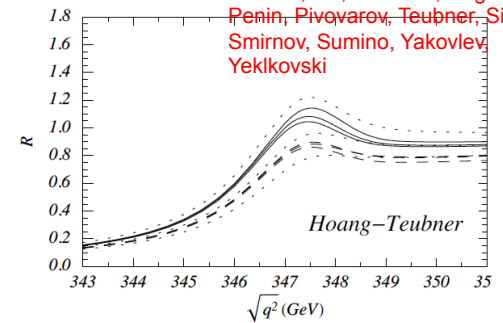
Mass schemes related to different computational methods

Relations computable in perturbation theory

Langenfeld, Moch, Uwer



Beneke, AH, Melnikov, Nagano, Penin, Pivovarov, Teubner, Signer, Smirnov, Sumino, Yakovlev, Yeklkovski



Fleming, AH, Mantry, Stewart

