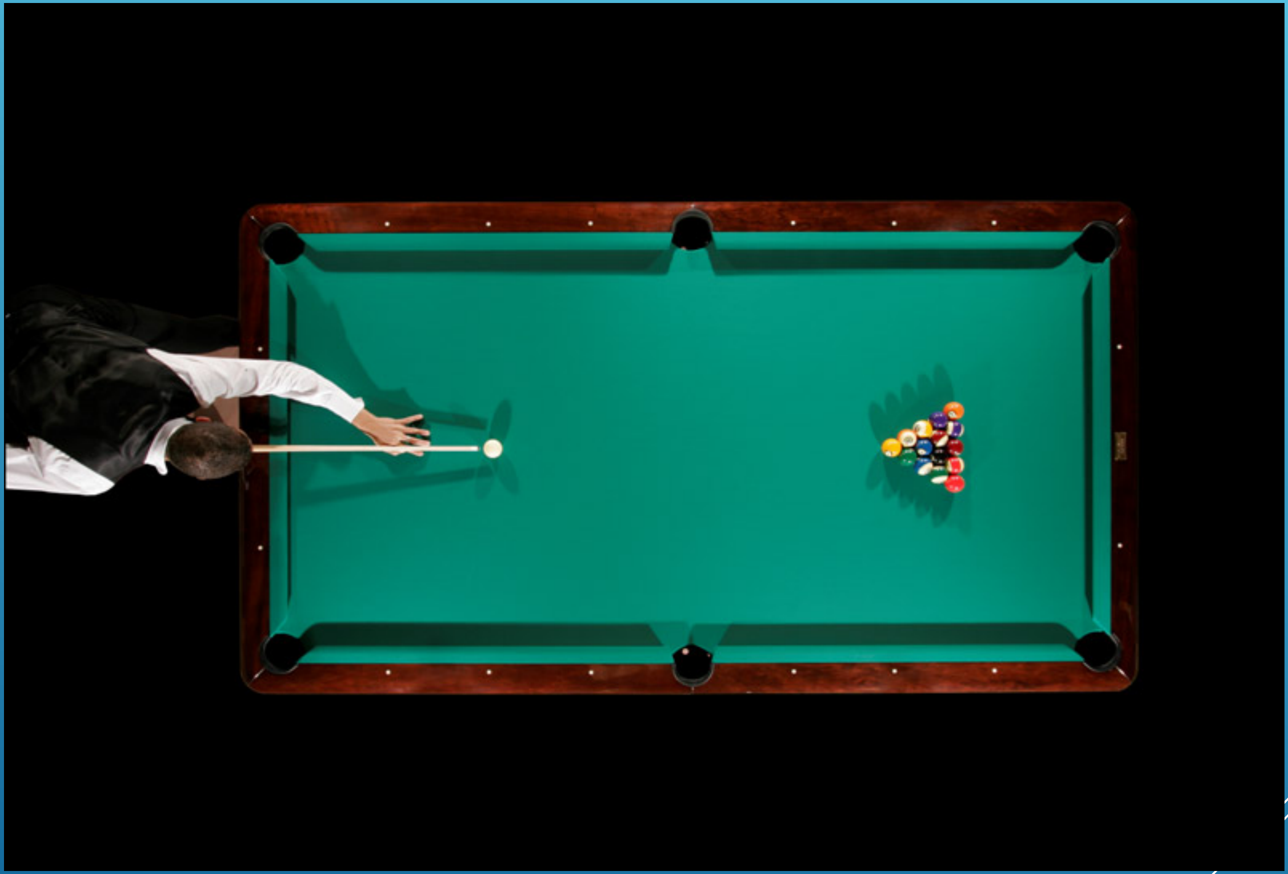
Quantum entanglement: from conclusive tests of Bell inequalities to quantum tech

Marco Genovese

I.N.RI.M.

CLASSICAL MECHANICS: OUR EVERY DAY LIFE

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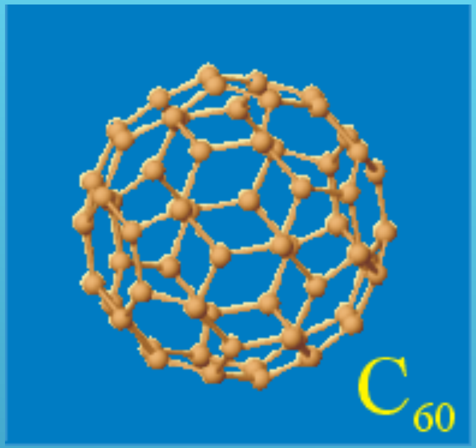
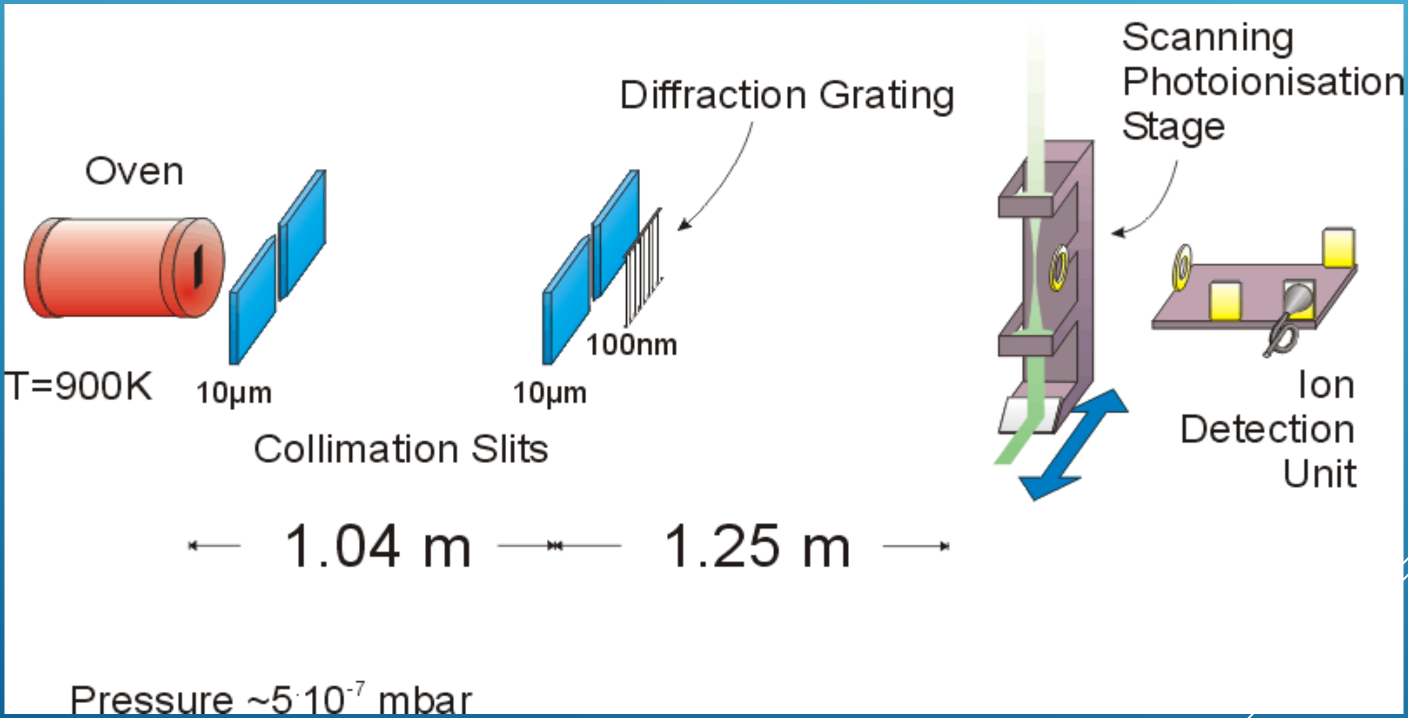
KIA
KIA MOTOR
25

Fullerene: a quantum ball





The measurement problem

The **von Neumann chain**:
(|a1>,|a2> quantum states,
|M0> initial state of detection apparatus
|M1>, |M2> final states of detection apparatus)

|a1> |M0> → |a1> |M1>
|a2>|M0> → |a2>|M2>

QM is linear:

(a |a1> + b |a2>) |M0> → a |a1> |M1> + b |a2> |M2>

The detection apparatus is entangled as well!!

The sad story of Schrödinger cat

“About your cat, Mr. Schrödinger—I have good news and bad news.”

Schrödinger cat paradox

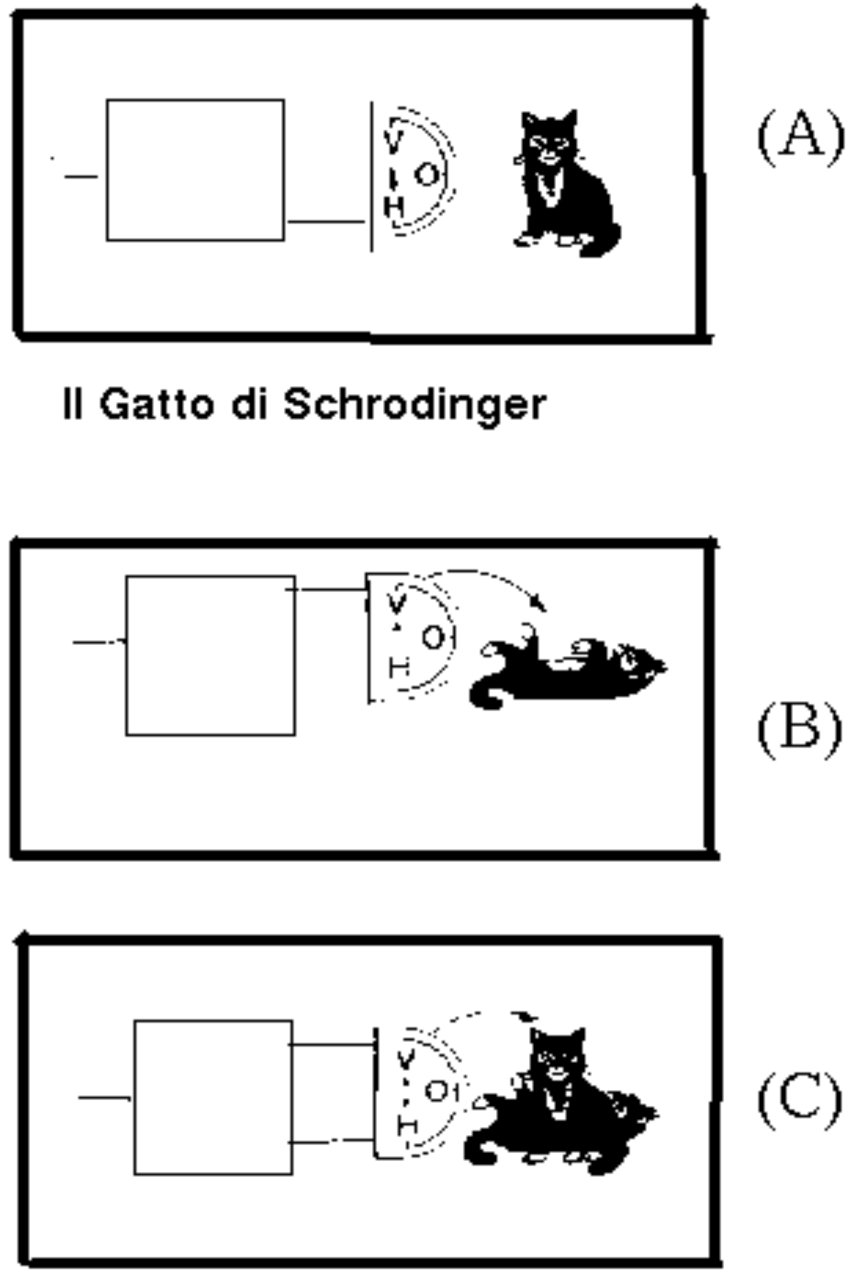
$(a\,|H\rangle + b\,|V\rangle)\ \ |M0\rangle\,|cat\rangle$

$\rightarrow$

$(a\,|H\rangle\,|M1\rangle +$
$b\,|V\rangle|M2\rangle)\qquad|cat\rangle$

$\rightarrow$

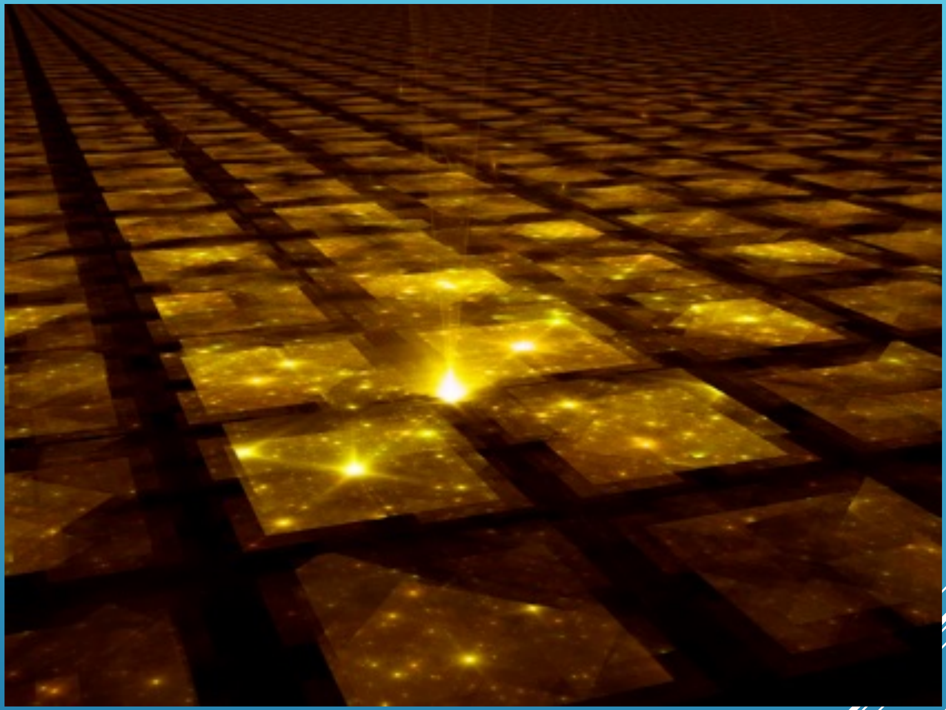
$a\,|H\rangle\,|M1\rangle|cat\ alive\rangle +$
$b\,|V\rangle|M2\rangle|dead\ cat\rangle$




Shhh, non dire un solo "miao"

Alternatives:

- *Hidden variable Theories*
- *Collapse models*
- *Many worlds*
- *[Qubism (?)]*



Spontaneous localization models

Ghirardi – Rimini- Weber

$$\Psi(x_1, ..., x_N) j(x - x_i)/R$$

$$j(x - x_i) = A \exp[-(x - x_i)^2/(2a)^2]$$

$$|R(x)|^2 = \int dx_1 ... dx_N |\Psi(x_1, ..., x_N) j(x - x_i)|^2$$

a ~ 10^{-7} m , rate ~ 10^{-17} s^{-1}

Possible experimental tests:

Upper bounds on λ

Laboratory experiments	Decades above the conventional value
Fullerene diffraction experiments	13
Decay of supercurrents	14
Spontaneous x-ray emission from Ge	6
Proton decay	18
Mirror cantilever interferometric experiment	9
Cosmological data	**Decades above the conventional value**
Dissociation of cosmic hydrogen	17
Heating of intergalactic medium (IGM)	8
Heating of interstellar dust grains	15

From S. Adler and A. Bassi Science 325 (2009) 275

Is QM a complete theory?

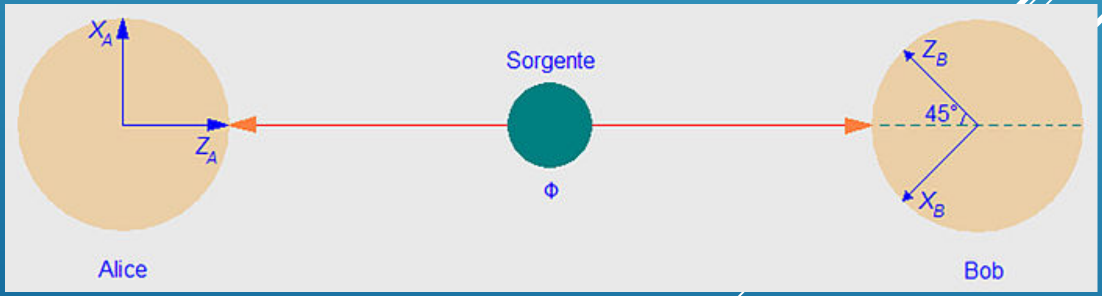
1935 **Einstein, Podolsky, Rosen** posed the question if is Quantum Mechanics a complete theory ?.

i) Element of Physical reality : If we can predict with certainty the value of an observable without disturbing the system

ii) No action at distance

$$\frac{|H\rangle\,|V\rangle - |V\rangle\,|H\rangle}{\sqrt{2}}$$

$$\frac{|45\rangle\,|-45\rangle - |-45\rangle\,|45\rangle}{\sqrt{2}}$$



Quantum Non Locality compatible with special relativity

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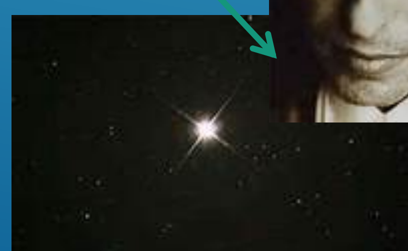
$$\frac{|H\rangle\,|V\rangle - |V\rangle\,|H\rangle}{\sqrt{2}}$$



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In SMQ no superluminal communication [Ghirardi,Rimini,Weber LNC 27 (80) 293.]

$$W_{12} \to W'_{12} = \sum_s P_s^1 W_{12} P_s^1$$

The reduced trace is:

$$W_2 = Tr_1[W'_{12}] = Tr_1[\sum_k P_k^1 W_{12} P_k^1] = \sum_k Tr_1[P_k^1 W_{12} P_k^1]$$

By using trace properties:

$$W_2 = \sum_k Tr_1[P_k^1 W_{12} P_k^1] = \sum_k Tr_1[P_k^1 W_{12}] = Tr_1[\sum_k P_k^1 W_{12}] = Tr_1[W_{12}]$$

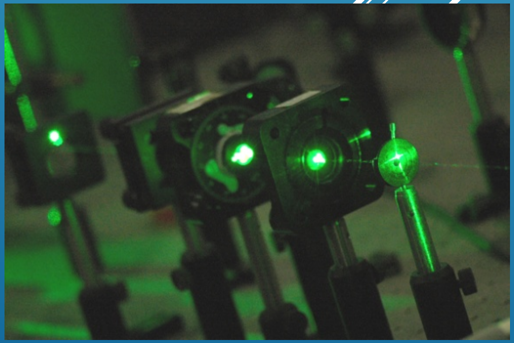
That is exactly the same reduced density operator we would have obtained without any measurement

1964 Bell Theorem

entangled states

Bell inequalities

Possible testing every Local Hidden Variables Theory against **Standard QM.**



Example : the CH inequality

$$P(\theta_1,\theta_2)+P(\theta'_1,\theta'_2)+P(\theta'_1,\theta_2)-$$
$$-P(\theta_1,\theta'_2)-P(\theta'_1)-P(\theta_2)\leq 0$$

$P(\theta_i)=$ **Probability of finding a single particle in detector** i **with a certain property** θ_i **(e.g. spin/polarization direction with respect to a selected axis);**

$P(\theta_i,\theta_j)=$ **Joint probability of observing both one particle in** i **with a property** θ_i **and the other in** j **with** θ_j **.**

In a Local HVT:

$$P(\theta_i) = \int P(\theta_i, x)\rho(x)dx$$

$$P(\theta_i,\theta_j) = \int P(\theta_i,\theta_j,x)\rho(x)dx = \int P(\theta_i,x)\cdot P(\theta_j,x)\rho(x)dx$$

Consider 4 real variables $x, x', y, y' \in [0,1]$

$$xy + x'y' + x'y - xy' - x' - y \leq 0 \qquad ??$$

$$x \geq x' \quad \Rightarrow \quad y(x-1) + x'(y-1) + y'(x'-x) \leq 0$$

$$x \leq x' \quad \Rightarrow \quad y(x'-1) + x'(y'-1) + x(y-y') \leq x'(y'-1) + x(y-1) \leq 0$$

$$\Downarrow$$

$P(\theta_i), P(\theta_i,\theta_j) \in [0,1] \quad \Rightarrow$ ***The CH inequality holds*** but

CH > 0

For certain values of parameters in SQM

Various other inequalities, (Bell 1 & 2, CHSH, …)

Many experimental tests :

A. Aspect et al., PRL. 49 (1982) 1804; J. P. Franson, PRL 62 (1989) 2205; J. G. Rarity and P. R. Tapster, PRL 64 (1990) 2495; J. Brendel et al., EPL 20 (1992) 275; P. G. Kwiat et al., PRA 41 (1990) 2910; T.E. Kiess et al., PRL 71 (1993) 3893; P.G. Kwiat et al., PRL 75 (1995) 4337; …

All confirm SQM: but low detection efficiency ⇒

Detection loophole

In 70's experiments with **cascade atomic decay**

82 Orsay experiment *[A. Aspect et al., PRL. 49 (1982) 1804]*

Entangled photons from J=0 → J=1 → J=0 Calcium 40 decays

Addressed to detectors separated of 6 m

Space-like separation through acousto-optic switches

$$CH = 0.101 \pm 0.020$$

Very low detection efficiency
(e.g. 40 coincidence per second against typical production rate of 10^7 pairs per second)

Other systems?

i) **Ions:** Experiment with Berillum ions
High efficiency (98%), but subsystems are not separated during measurement *(Rowe et al., Nature 409 (01) 791)*
Improvement more recently: 1 m [Monroe et al., qph 0801.2184]

One needs many km (detection time around 50 μs)

ii) **Neutrons** *[Rauch]*

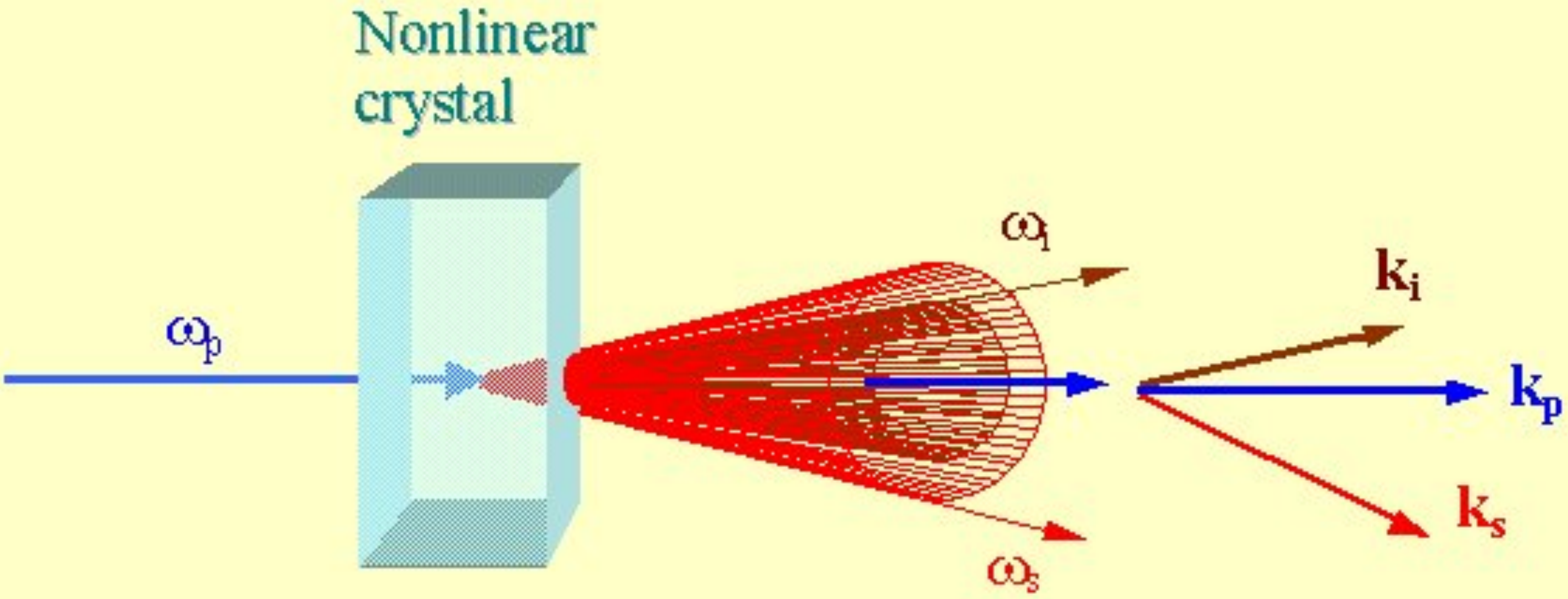
iii) **Mesons (K,B)** *[Foadi,Selleri PRA 61 (99) 012106-1,EPJ C14 (00) 469; Di Domenico NP B 450 (95) 293;Bramon,Garbarino, PRL 89 (02) 160401,Hiesmayr Fpl 14 (01)231]*

$$|\Psi\rangle = \frac{|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle}{\sqrt{2}} = \frac{|K_L\rangle|K_S\rangle - |K_S\rangle|K_L\rangle}{\sqrt{2}}$$

Some violation of Bell inequalities osserved by Belle [A.Go, JMO 51 (04) 991]

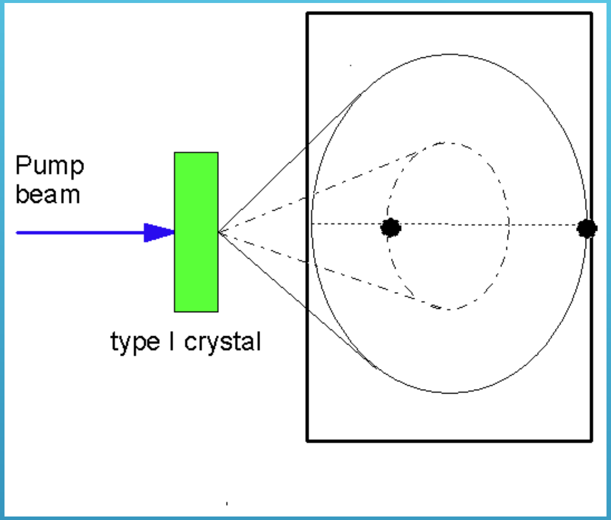
detection loophole reappears as HV can also determine
a) decay channel *[M.G. et al., PLB 513 (01) 401, FP 32 (02) 589]*
b) time of decay *[MG, PRA 69 (04) 022103]*

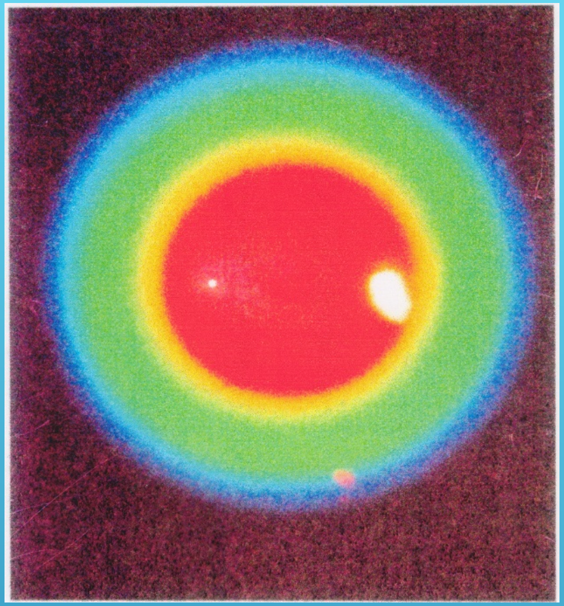
Parametric Down Conversion



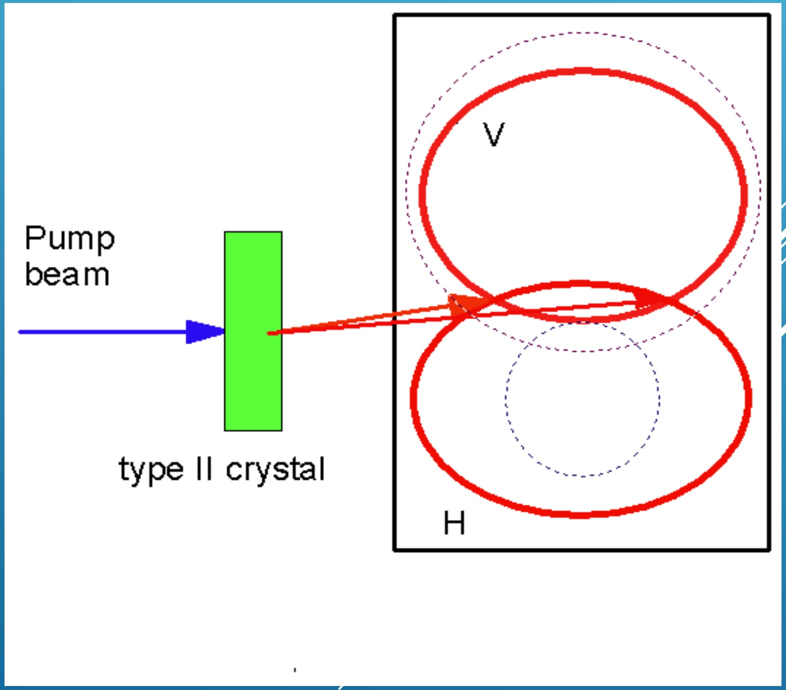
- Energy conservation: $\omega_p = \omega_s + \omega_i$
- Momentum conservation: $\vec{k}_p = \vec{k}_s + \vec{k}_i$
- ω_s and ω_i are emitted **at the same time**

type I PDC

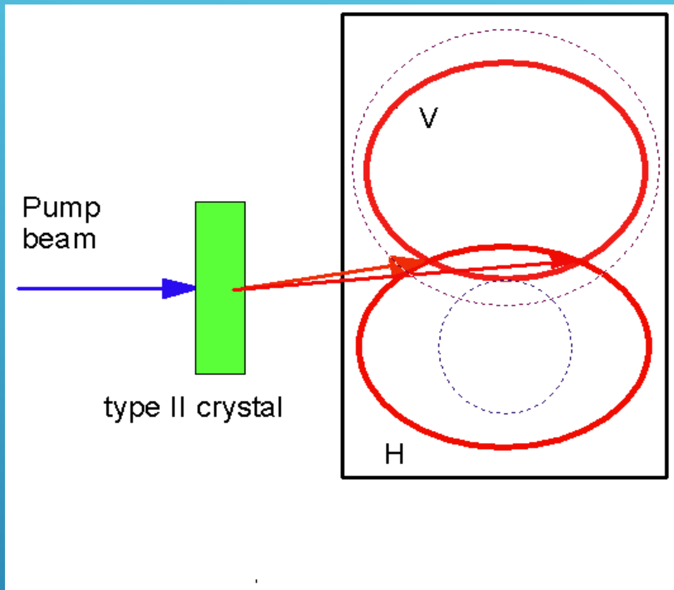




type II PDC



Brilliant sources:

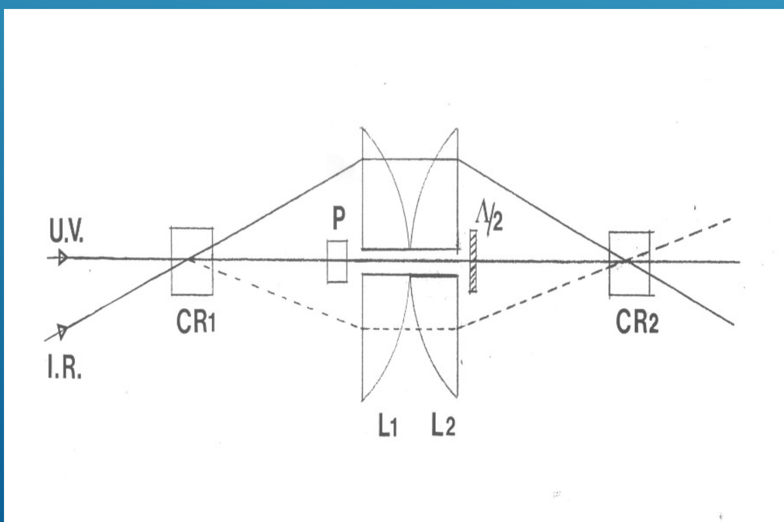


Type II PDC

$$\frac{[(|H>|V>+|V>|H>)]}{\sqrt{(2)}}$$

Th: A. Garuccio
EXP: Zeilinger,
Sergienko, Kwiat et al.PRL 75 (95) 4337

102 standard deviations violation of CHSH ineq. [P. Kwiat et al.,]

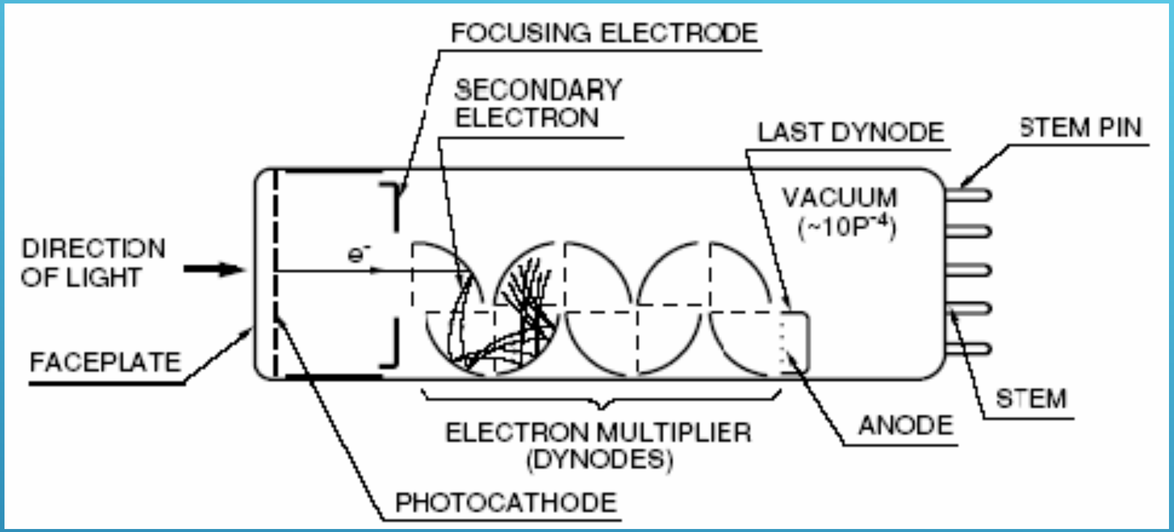


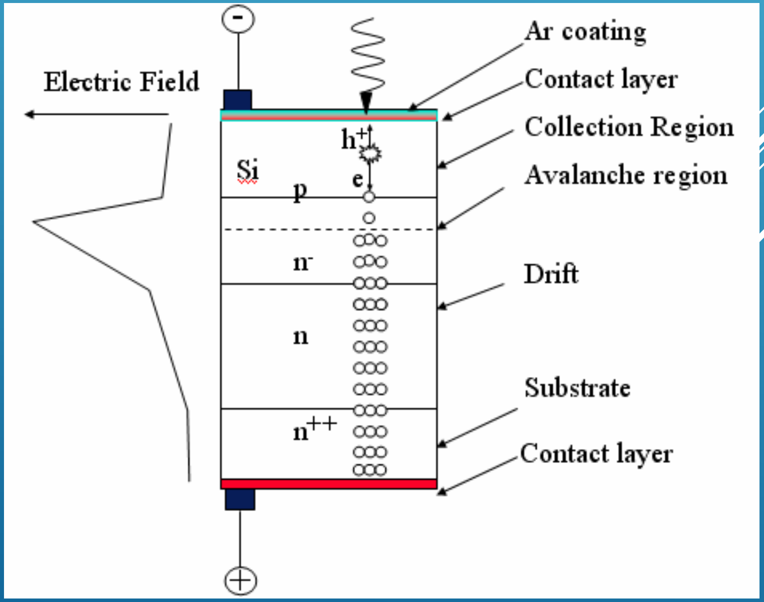
$$\frac{[(|H>|H>+f|V>|V>)]}{\sqrt{(1+|f|^2)}}$$

Two type I PDC
Th: Hardy
Exp: P. Kwiat et al., PRL 83 (99) 3103
G. Brida, M.G., C. Novero and E. Predazzi, PLA 268 (2000) 12

CH = 513 ± 25

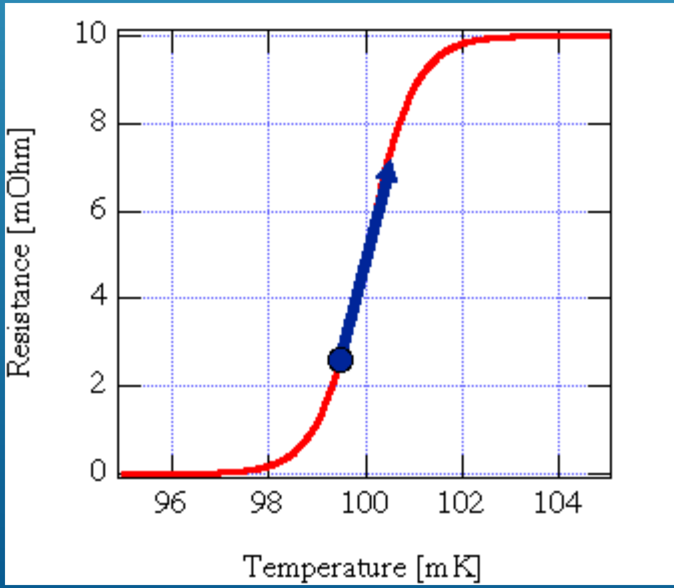
PHOTODETECTORS:DETECTION LOOPHOLE





TES

A transition-edge sensor is a thermometer made from a superconducting film operated near its transition temperature Tc.



Bell violation using entangled photons without the fair-sampling assumption

Marissa Giustina[1,2*], Alexandra Mech[1,2*], Sven Ramelow[1,2*], Bernhard Wittmann[1,2*], Johannes Kofler[1,3], Jörn Beyer[4], Adriana Lita[5], Brice Calkins[5], Thomas Gerrits[5], Sae Woo Nam[5], Rupert Ursin[1] & Anton Zeilinger[1,2]

PRL **111,** 130406 (2013) PHYSICAL REVIEW LETTERS week ending 27 SEPTEMBER 2013

Detection-Loophole-Free Test of Quantum Nonlocality, and Applications

B. G. Christensen,[1,*] K. T. McCusker,[1] J. B. Altepeter,[1] B. Calkins,[2] T. Gerrits,[2] A. E. Lita,[2] A. Miller,[2,3] L. K. Shalm,[2] Y. Zhang,[2,4] S. W. Nam,[2] N. Brunner,[5,6] C. C. W. Lim,[7] N. Gisin,[7] and P. G. Kwiat[1]

Experimental loophole-free violation of a Bell inequality using entangled electron spins separated by 1.3 km

B. Hensen,[1,2] H. Bernien,[1,2,*] A.E. Dréau,[1,2] A. Reiserer,[1,2] N. Kalb,[1,2] M.S. Blok,[1,2] J. Ruitenberg,[1,2] R.F.L. Vermeulen,[1,2] R.N. Schouten,[1,2] C. Abellán,[3] W. Amaya,[3] V. Pruneri,[3] M.W. Mitchell,[3,4] M. Markham,[5] D.J. Twitchen,[5] D. Elkouss,[1] S. Wehner,[1] T.H. Taminiau,[1,2] and R. Hanson[1,2,†]

- The two measurements must be set independently (locality loophole).
- The choice of the setting must be truly random (freedom-of-choice loophole)
- One should be able to detect all the pairs involved in the experiment or, at least, a sufficiently large fraction of them (detection loophole).

Furthermore:

- the number of emitted particle must be independent by measurement settings (production rate loophole)
- the presence of a coincidence window must not allow in a hidden variable scheme a situation where local setting may change the time at which the local event happens (coincidence loophole)
- an eventual memory of previous measurements must be considered in the statistical analysis since the data can be not-independent and identically distributed (memory loophole).

When all these conditions are satisfied, no room is left for local realistic hidden variable theories.

- the two measurements clearly space like separated (keeping in to account delays in transmission etc.) of setting choices and measurements is done. Thus, locality loophole is overcome
- the use of high detection efficiency TES together with non-maximally entangled states (as suggested by Eberhard) allowed a detection loophole free experiment.
- Independent random number generators based on laser phase diffusion guarantee the elimination of freedom-of-choice loophole (except ,as mentioned, in presence of superdetermininsm or other hypotheses that, by definition, do not allow a test through Bell inequalities).
- A perfect random choice of settings, as realized, does not permit production rate loophole.
- The use of a pulsed source eliminates coincidence loophole.
- An involved statistical analysis does not leave room for memory loophole.

Is determinism excluded?

-Non local HVT (de Broglie Bohm theory, Nelson stocastic model, …)

- Determinism at Planck scale [t' Hooft]

A physical system can evolve deterministically at Planck scale, but a probabilistic theory can derive at larger spatial scales due to loss of information (a quantum state is defined as a class of equivalence of states all having the same future).
Nowadays Bell inequalities do not involve the rigth degrees of freedom.

[Elze, Biro', Blasone et al., …]

Let us consider a discrete system with four states $e_1; e_2; e_3; e_4$ whose deterministic evolution is after every step

$$e_1 \to e_2,\ e_2 \to e_1,\ e_3 \to e_3,\ e_4 \to e_1,$$

$$U = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

After a short lapse of time only the states $e_1; e_2; e_3$ survive. Thus one can simply erase the state e_4 and considering $e_1; e_2; e_3$ as the "quantum" system with a unitary evolution described by the upper 3x3 part of U

This system may therefore be described in three equivalence classes:

$$E_1 = \{e_1\},\ E_2 = \{e_2, e_4\},\ E_3 = \{e_3\},$$

with unitary evolution operator

$$U' = e^{-iH} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

This simple model shows how, if information is allowed to dissipate, one has to define quantum states as equivalence classes of states, where two states are equivalent if, some time in the future, they evolve into one and the same state.

Quantum Technologies

Quantum INFORMATION

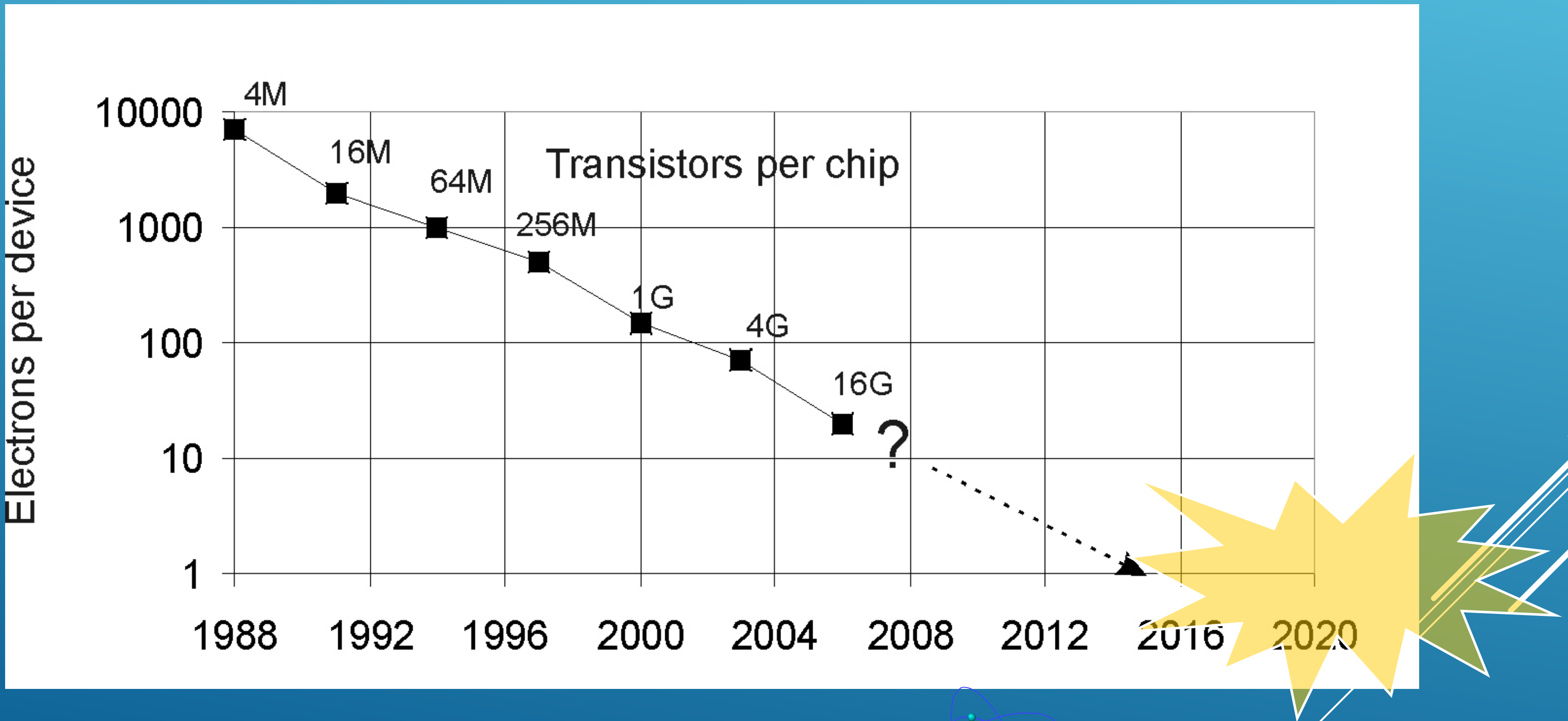
From bit (0,1), to quantum-bit (qubit) $|0>$ $|1>$

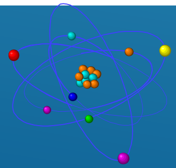
$$\mathbf{a\ |0> + b\ |1>}$$

Many particles: entanglement

$$a_1\ |0\ 0\ \ldots\ 0> + \ldots + a_N|1\ \ldots\ 1>$$

Quantum computer



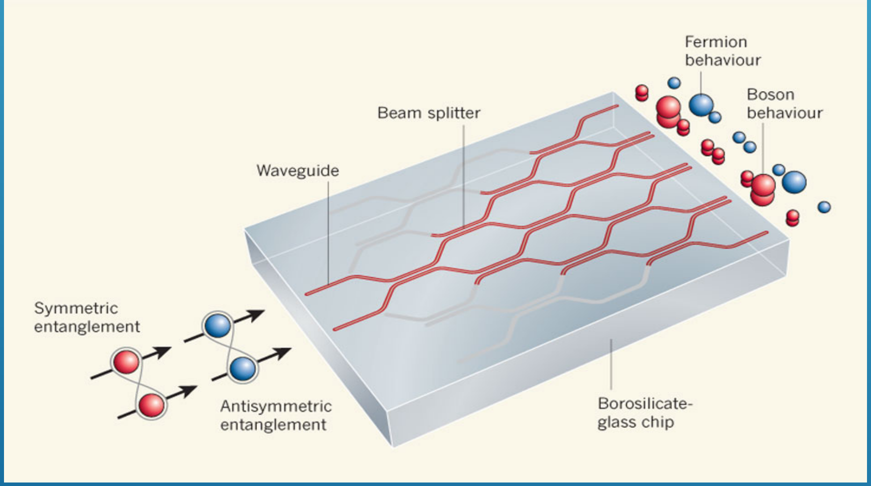


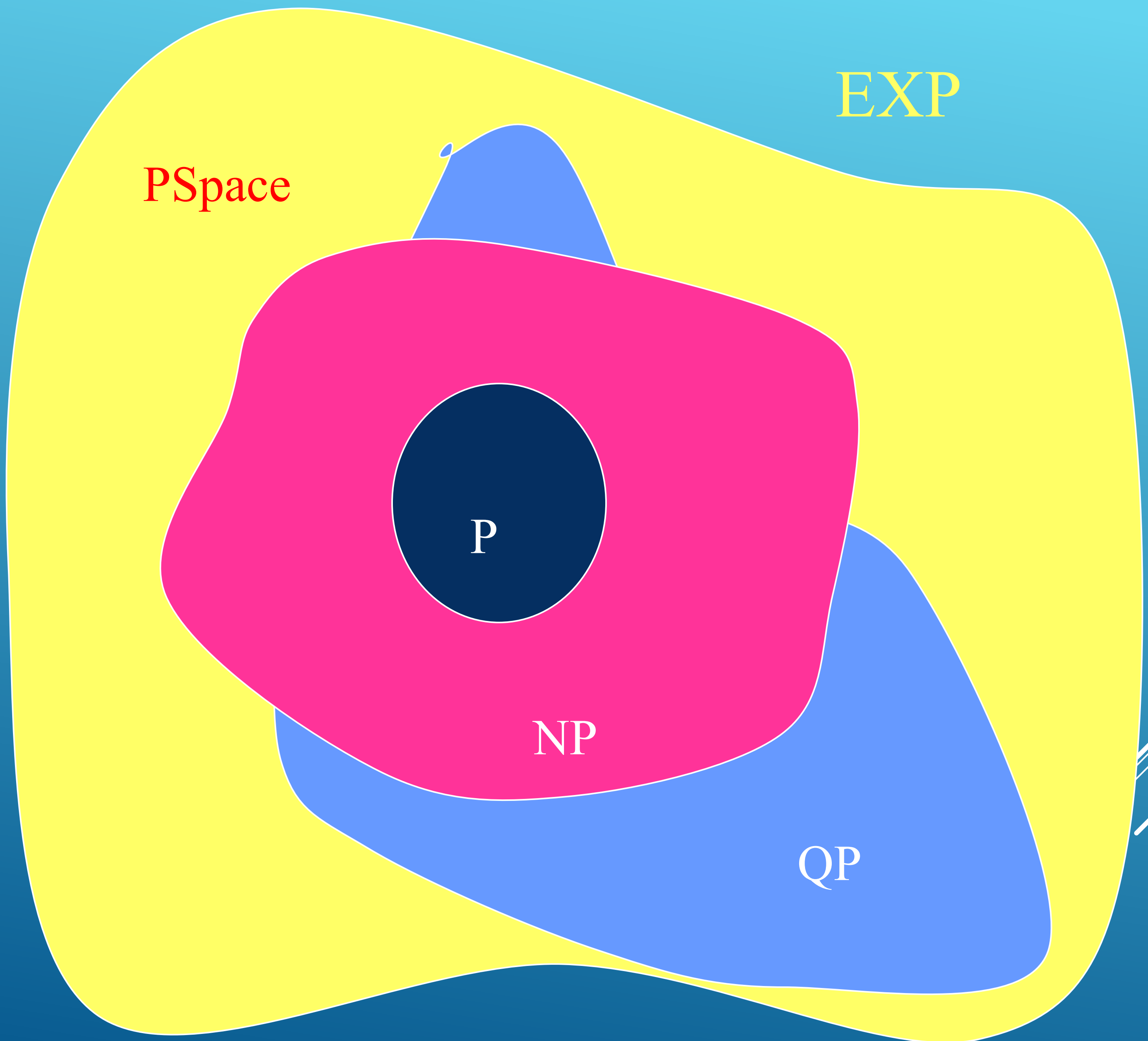
Quantum Technology

quantum parallelism

$$a_1\,|0\;0\;\ldots\;0>+\;\ldots+a_N|1\;\ldots\;1>$$

$$\mathbf{O}\,[a_1\,|\,0\;\ldots..\;0>+\,a_2\,|\,0\;\ldots\;0\;1>+\;\ldots\;a_N\,|\,1\;\ldots\;1>]=$$
$$[a_1\;\mathbf{O}|\,0\;\ldots..\;0>+\,a_2\,\mathbf{O}|\,0\;\ldots\;0\;1>+\;\ldots\;a_N\,\mathbf{O}\,|\,1\;\ldots\;1>]$$




EXP
PSpace
P
NP
QP

Factorization (100=5x5x2x2):
Is a NP problem

10 GHz processor for a 100 digits number

$10^{100/2} : 10^{10} = 10^{40}$ seconds (universe life time= 10^{18} s)

Cryptographic codes are sicure?

No with a quantum PC!!!!

ELEMENTS OF A QUANTUM COMPUTER

Different logical gates are necessary, both operating on single qubits and on more qubits (two).

An interaction among different qubits is needed for realising multi-qubits gates.

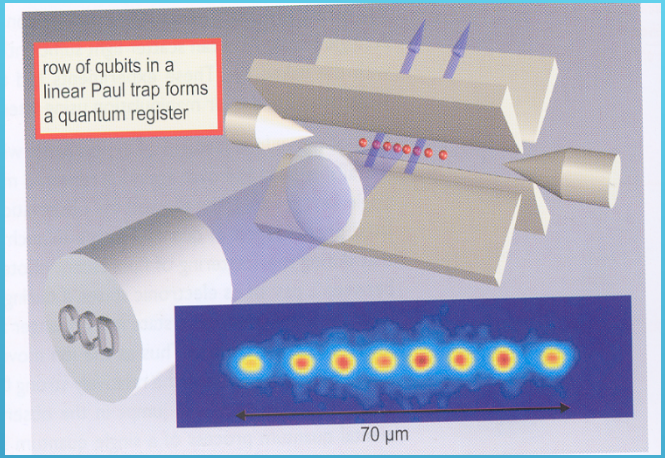
Single qubits gates + controlled not are a universal set of gates -> they allow any possible operation

Controlled not

$$|0\rangle\,|0\rangle \;\rightarrow\; |0\rangle\,|0\rangle$$
$$|0\rangle\,|1\rangle \;\rightarrow\; |0\rangle\,|1\rangle$$
$$|1\rangle\,|0\rangle \;\rightarrow\; |1\rangle\,|1\rangle$$
$$|1\rangle\,|1\rangle \;\rightarrow\; |1\rangle\,|0\rangle$$

- HOW will quantum pc be?

- Ion Trap



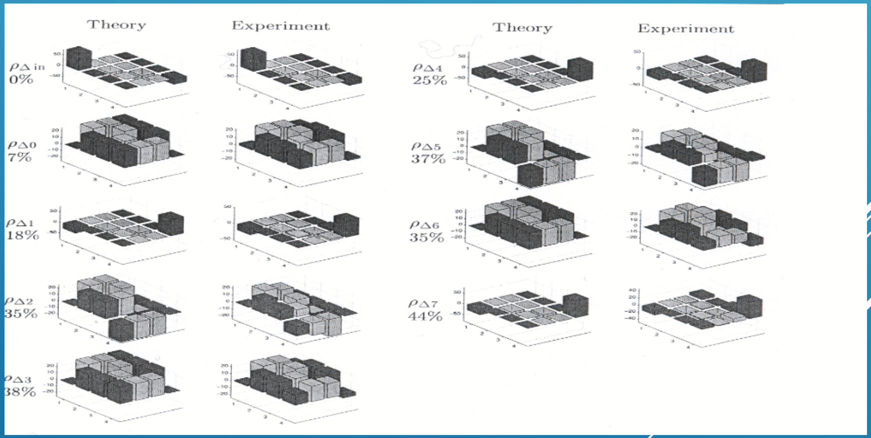
- qubit : hyperfine state, phonons

- - evoluzione: laser. Interazione con fononi

Results: C-not [Wineland et al.]

2 qubits Deutsch-Josza alghoritm

- NMR



- qubit : nucleus spin

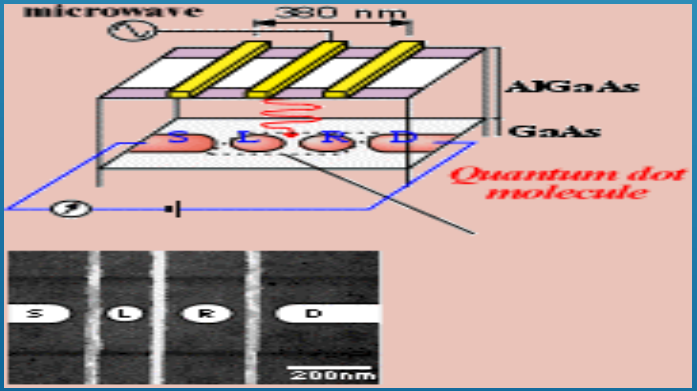
Results: Grover search algorithm 7 elements.
Factorization N=15 [Vandersypen et al., Nature (01)]

- Solid State: Quantum dots, Superconductors etc.

- qubit representation: charge
- evolution: electrostatic gates, Coulomb interaction
- problems: decoherence time?

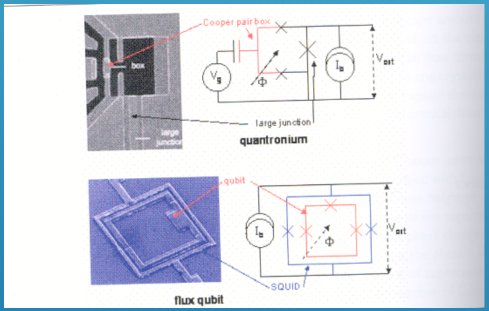
Practical realisation

Quantum Dots



Controlled Rotation (equivalent to C-not) [X.Li et al., Science (03)]

Josephson junction



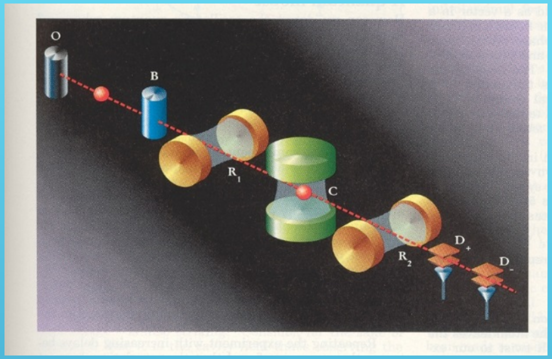
- qubit realisation [Martinis NIST (02), Ioffe, Nature (02)]

- coupling of 2 qubits [Pashkin et al., Nature (03)] – **Cnot** [Platenburg, Nature (07)]

- QED cavity

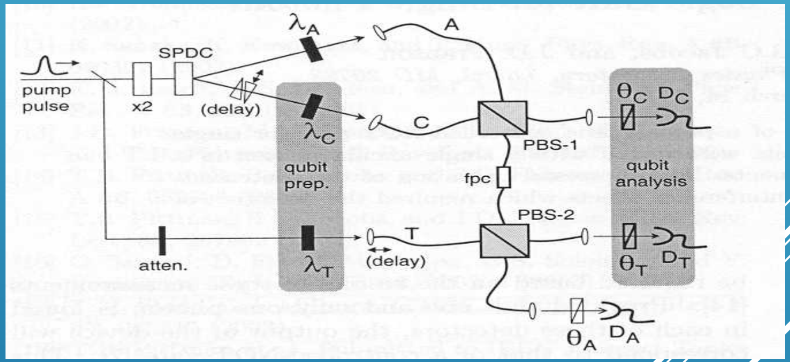
photon – atom interaction in cavity

- qubit : atomic state, EM field (micro-wave, optical)



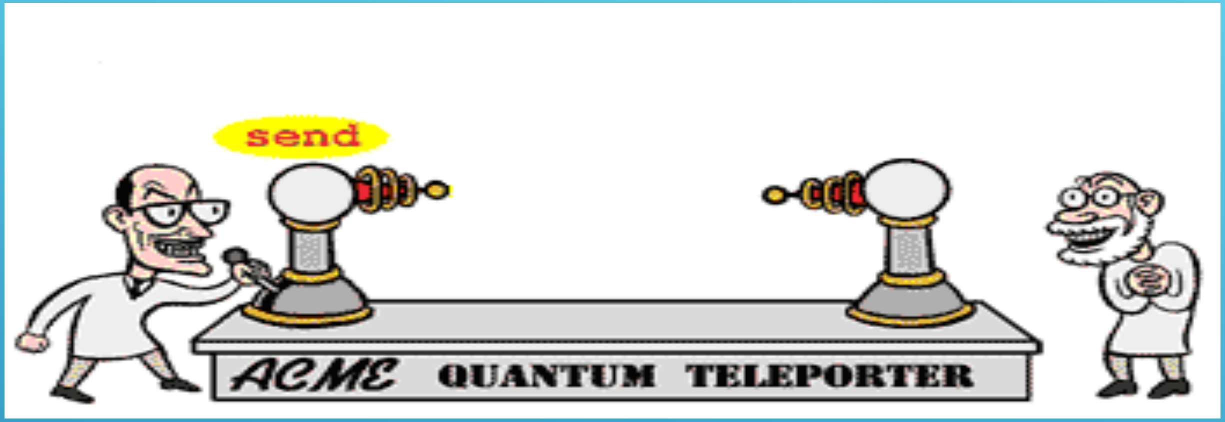
Results: conditional phase shift [Turchette et al., Haroche et al.]

- Linear optics QC

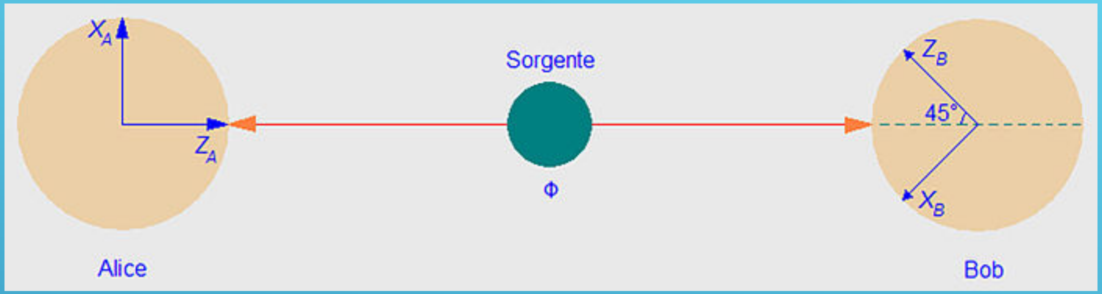


Results: Probabilistic C-not
[Pittmann et al., Phys. Rev. A 68, 032316 (2003)]

Quantum communication



QKD



key = 010001110 1110011110000

+

Message = 1110101010101010011111

→

key = 010001110 1110011110000

Whys QKD is secure?

-Single quantum state cannot be determined

a |0> + b |1>

- no quanstum xerox

a |0> + b |1>

a |0> + b |1> ->

a |0> + b |1>

………

a |0> + b |1>

- Eavesdropping adds noise

No cloning theorem

cloning arbitrary states $|\psi_1>\ |\psi_2>$

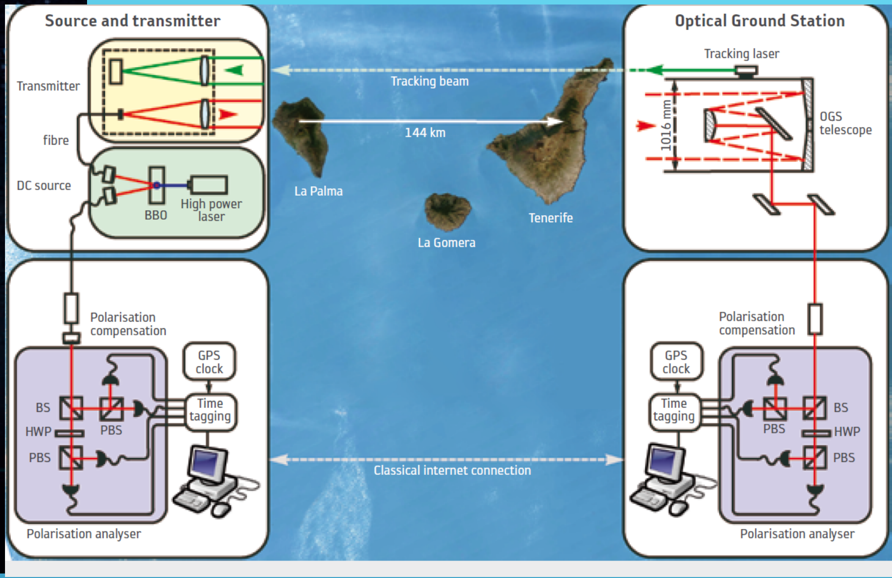
$$U|\psi_0>\ |\psi_1>\ \rightarrow |\psi_1>\ |\psi_1>$$

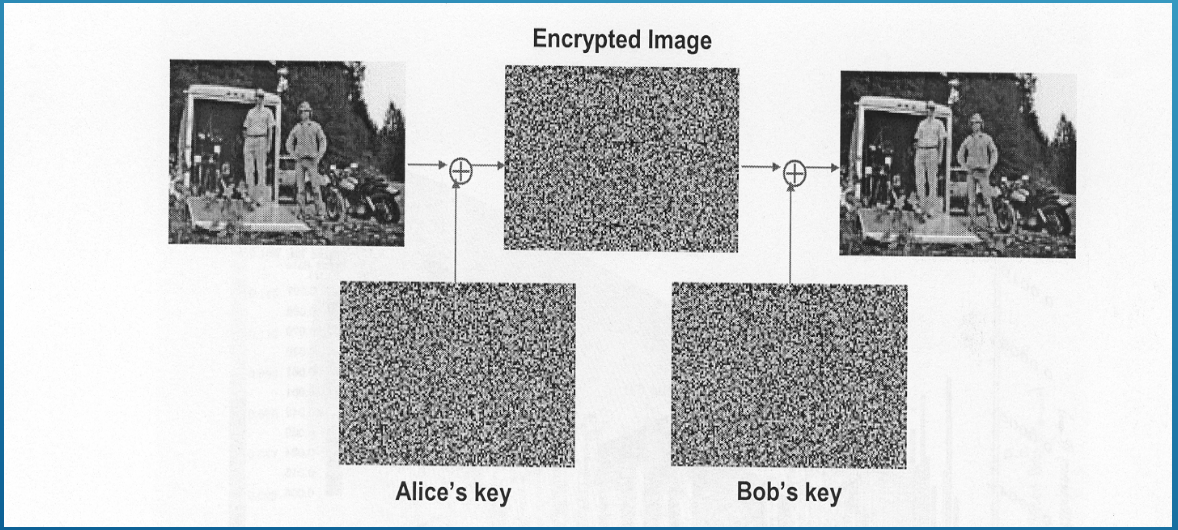
$$U|\psi_0>\ |\psi_2>\ \rightarrow |\psi_2>\ |\psi_2>$$

$$<\psi_1|\psi_2> = \ |<\psi_1|\psi_2>|^2$$

$$\rightarrow \qquad <\psi_1|\psi_2>\ = 0 \qquad \text{or} \qquad <\psi_1|\psi_2>\ = 1$$

Open space QKD





Over140 km (Tenerife-La Palma)

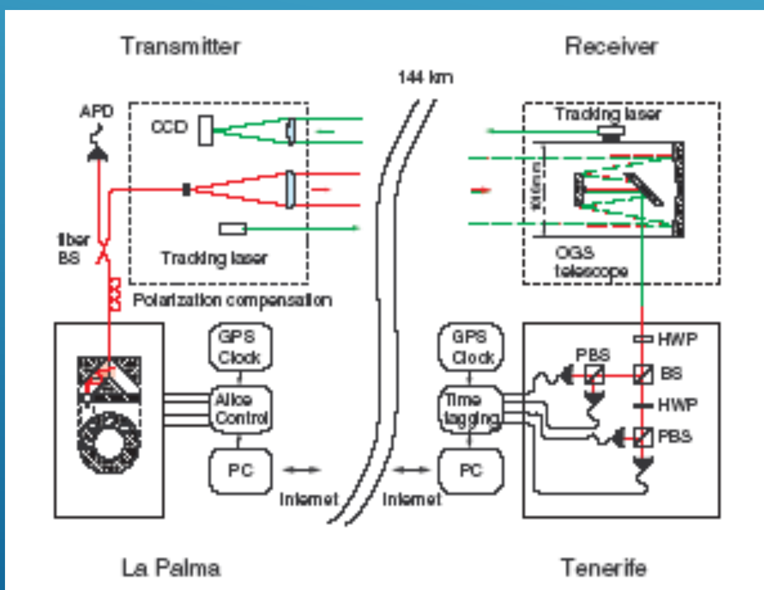
- Violation of Bell inequality, $S = 2.508 \pm 0.037$

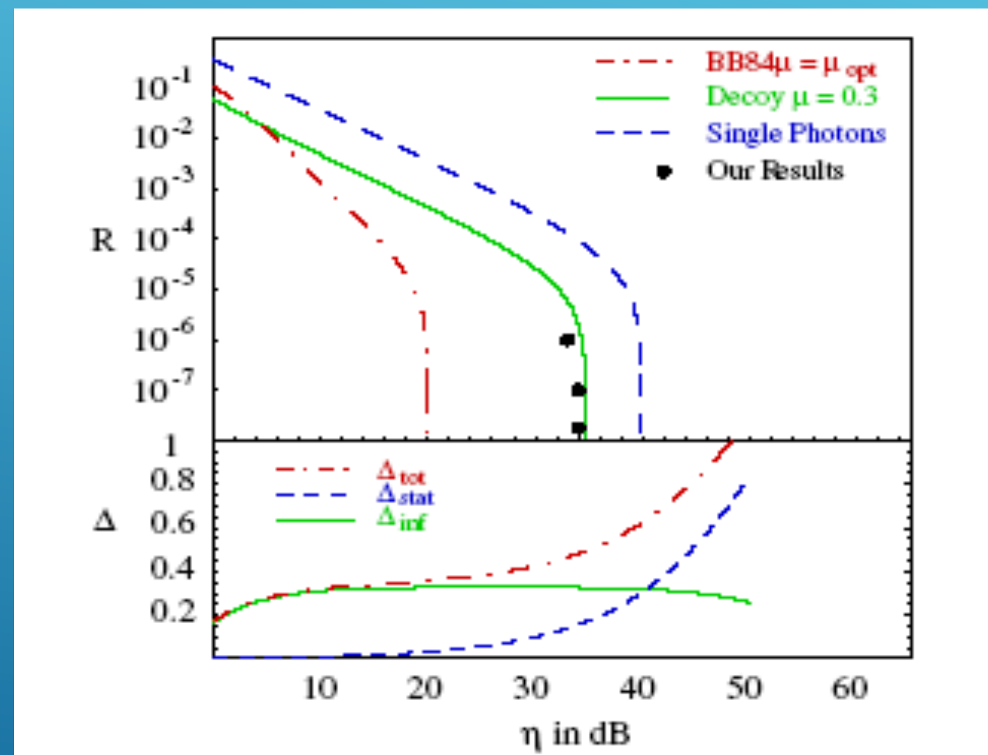
- BB84

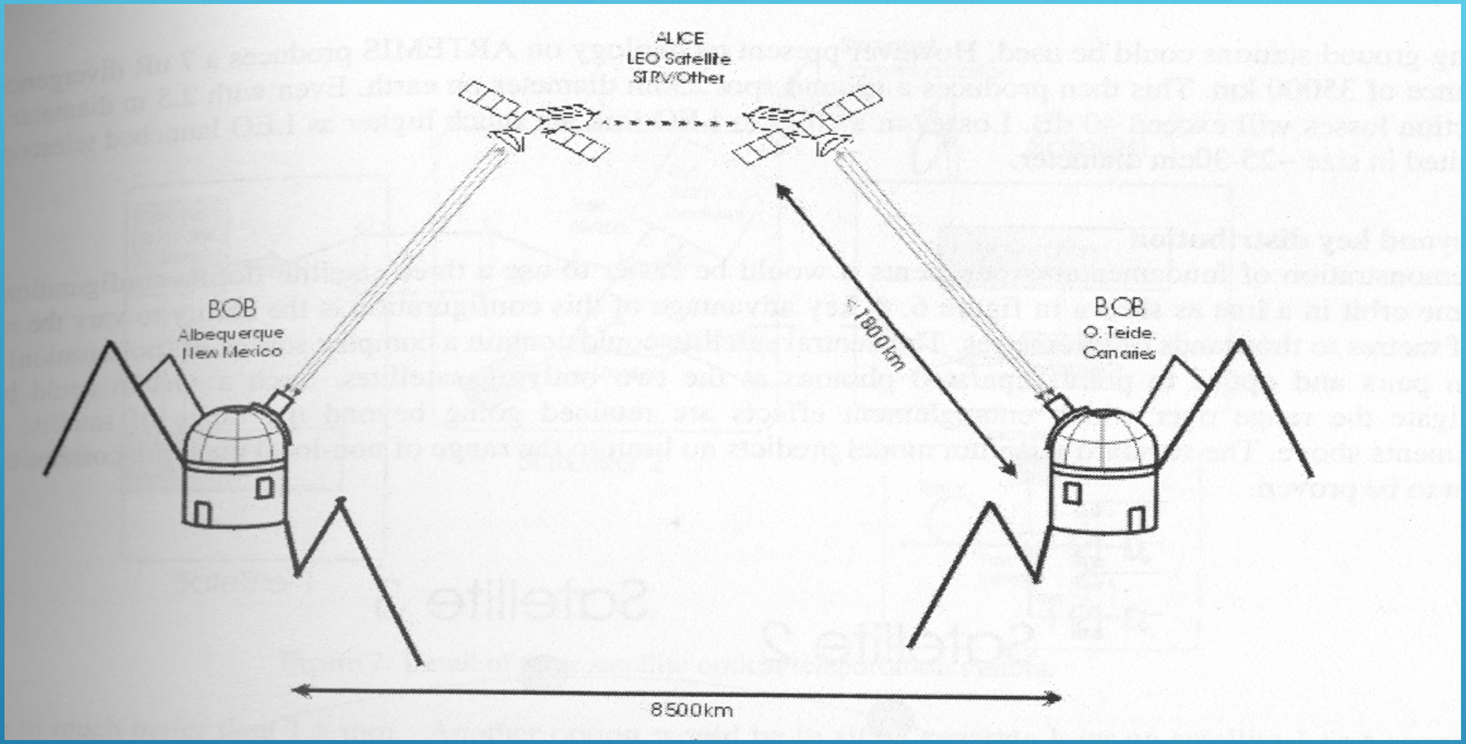
($\langle n \rangle = 0.27$ signal, $\langle n \rangle=0.39$ decoy states)

Rate: 28 bit/s

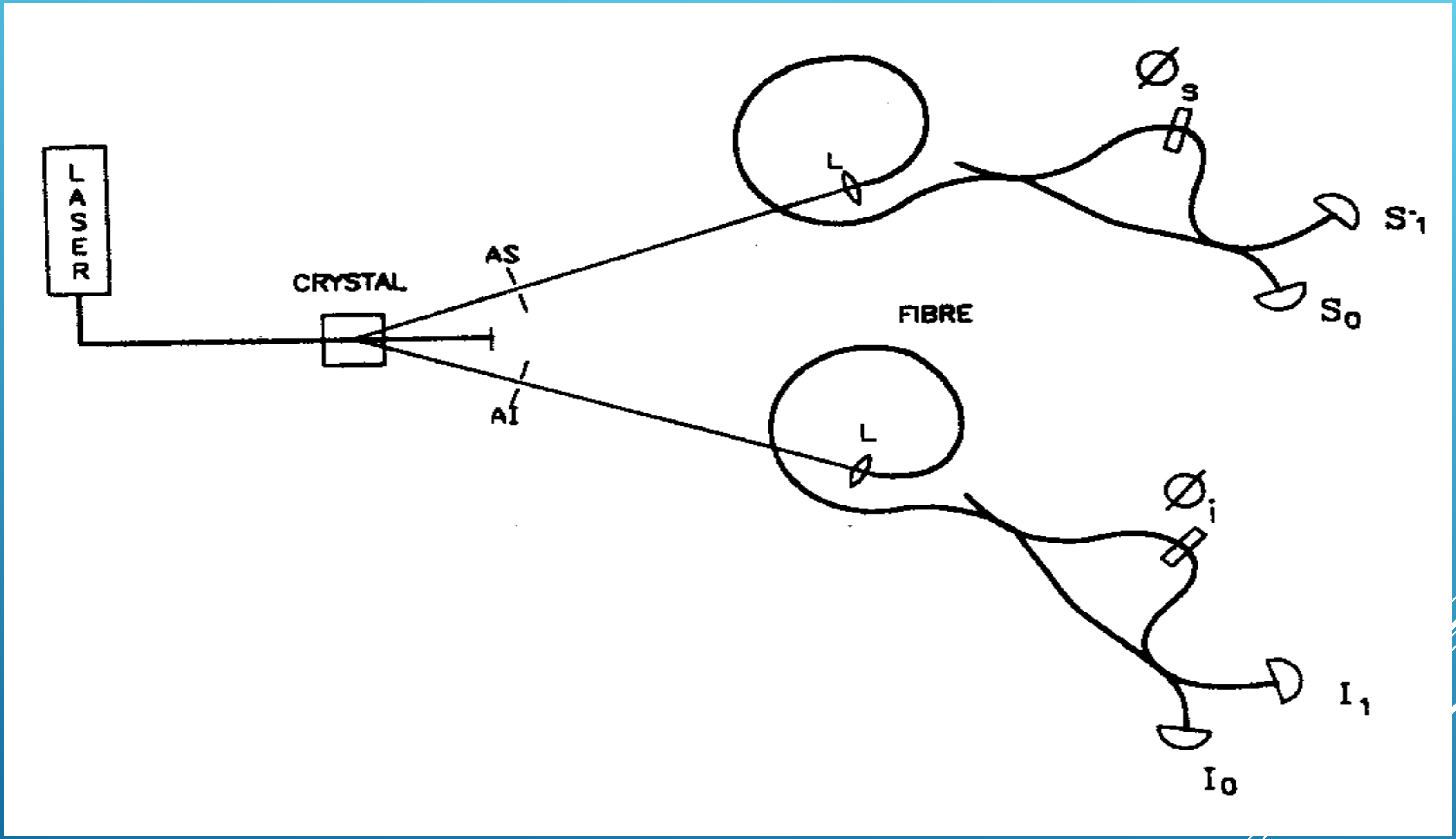
QBER = 6.77 %






ALICE
LEO Satellite
STRV/Other
BOB
Albequerque
New Mexico
BOB
O. Teide.
Canaries
1800km
8500km

Fiber communication



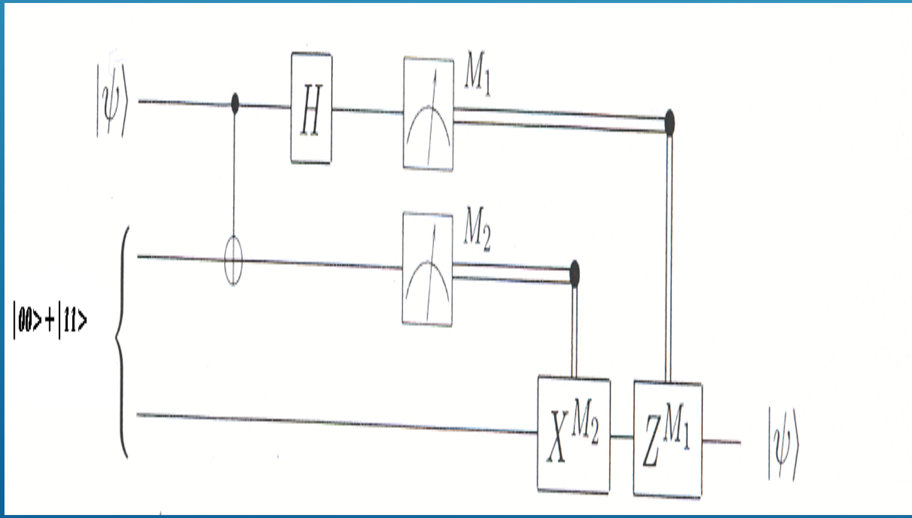
Over 200 km in telecom fibre

- Some groups are selling plug and play QKD systems.

Id Quantique

Teleportation

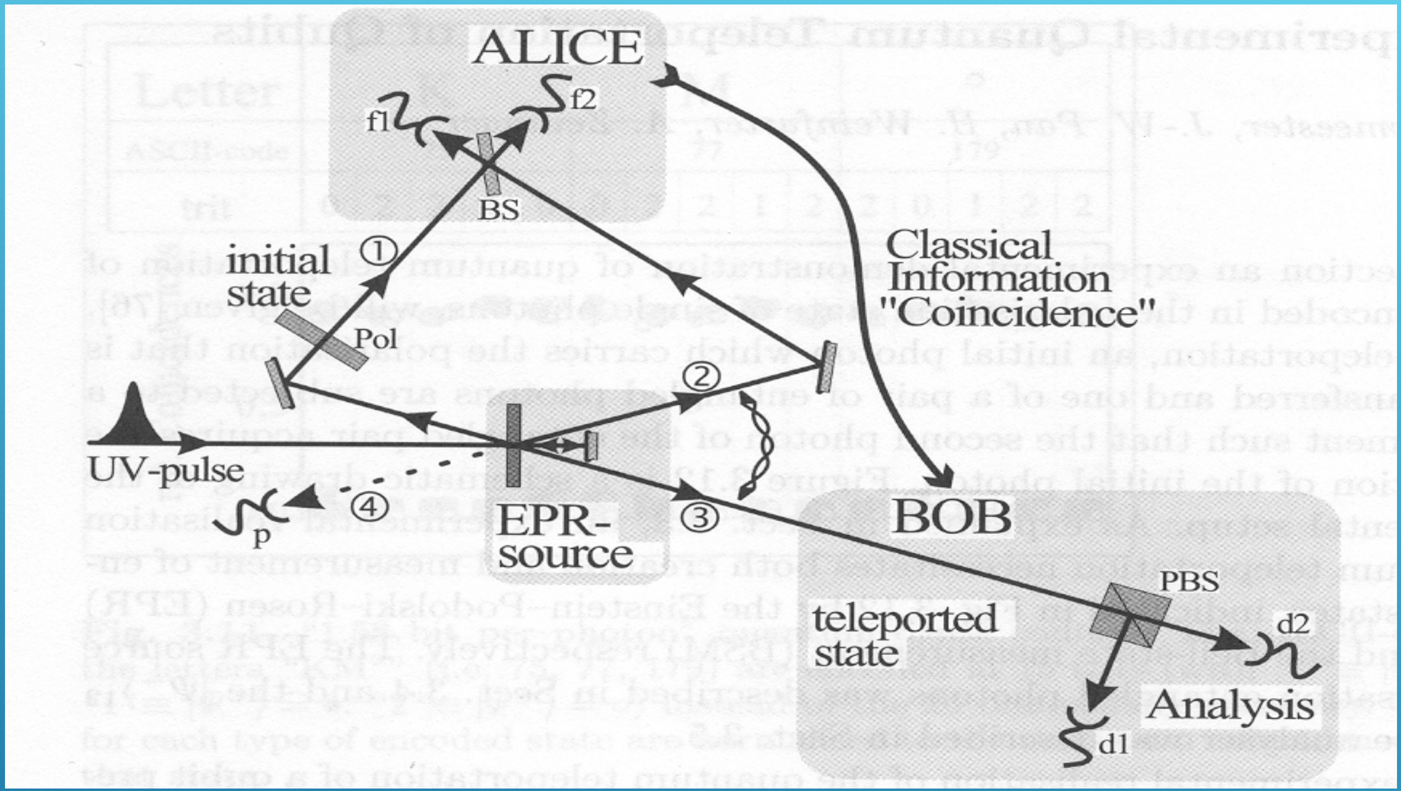
Teleportation is a protocol where an unknown state is measured in a laboratory (Alice) together with a member of an entangled state; then, by applying a unitary operation on the other member of the entangled state according to the result of this measurement (communicated by a classical channel) it is reconstructed in the second lab

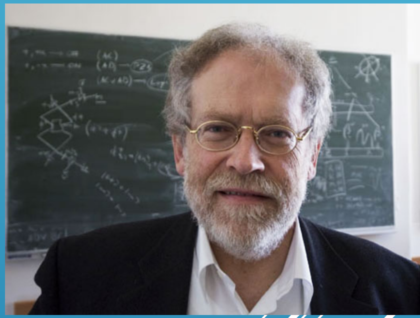


a |0> + b |1> (|00> + |11>) ->

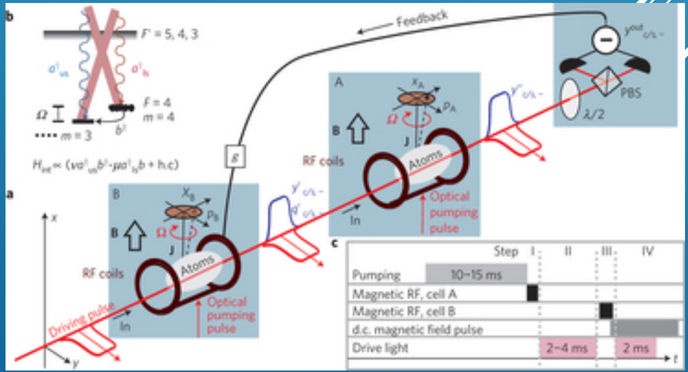
a|000 > + a |011> + b |110> + b |101> ->

½ (|00 > [a |0> + b |1>] +
|01> [a |1> + b |0>] +
|10> [a |0> - b |1>] +
|11> [a |1> - b |0>])

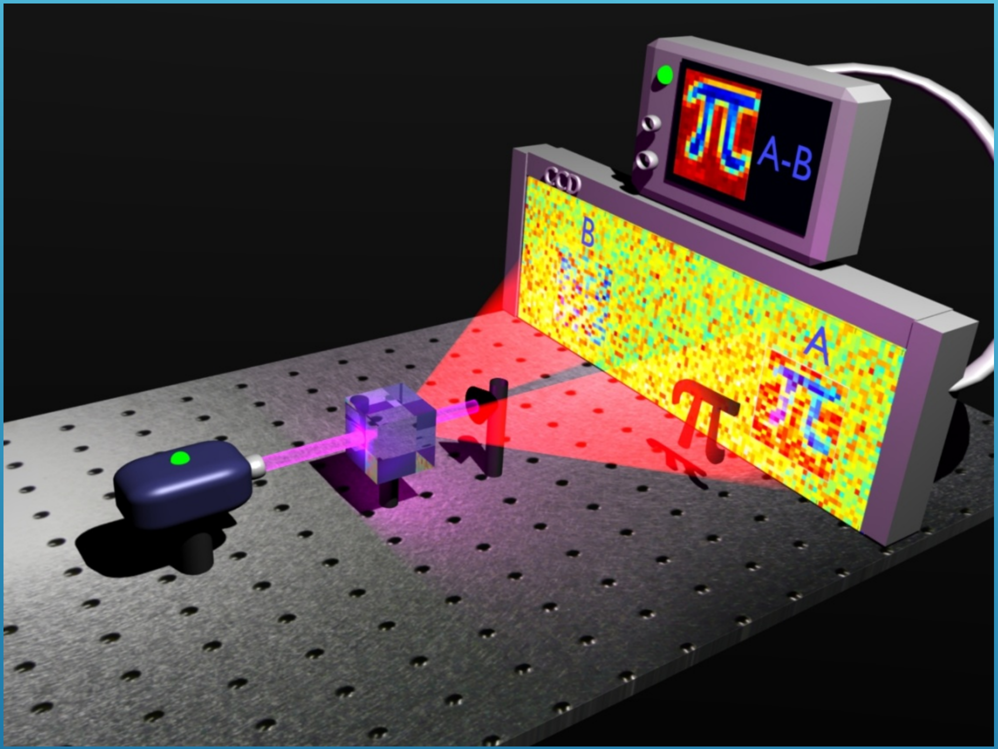


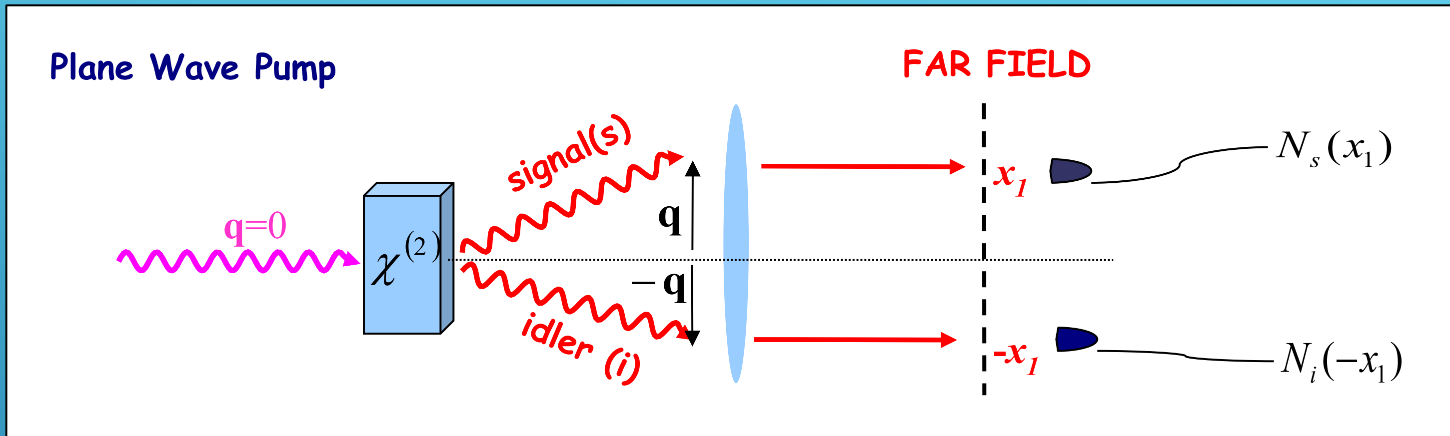


10^12 cold atoms



Quantum Imaging

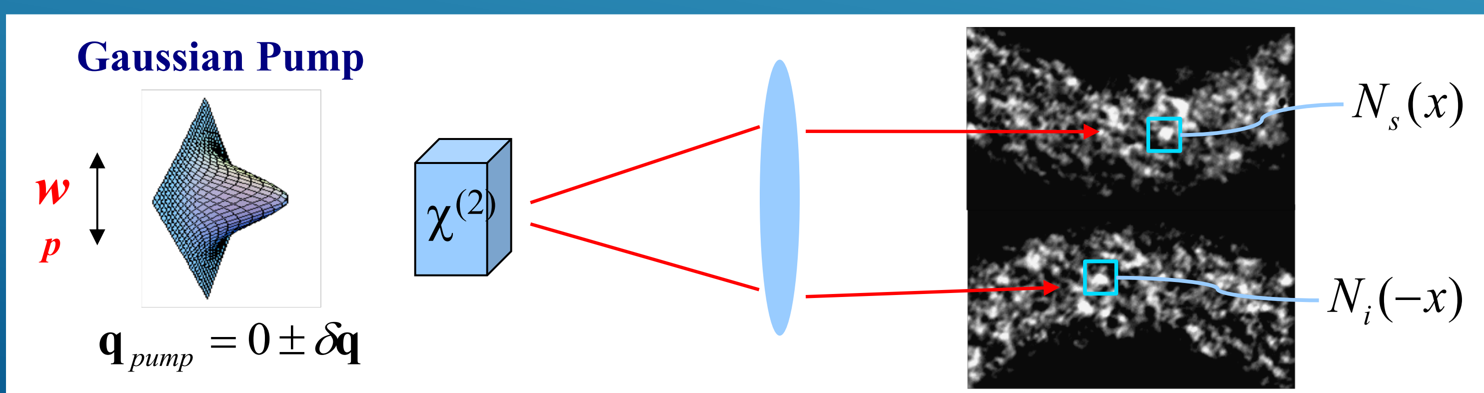




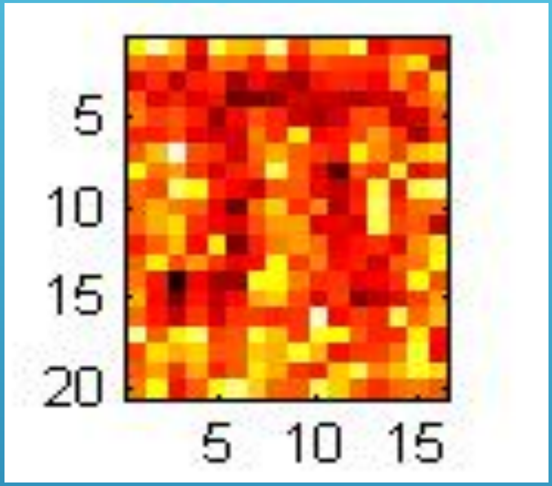
Two-Mode Entangled State
(squeezed vacuum)

Two-Mode Photon
number correlation

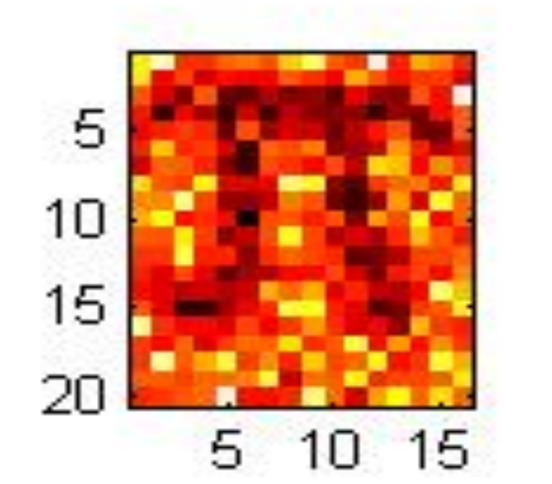
$$\left|\psi_{\mathbf{q}}\right\rangle = \exp(-z a_1 a_2 + h.c)\left|0\right\rangle = \sum_{n=0}^{\infty} c_n(\mathbf{q})\left|n_{\mathbf{q}}\right\rangle\left|n_{-\mathbf{q}}\right\rangle \Rightarrow N_s(\mathbf{x}_1) = N_i(-\mathbf{x}_1)$$

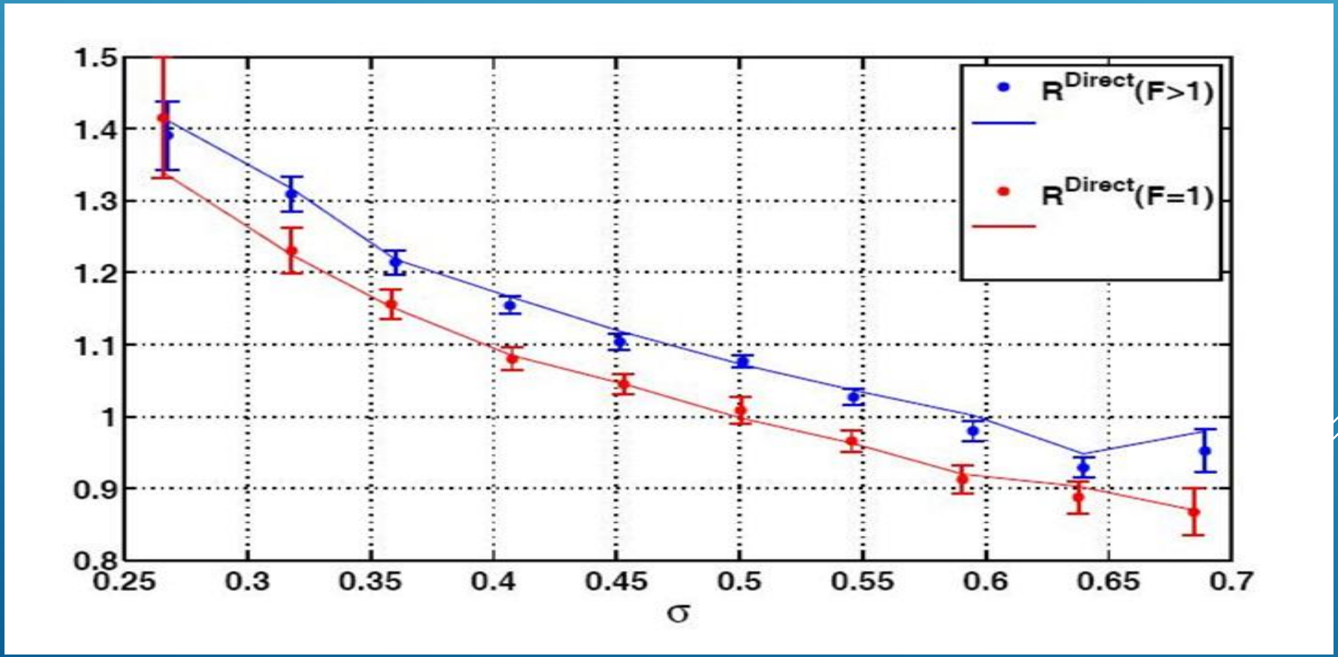


Classical differential measurement

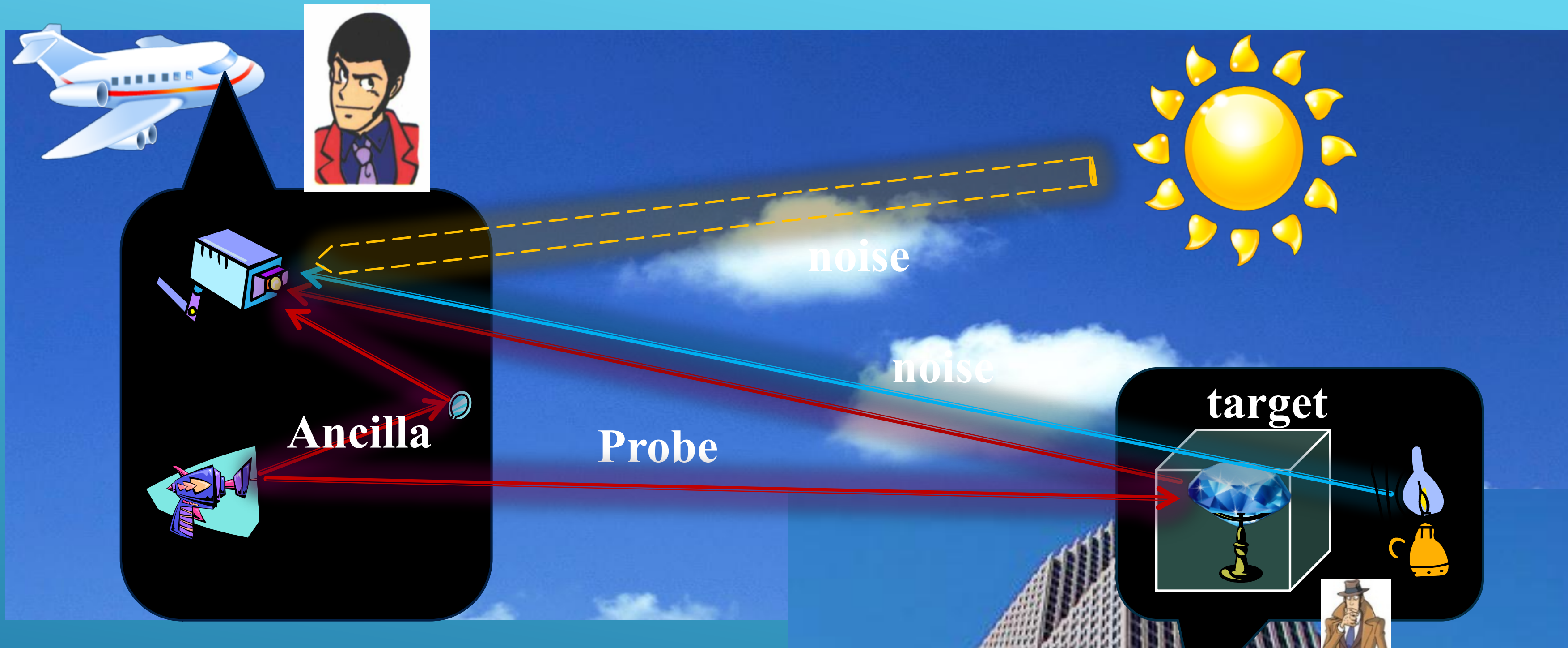


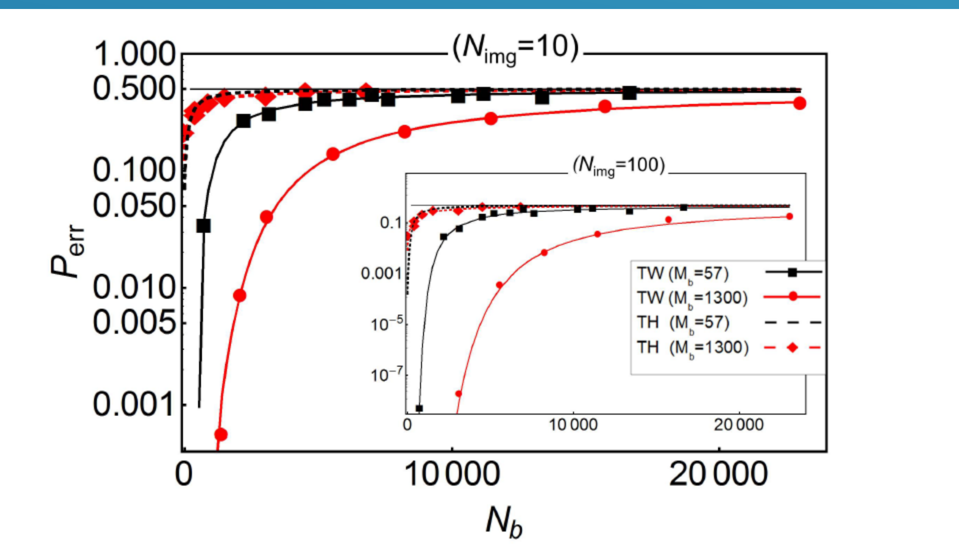
With PDC correlation

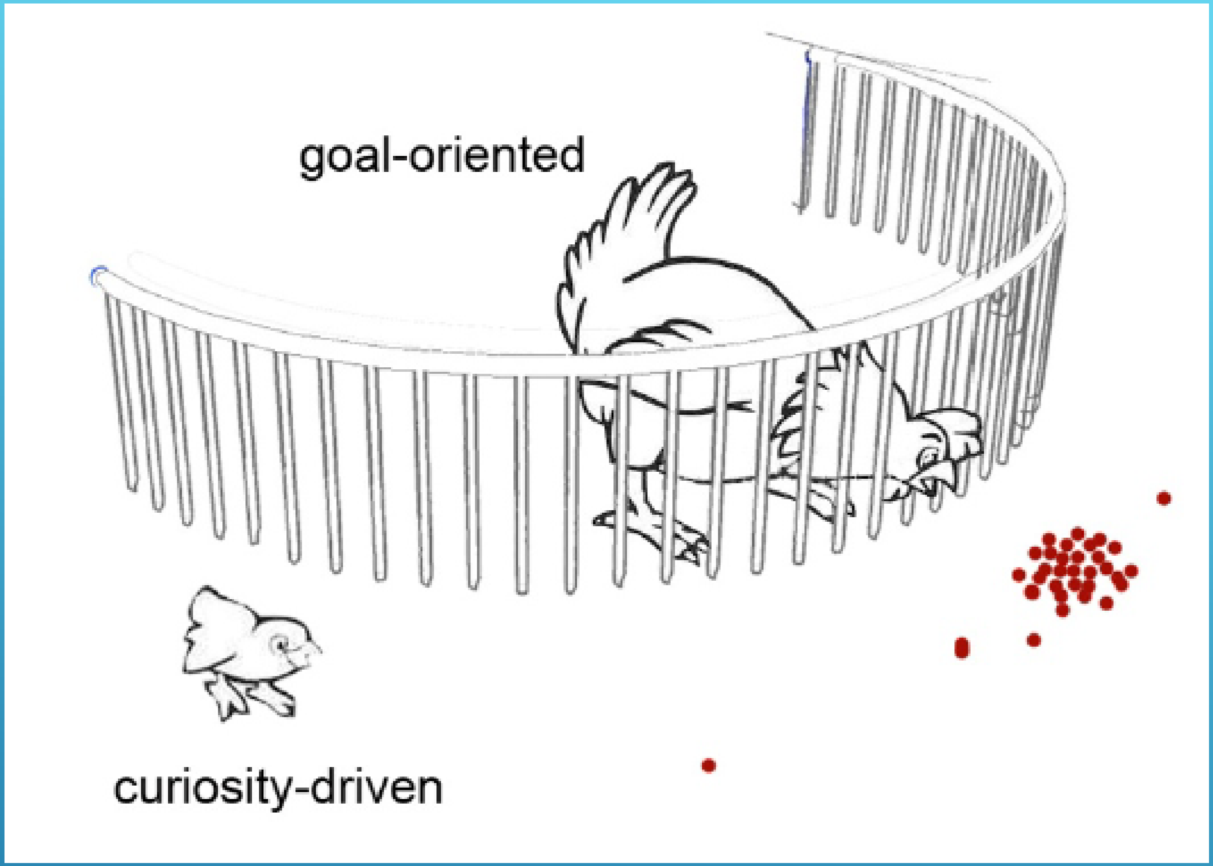




Quantum Radar







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