The top-quark mass: interpretation of the measurements and theoretical uncertainties

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1. Introduction
2. Standard top-mass measurements and Monte Carlo generators
3. Mass definitions and interpretation of the measurements
4. Alternative methods for mass measurements and interpretations
5. Hadronization systematics on the top mass
6. Top mass determination at lepton colliders
7. Conclusions

Frascati workshop on 'Top mass: challenges in definition and determination', May 2015
https://agenda.infn.it/conferenceDisplay.py?confId=9202

Top 2015 workshop: http://top2015.infn.it/

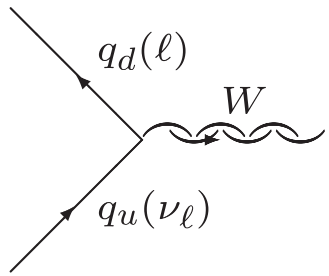
Standard Model relativistic QFT with a gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$

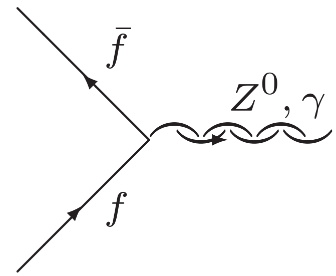
Spontaneous symmetry breaking: $SU(3)_C \times SU(2)_L \times U(1)_Y \longrightarrow SU(3)_C \times U(1)_{\rm em}$

Quarks: $\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \qquad u_R \quad d_R \quad c_R \quad s_R \quad t_R \quad b_R$

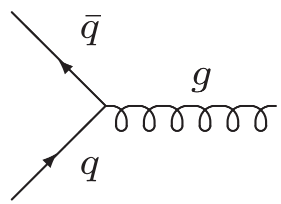
Leptons: $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \qquad e_R \quad \mu_R \quad \tau_R$

Electroweak interactions mediated by $W^\pm$, Z^0 and γ

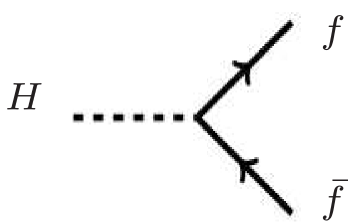


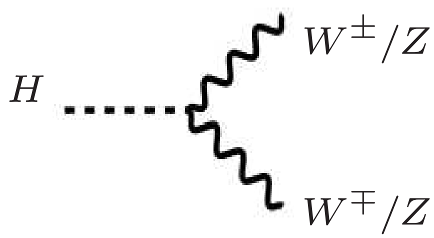


Gluon mediates strong interactions



Masses via interactions with Higgs g, g' y - SU(2)$_L$, U(1)$_Y$ and Yukawa couplings





$m_W^2 = \frac{1}{2} g^2 v^2 \;\; ; \;\; m_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2$

$m_f = y_f v/\sqrt{2}$ ($v \simeq 247$ GeV: Higgs v.e.v.)

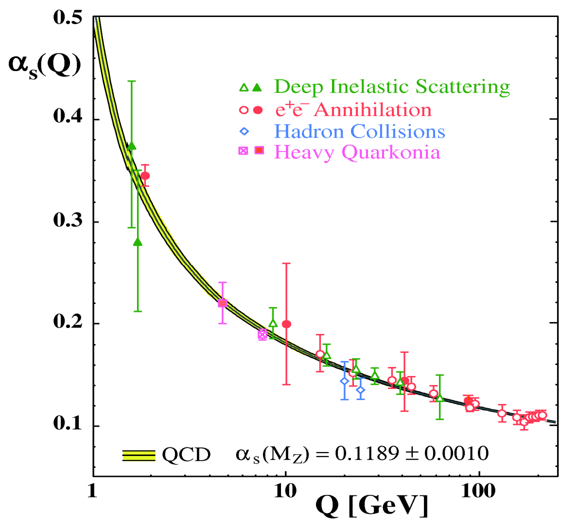
Heavy-quark (c, b, t) production interesting from theoretical and experimental viewpoints

Tests of QCD, electroweak interactions and parton model

Investigation of power corrections and hadronization to predict heavy-hadron spectra

$m \gg \Lambda_{\mathrm{QCD}} \sim \mathcal{O}(100$ MeV: perturbative QCD can work

Asymptotic freedom allows perturbative expansion at large energies:

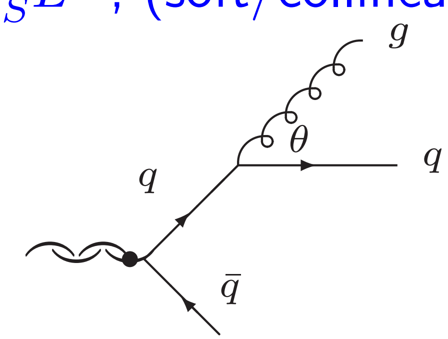


$$\sigma(Q) = c_0 + c_1\alpha_S(Q) + c_2\alpha_S^2(Q) + \ldots \text{ (LO, NLO, etc.)}$$

$$\alpha_S(Q) = \frac{1}{\beta_0 \ln \frac{Q^2}{\Lambda^2}} \left\{ 1 - \frac{\beta_1 \ln[\ln(Q^2/\Lambda^2)]}{\beta_0^2 \ln(Q^2/\Lambda^2)} \right\} \quad (\overline{\mathrm{MS}} \text{ ren. scheme})$$

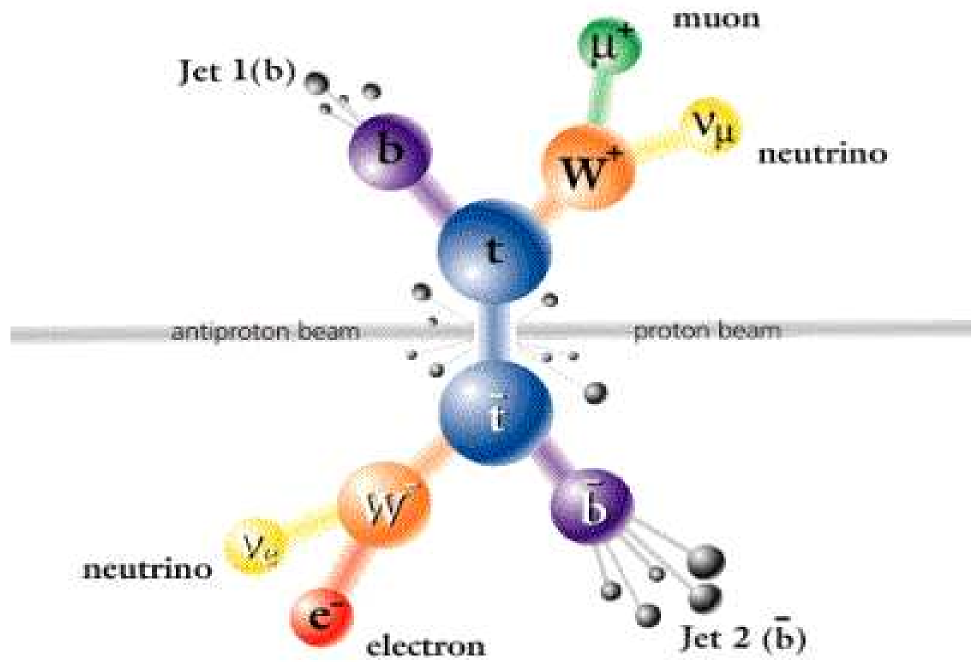
Fixed-order calculations reliable for total cross sections

Differential distributions $\sim \alpha_S^n L^m$, (soft/collinear emission): all-order resummation



$E_g \to 0$: soft radiation $\quad \theta \to 0$: collinear radiation ($\ln(m/Q)$ for heavy quarks)

The top quark was discovered in 1995 by CDF and D0 experiments at Tevatron (FNAL)



$Q = (2/3)\ e$, $T_3 = +1/2$, phenomenology driven by its large mass: $m_t \simeq 173$ GeV

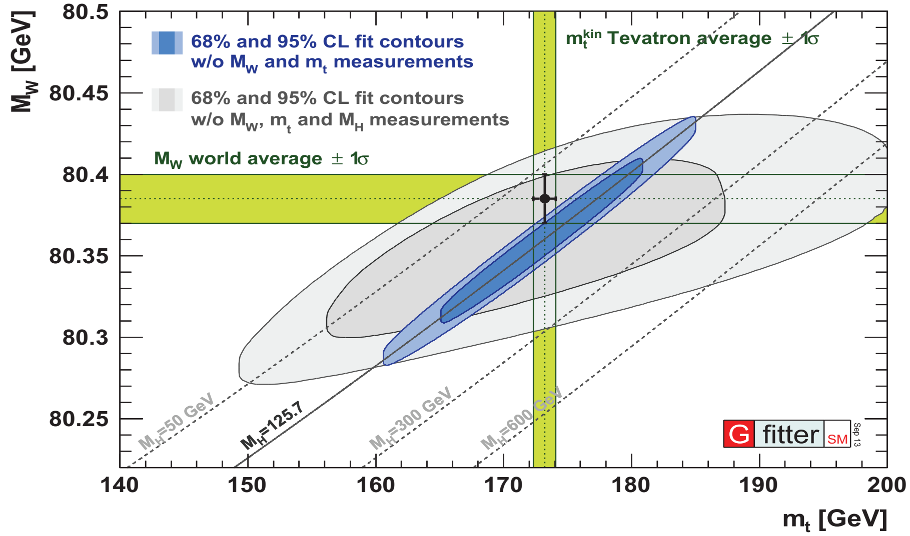
Large width $\Gamma_t \simeq 1.35\ \mathrm{GeV} \Rightarrow \tau_t \simeq 0.5 \times 10^{-24}\ \mathrm{s}$ (PDG'14)

The top quark decays before forming any T-hadron or $t\bar{t}$ resonance

Being $m_t \sim m_H$, the top Yukawa coupling is the only of order 1

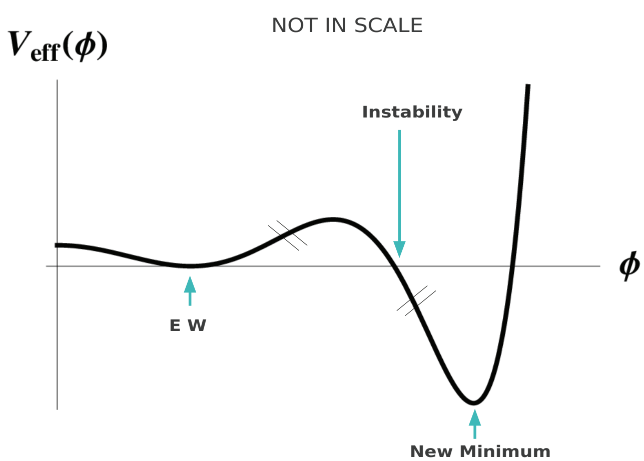
Processes with top quarks are background for many New Physics searches

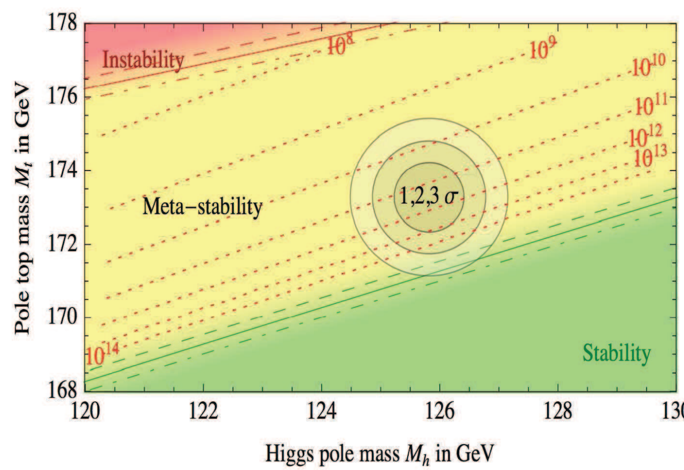
The top quark mass plays a crucial role in the electroweak symmetry breaking



Stability of the SM vacuum depends on top and Higgs masses (Degrassi et al, JHEP'12)

$$V_{RG}(\phi) \simeq \frac{1}{2} m^2(\Lambda)\phi^2(\Lambda) + \frac{1}{4}\lambda^4(\Lambda)\phi^4(\Lambda)\ ,\ \phi \sim \Lambda \gg v$$



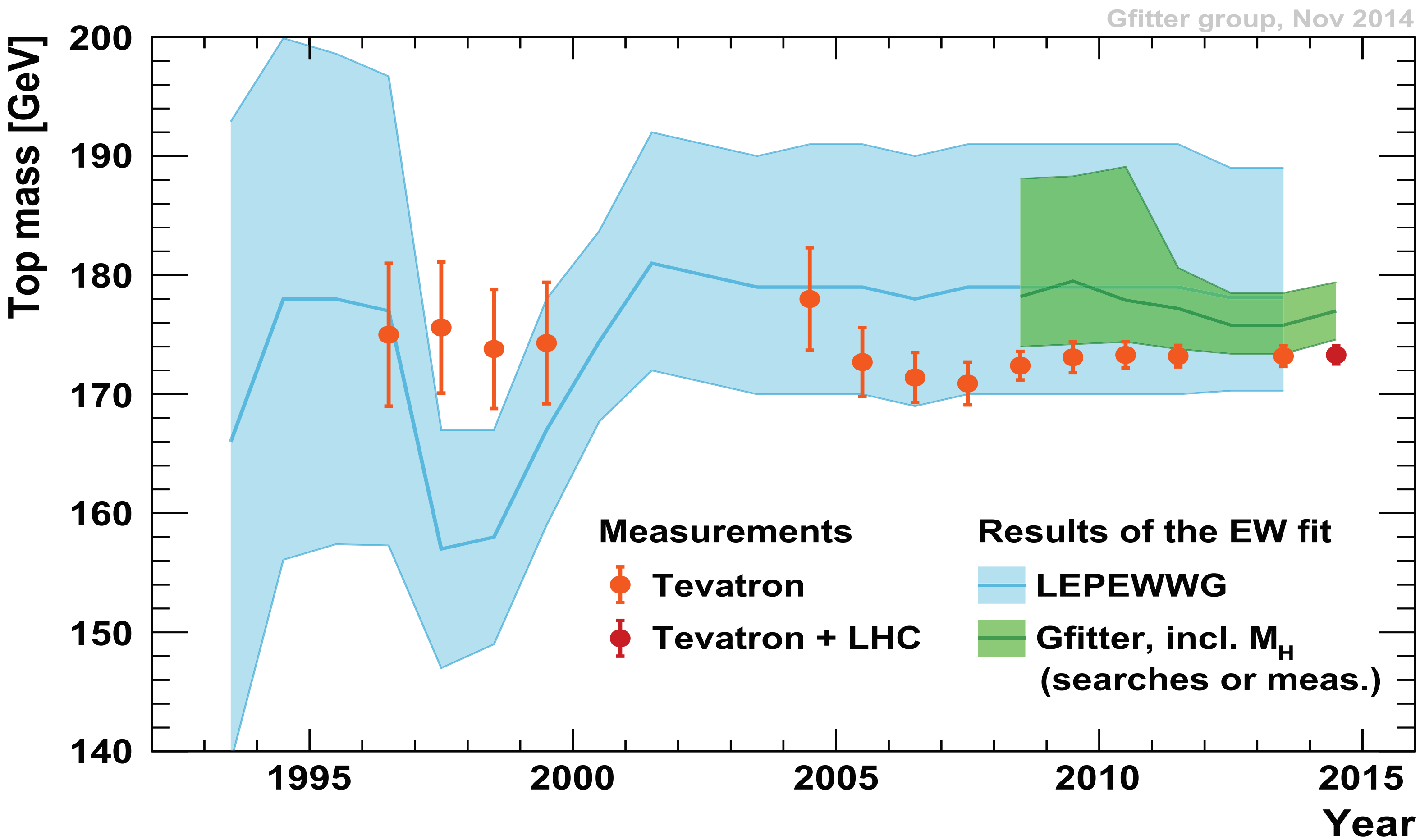


Stability: $V_{\rm eff}(v) < V_{\rm eff}(v')$; Instability: $V_{\rm eff}(v) > V_{\rm eff}(v')$; Metastability: $\tau > T_U$

Top mass world average as pole mass in determination of Yukawa coupling

World average (Tevatron and LHC):

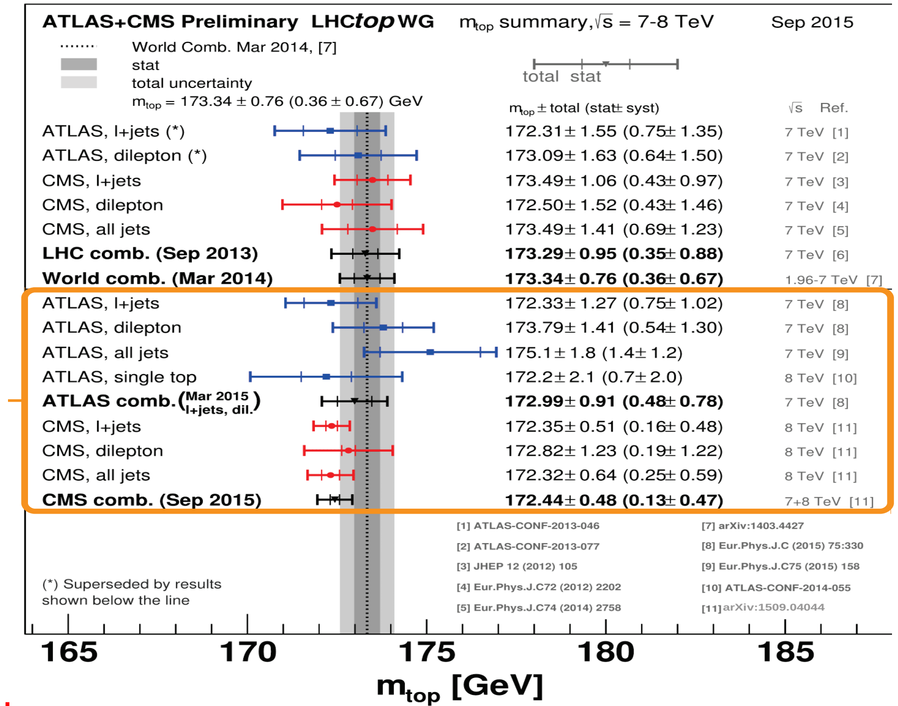
$m_t^{\mathrm{TeV+LHC}} = [173.34 \pm 0.27(\mathrm{stat}) \pm 0.71(\mathrm{syst})]$ GeV (arXiv:1403.4427)



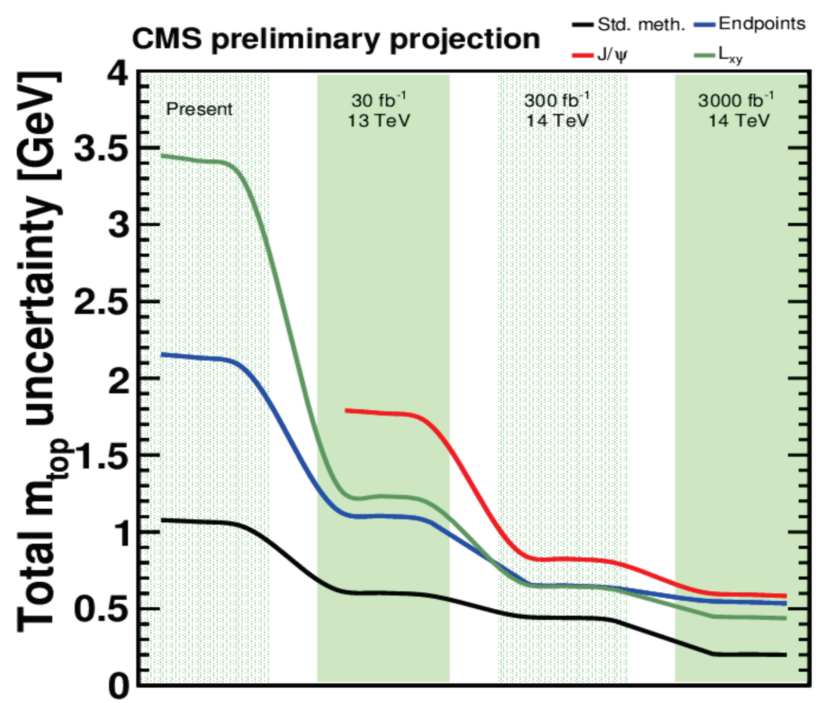
Recent CMS analysis (1509.04044), $\mathcal{L} = 19.7\ \mathrm{fb}^{-1}$, 8 TeV+7 TeV data:

$m_t = [172.44 \pm 0.13\ (\mathrm{stat}) \pm 0.47\ (\mathrm{syst})]$ GeV

Top mass measurement overview



Prospects on precision mass measurements



Theoretical uncertainties on the top mass measurement

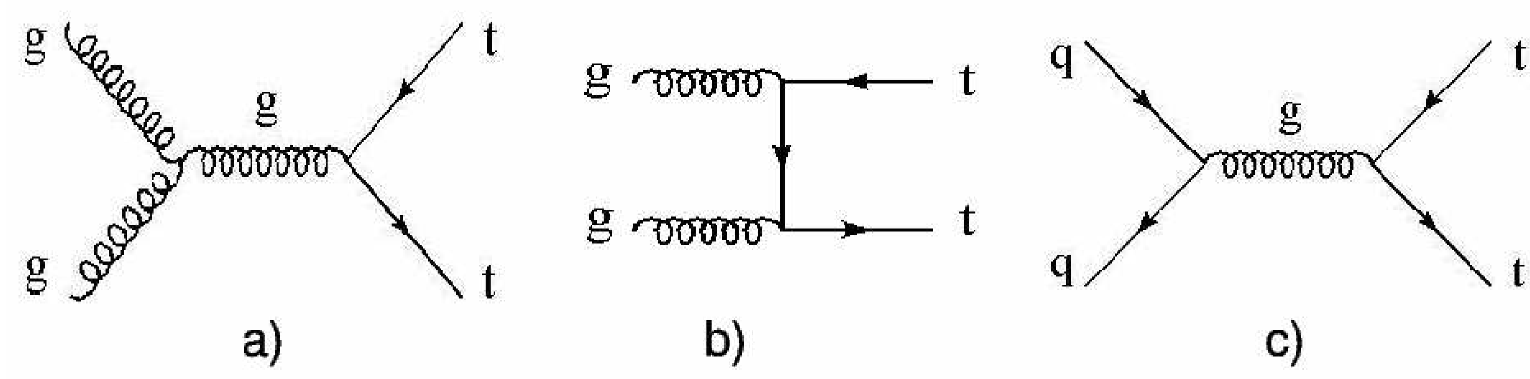
Monte Carlo systematics (MC); modelling QCD radiation effects (Rad); colour reconnection (CR); parton distribution functions (PDF)

All values in GeV	CDF	D0	ATLAS	CMS	Tevatron	LHC	WA
$m_{\rm top}$	173.19	174.85	172.65	173.58	173.58	173.28	173.34
Stat	0.52	0.78	0.31	0.29	0.44	0.22	0.27
iJES	0.44	0.48	0.41	0.28	0.36	0.26	0.24
stdJES	0.30	0.62	0.78	0.33	0.27	0.31	0.20
flavourJES	0.08	0.27	0.21	0.19	0.09	0.16	0.12
bJES	0.15	0.08	0.35	0.57	0.13	0.44	0.25
MC	0.56	0.62	0.48	0.19	0.57	0.25	0.38
Rad	0.09	0.26	0.42	0.28	0.13	0.32	0.21
$\Longrightarrow$ CR	0.21	0.31	0.31	0.48	0.23	0.43	0.31 $\Longleftarrow$
PDF	0.09	0.22	0.15	0.07	0.12	0.09	0.09
DetMod	<0.01	0.37	0.22	0.25	0.09	0.20	0.10
b-tag	0.04	0.09	0.66	0.11	0.04	0.22	0.11
LepPt	<0.01	0.20	0.07	<0.01	0.05	0.01	0.02
BGMC	0.10	0.16	0.06	0.11	0.11	0.08	0.10
BGData	0.15	0.19	0.06	0.03	0.12	0.04	0.07
Meth	0.07	0.15	0.08	0.07	0.06	0.06	0.05
MHI	0.08	0.05	0.02	0.06	0.06	0.05	0.04
Total Syst	0.85	1.25	1.40	0.99	0.82	0.92	0.71
Total	1.00	1.48	1.44	1.03	0.94	0.94	0.76
χ^2/ndf	1.09 / 3	0.13 / 1	0.34 / 1	1.15 / 2	2.45 / 5	1.81 / 4	4.33 / 10
χ^2 probability [%]	78	72	56	56	78	77	93

Overall theory error on world average mass: $\Delta m_t \simeq 0.54$ GeV

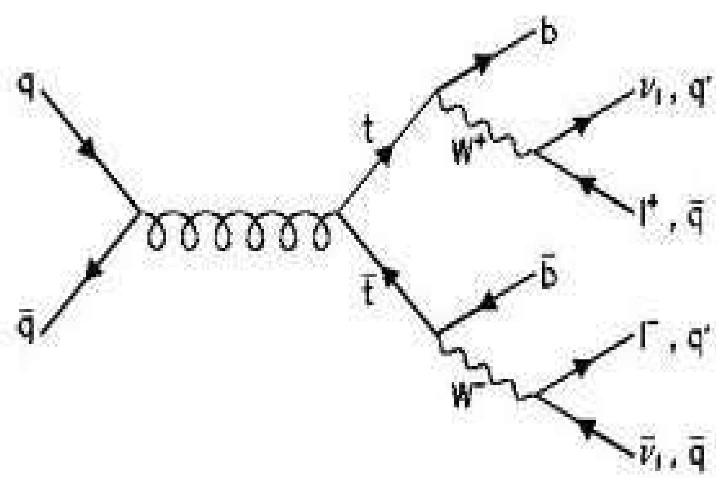
Issues: what mass is reconstructed; whether/why it is worth using any mass definition

Top production and decay ar hadron colliders ($t\bar{t}$ pairs)



Production via strong interaction (mainly $q\bar{q}$ at Tevatron, gg at LHC): LO is $\mathcal{O}(\alpha_S^2)\ldots$

Top decays via $t \to bW$ with $\mathrm{BR} \simeq 1$



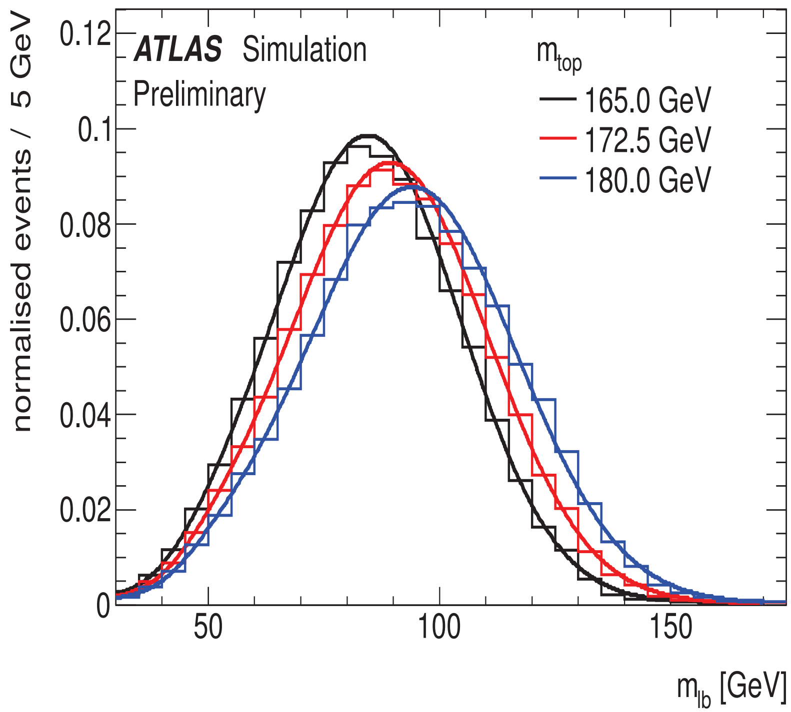
Final states as all-leptons, lepton+jets or all-jets according to W decays

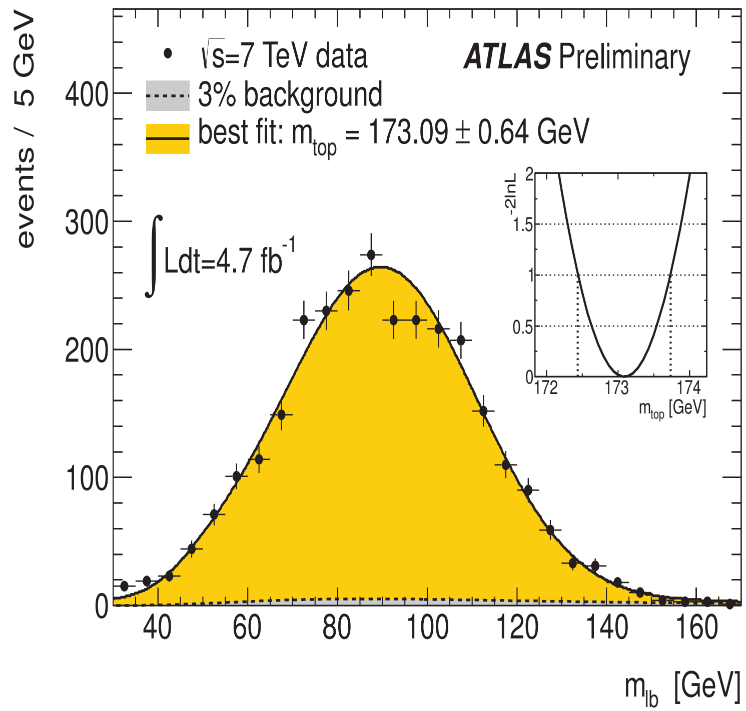
Top-mass measurements compare data with theory: the mass is a parameter in the theoretical prediction

Standard top mass reconstruction relies on top decays: template method

Distribution of observables sensitive to m_t and reconstruction under the assumption that the final state is $WbWb$ and the W mass is fixed

Data confronted with Monte Carlo templates and m_t is the value minimizing the χ^2



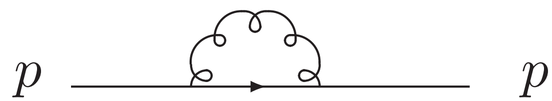


ATLAS: b-jet+lepton invariant mass in dilepton channel

m_t is the parameter in the event generator, often called Monte Carlo mass

Can we relate it to any theoretical mass definition?

Quark mass definitions - Subtraction of the UV divergences in the self energy $\Sigma(p)$



Renormalized propagator: $S(p) = -\dfrac{i}{\not p - m_0 + \Sigma^R(p, m_0, \mu)}$, $d = 4 - 2\epsilon$

$$\Sigma^R \sim \left[\frac{1}{\epsilon} - \gamma + \ln 4\pi - \ln\frac{m_0^2}{\mu^2} + A\right] \not p - \left[4\left(\frac{1}{\epsilon} - \gamma + \ln 4\pi - \ln\frac{m_0^2}{\mu^2}\right) + B\right] m_0 + (Z_2 - 1)\not p - (Z_2 Z_m - 1) m_0$$

On-shell renormalization (pole mass) - Z_2 and Z_m are determined by means of:

$$\Sigma^R(p) = 0 \ \ \text{and} \ \ \frac{\partial \Sigma^R}{\partial \not p} = 0 \ \ \text{for} \ \ \not p = m$$

$\overline{\rm MS}$ renormalization: counterterm to subtract $(1/\epsilon - \gamma_E + \ln 4\pi)$

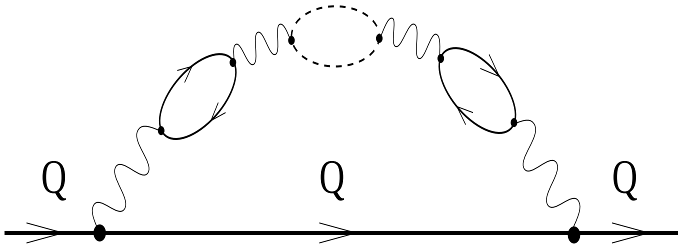
$$S^R_{\rm o.s.}(p) \sim \frac{i}{\not p - m_{\rm pole}} \ \ ; \ \ S^R_{\overline{\rm MS}} \sim \frac{i}{\not p - m_{\overline{\rm MS}} - (A - B) m_{\overline{\rm MS}}}$$

Pole mass is the pole of the propagator; $\overline{\rm MS}$ mass is quite far from the pole

Pole mass is a physical mass for free particles like electrons, but for heavy quarks it exhibits an ambiguity $\mathcal{O}(\Lambda_{\rm QCD})$ due to infrared renormalons (Braun,Beneke'94)

$\overline{\rm MS}$ mass suitable for off-shell quarks ($Z \to b\bar{b}$) at LEP, but $\sim (\alpha_S/v^2)^k$ at threshold

Higher-order corrections to the self energy



$$\Sigma(m,m) \sim m \sum_n \alpha_S^n \, (2\beta_0)^n \, n!$$

$$\delta m_{\rm pole} \approx \mathcal{O}(\Lambda)$$

Relation pole/$\overline{\rm MS}$ mass at 4 loops $[\bar{m} = \bar{m}(\bar{m})]$ (P.Marquard et al, PRL'15)

For top quarks:

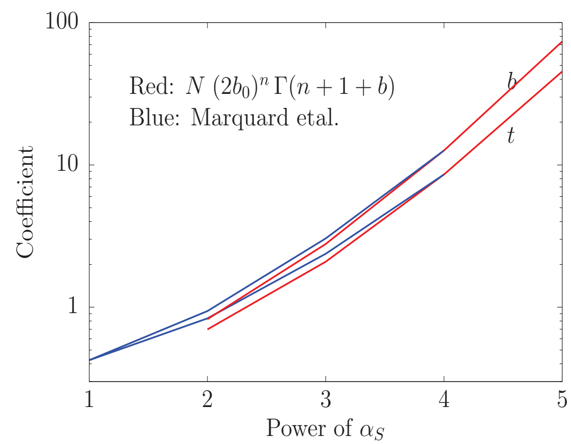
$$m_{\rm pole} = \bar{m}[1 + 0.42\,\alpha_S + 0.83\,\alpha_S^2 + 2.37\,\alpha_S^3 + (8.49 \pm 0.25)\,\alpha_S^4] \; ; \; \Delta m_{{\rm pole},\overline{\rm MS}} \simeq 195 \text{ MeV}$$

Renormalon calculation - large-n expansion: (M.Beneke'94)

$$m_{\rm pole} = \bar{m} \times \left(1 + \sum_{n=0}^{\infty} r_n \alpha_S^{n+1}\right) \; ; \; r_n \to N (2\beta_0)^n \Gamma\left(n + 1 + \frac{\beta_1}{\beta_0^2}\right)\left(1 + \sum_{k=1}^{\infty} \frac{s_k}{n^k}\right)$$

Fitting N: $N \simeq 0.726$

Can be used to predict higher orders:



$$m_{\rm pole} = (163.63 + 7.56 + 1.62 + 0.50 + 0.19 + 0.10 + \ldots) \text{ GeV}$$

Minimum for $n \sim$ 8-9: $\Delta m \approx |r_8 \alpha_S^8 - r_9 \alpha_S^9| \approx 68$ MeV (P.Nason, 1602.00443)

Other mass definitions subtract renormalons off the pole mass

Potential-subtracted mass: δm subtracts the IR divergences (M.Beneke)

$$m_{\rm PS}(\mu_F) = m_{\rm pole} - \delta m(\mu_F) \;\; ; \;\; \delta m(\mu_F) = \frac{1}{2}\int_{|q|<\mu_F} \frac{d^3q}{(2\pi)^3}\tilde{V}(q)$$

$V(q)$: $t\bar{t}$ Coulomb potential (e.g. $e^+e^- \to t\bar{t}$ at threshold)

MSR masses in terms of an infrared scale R, e.g. μ_F, $\bar{m}(\mu)$, etc., in SCET (A.Hoang et al)

$$m^{\rm MSR}(R) = m^{\rm pole} - \Sigma^{\rm fin}(R,\mu) \;\; , {\rm for} \;\; R < m$$

Like pole mass, but renormalons at scales $< R$ are not absorbed in the mass

$m_t^{\rm MSR}(R) \to m_{\rm pole}$ for $R \to 0$; $m^{\rm MSR}(R) \to \bar{m}_t(\bar{m}_t)$ for $R \to \bar{m}_t(\bar{m}_t)$

$$\frac{dm_{\rm pole}}{d\ln\mu} = 0 \Rightarrow \frac{dm^{\rm MSR}(R,\mu)}{d\ln\mu} = -R\gamma[\alpha_S(\mu)]$$

$m_t(R = 1 \text{ GeV})$ could be a suitable kinematic mass, but without renormal problem

NLO top production+decays necessary to have a consistent top mass definition

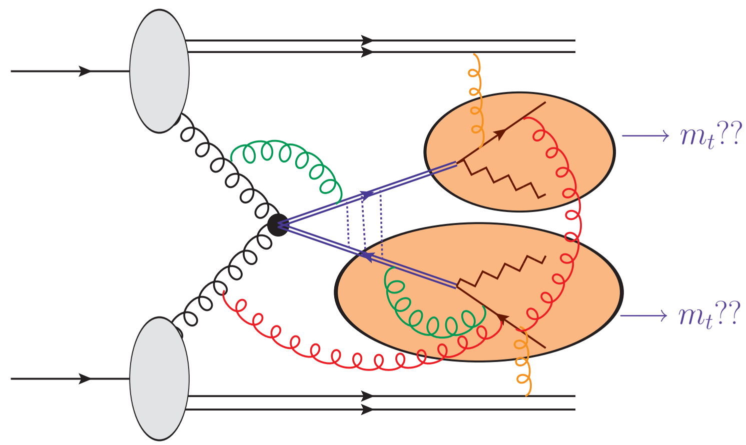
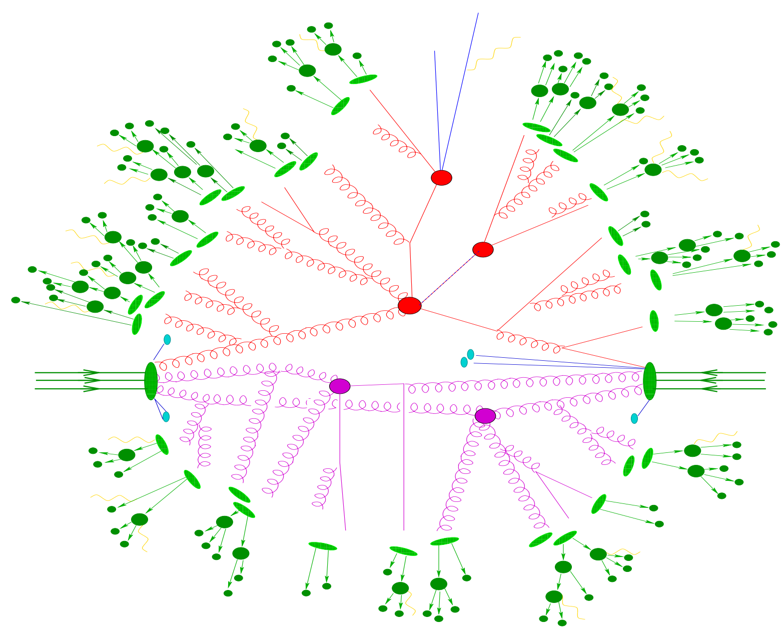


Figure by A. Signer

Monte Carlo generators:

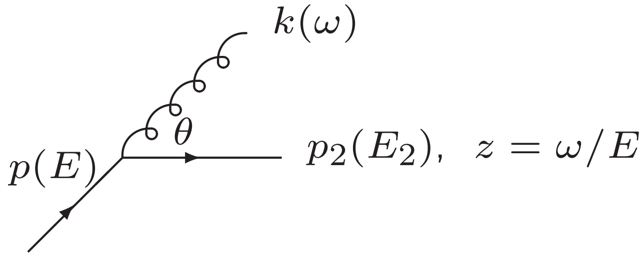


Hard scattering at LO (HERWIG, PYTHIA) or NLO (aMC@NLO, POWHEG)

Parton showers in soft/collinear approximation

Models for hadronization and underlying event

Parton shower algorithms



$$dP = \frac{\alpha_S}{2\pi} \, P(z) dz \, \frac{dQ^2}{Q^2} \, \Delta_S(Q^2_{\rm max}, Q^2)$$

Q^2: ordering variable

$\Delta_S(Q^2_{\rm max}, Q^2)$: no radiation in $[Q^2, Q^2_{\rm max}]$ (soft/collinear virtual corrections)

$$\Delta_S(Q^2_{\rm max}, Q^2) = \exp\left[-\int_{Q^2}^{Q^2_{\rm max}} \frac{\alpha_S}{2\pi} \frac{dQ'^2}{Q'^2} \int_{z_{\rm min}}^{z_{\rm max}} dz P(z) \right] \simeq 1 - \int_{Q^2}^{Q^2_{\rm max}} \frac{\alpha_S}{2\pi} \frac{dQ'^2}{Q'^2} \int_{z_{\rm min}}^{z_{\rm max}} dz P(z) + \dots$$

HERWIG : $Q^2 = E^2(1-\cos\theta) \simeq E^2\theta^2/2$ (angular ordering);

PYTHIA: $Q^2 = p^2$ or k_T^2

$\alpha_S(k_T^2)$ NLO in HERWIG and LO in PYTHIA

Total cross section LO thanks to unitarity $(1 = R + V)$

Distributions equivalent to threshold LL resummation, + some NLLs

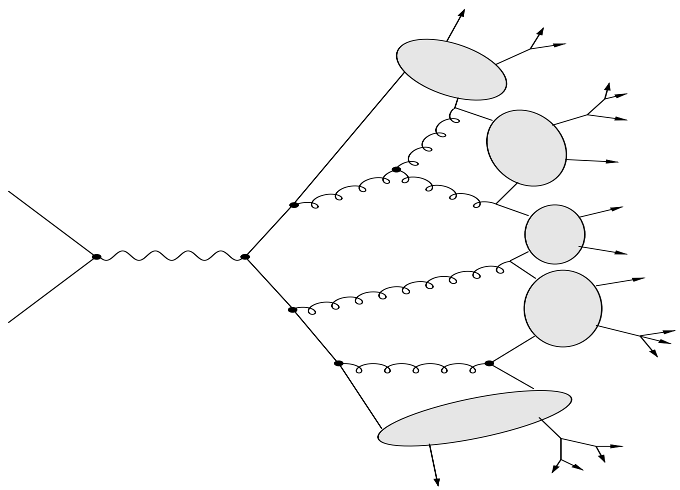
Interface with matrix-element generators (ALPGEN, MadGraph, CalcHeP)

NLO total cross section when hard-scattering via MC@NLO or POWHEG

NLO top decays in POWHEG, with approximate treatment of top width

Hadronization in Monte Carlo event generators

$$D_{\rm K}(x,\alpha) = (1+\alpha)(2+\alpha)x(1-x)^{\alpha} \;\; ; \;\; D_{\rm P}(x,\epsilon) = \frac{N_P}{x\,[1-1/x-\epsilon/(1-x)]} \;\; ; \;\; x \simeq \frac{E_h}{E_q}$$



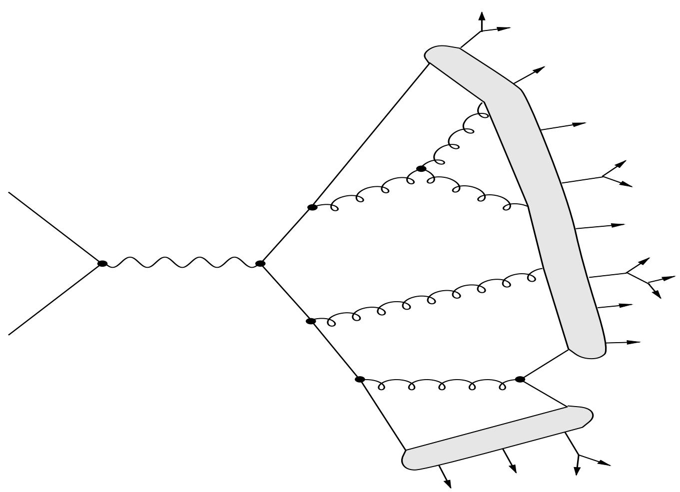
HERWIG: cluster model

Perturbative evolution ends at $Q^2 = Q_0^2$

Angular ordering $\Rightarrow$ colour preconfinement

Forced gluon splitting $(g \to q\bar{q})$

Colour-singlet clusters decay into the observed hadrons



PYTHIA: string model

q and $\bar{q}$ move in opposite direction

The colour field collapses into a string with uniform energy density

$q\bar{q}$ pairs are produced

The string breaks into the observed hadrons

Possible interface with NP fragmentation functions

Tuning involves hadronic and perturbative parameters: Q_0, $\Lambda_{\rm MC}$, m_g, etc. and relies on precise e^+e^- data (LHC data in future?)

Monte Carlo differences in parton showers and hadronization lead to the so-called Monte Carlo systematics

No unique way to estimate it: comparing two codes or one code and varying its parameters

CDF: comparing HERWIG and PYTHIA;

D0: ALPGEN+HERWIG vs. ALPGEN+PYTHIA;

Both CDF and D0 PYTHIA vs. MC@NLO to gauge the impact of NLO corrections

ATLAS: MC@NLO vs. POWHEG (NLO corrections), HERWIG vs. PYTHIA for hadronization

CMS: MadGraph vs. POWHEG

Radiation systematics: how much ISR and FSR affect the measurement of the top mass

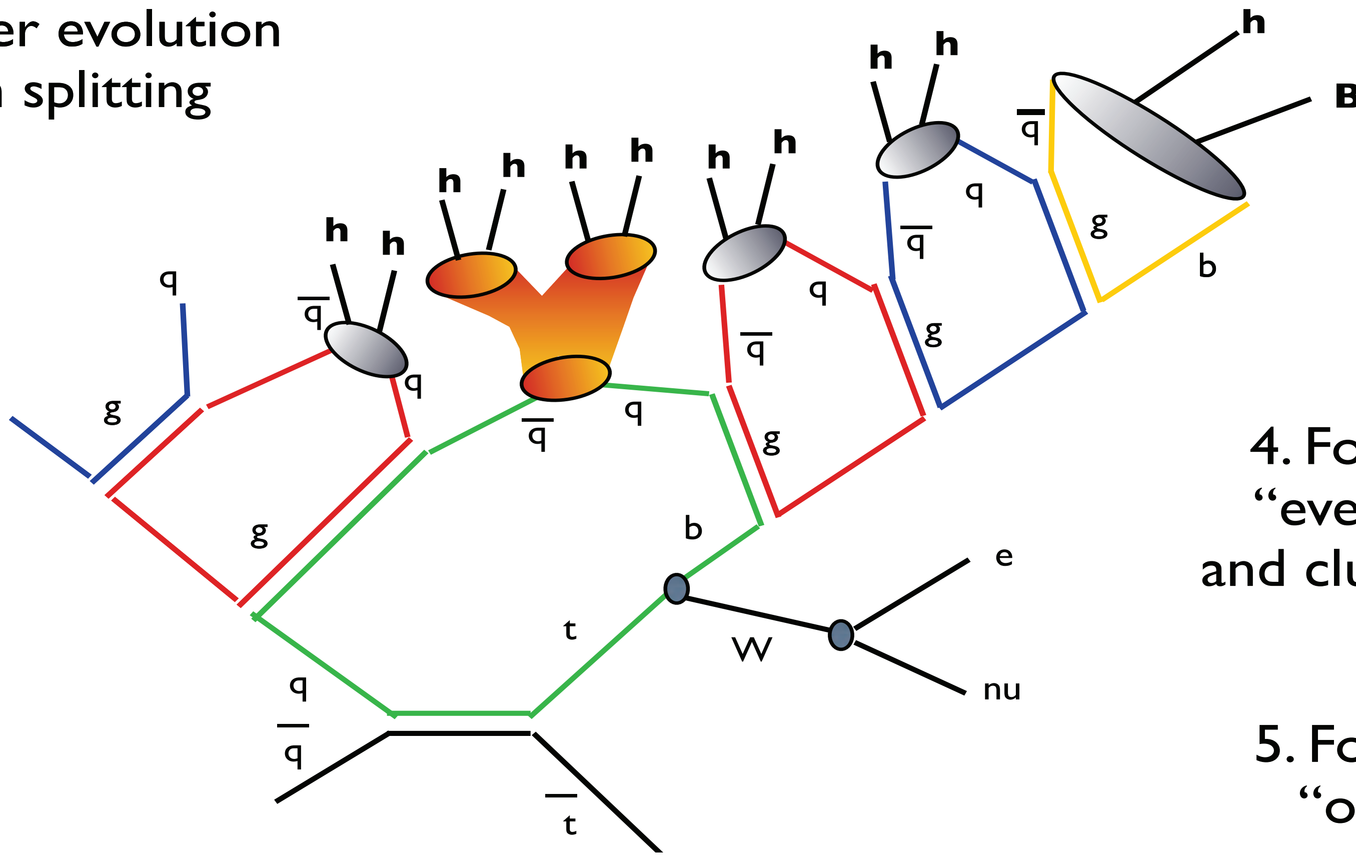
CDF and D0: tuning PYTHIA $\Lambda_{\rm QCD}$ and initial Q^2 scale to Drell–Yan events ($q\bar{q}$ initial state) and propagate to $t\bar{t}$ events

ATLAS: playing around with ISR and FSR parameters of PYTHIA

CMS: varying factorization and renormalization scales of MadGraph, then matching scales for matching with parton showers

Colour reconnection: hadrons can be formed via 'even' or 'odd' clusters: effects already studied at LEP for $e^+e^- \to W^+W^- \to 4$ jets M.L.Mangano, talk at TOP2013

1. Hard Process
2. Shower evolution
3. Gluon splitting



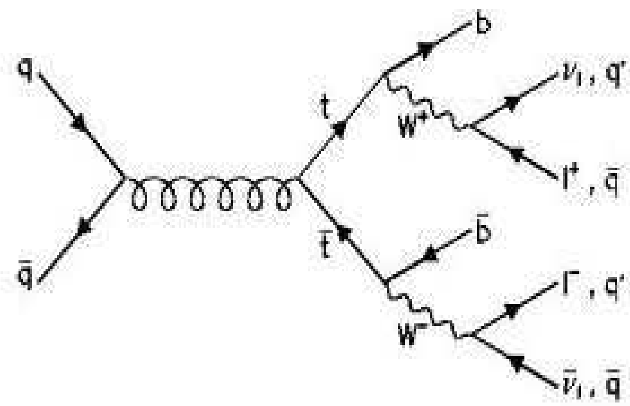
4. Formation of "even" clusters and cluster decay to hadrons

5. Formation of "odd" cluster

$m_t = (p_W + p_{b-\rm jet})^2$: the b-jet may come from either an even or an odd cluster

Colour reconnection can be investigated by varying relevant parameters in HERWIG and PYTHIA (so-called Perugia tunings) and affects the top mass about $\Delta m_t \leq 1$ GeV

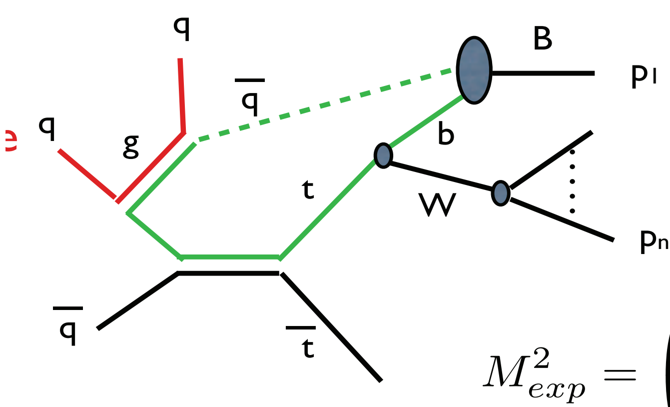
Measured mass must be close to $m_{\rm pole}$: top-decay kinematics is driven by $m_{\rm pole}$



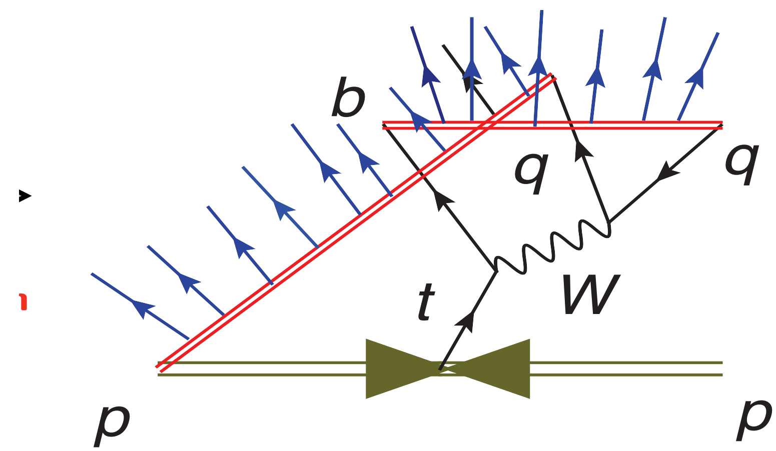
$$\frac{1}{[(p_W+p_b)^2-m_t^2]^2+m_t^2\Gamma_t^2} \sim \frac{1}{\pi}\,\delta[(p_W+p_b)^2-m_t^2] \text{ for } \Gamma_t \ll m_t$$

Reconstructed mass $p^2 = (p_{b-\rm jet}+p_\nu+p_\ell)^2$ (with cuts on jets and leptons) with on-shell tops should be close to the pole mass, up to widths, NP and higher-order corrections

Colour-reconnection effects can spoil this picture



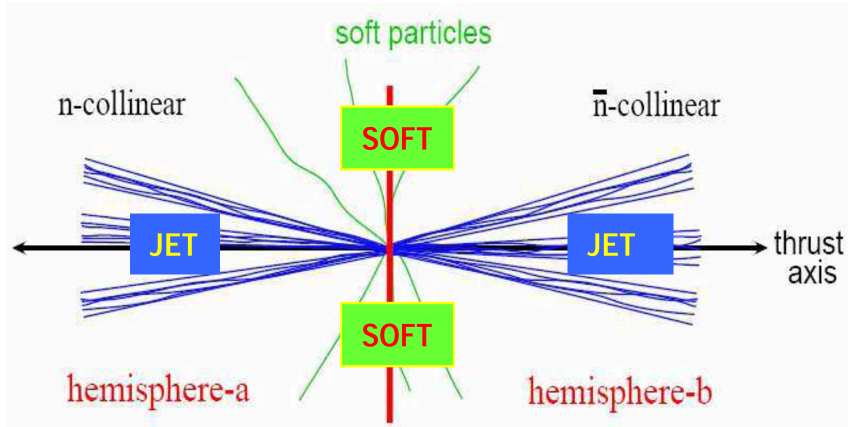
$$M^2_{exp} = \left(\sum_{i=1,\dots,n} p_i\right)^2$$



Left: M.L.Mangano, TOP 2013 workshop, Right: S.Argyropoulos, LNF'15 workshop

Leptonic observables without reconstructing top decay products minimize such effects

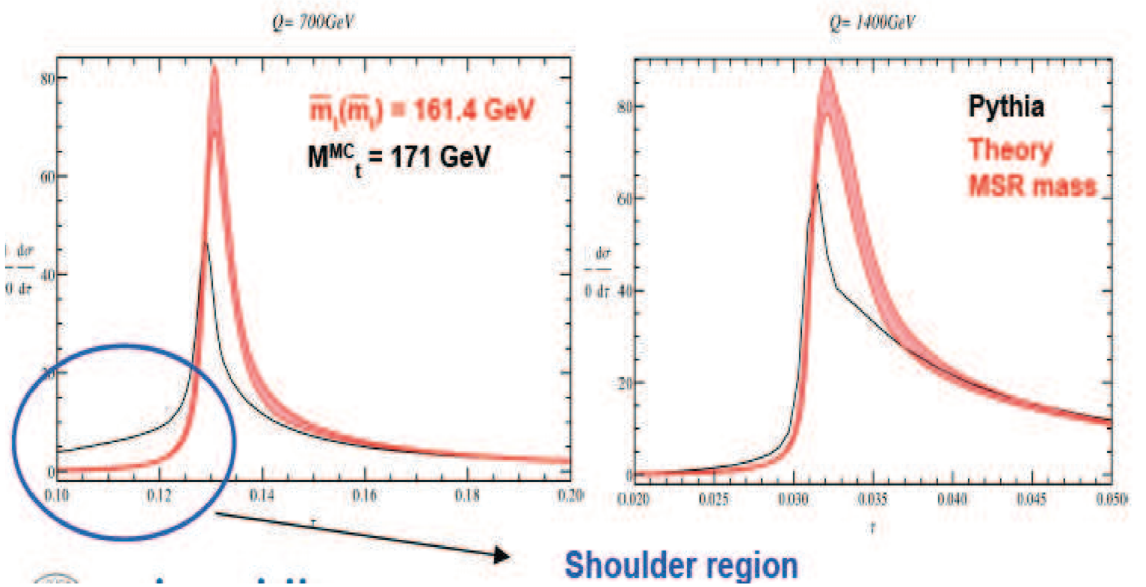
Attempt to address the MC mass using the SCET formalism $\sqrt{s} \gg m_t \gg \Gamma_t \gg \Lambda_{\rm QCD}$



Jet mass: MSR mass with $R \sim \Gamma_t$ $\quad m_{\rm pole} = m_J(\mu) + e^{\gamma_E}\Gamma_t \frac{\alpha_S(\mu) C_F}{\pi}\left(\ln\frac{\mu}{\Gamma_t} + \frac{1}{2}\right) + \mathcal{O}(\alpha_S^2)$

$\mu = Q_0 \sim$ 1-2 GeV $\Rightarrow m_{\rm pole} - m_J(Q_0) \simeq 150 - 200$ MeV

$m_t^{\rm MC} = m_t^{\rm MSR}(R) + \Delta_t(R)$ $\Rightarrow$ fix PYTHIA mass and calibrate $m_t^{\rm MSR}$



Thrust distribution

$e^+e^- \to t\bar{t}$ (A.Hoang, LNF workshop)

Other attempts: run HERWIG with top-hadron states (G.C., R.Chierici and M.Mangano):

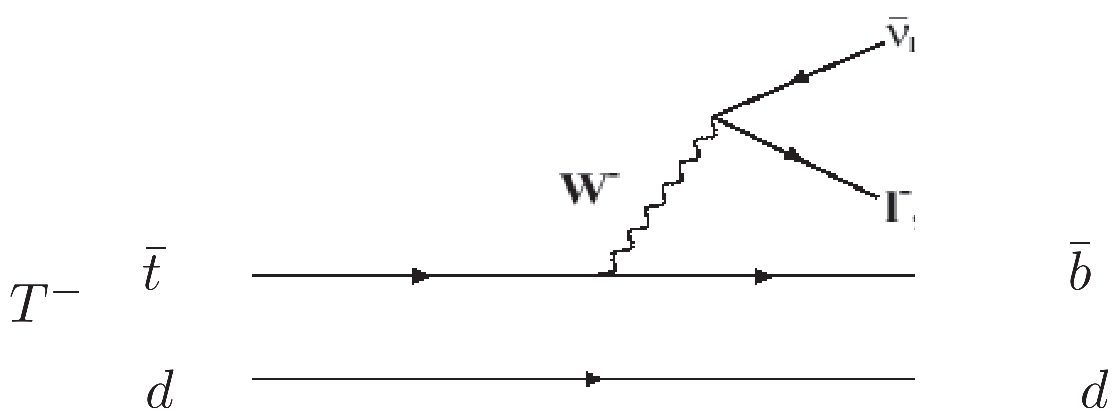
Pretend that top quarks hadronize and decay via the spectator model

From a given observable R extract the Monte Carlo mass $m_{t,T}^{\rm MC}$

Study the same observable R with standard top samples, get $m_t^{\rm MC}$ and compare the extracted masses : $m_{t,T}^{\rm MC} = m_t^{\rm MC} + \Delta m_{t,T}$

In the hadronized samples, the Monte Carlo mass can be related to the T-meson mass M_T and ultimately to the pole or $\overline{\rm MS}$ top-quark masses by using lattice, potential models, NRQCD, etc.

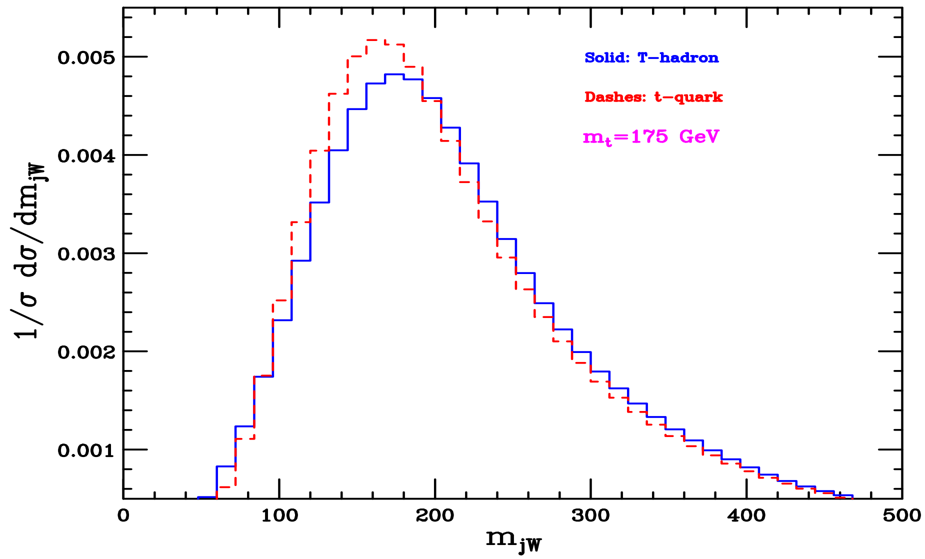
Spectator model decays: $T^- \to (\bar{b}d)\ell^-\bar{\nu}_\ell + X \ldots \quad p_T^2 = (p_{\bar{b}} + p_W + p_q + p_X)^2$

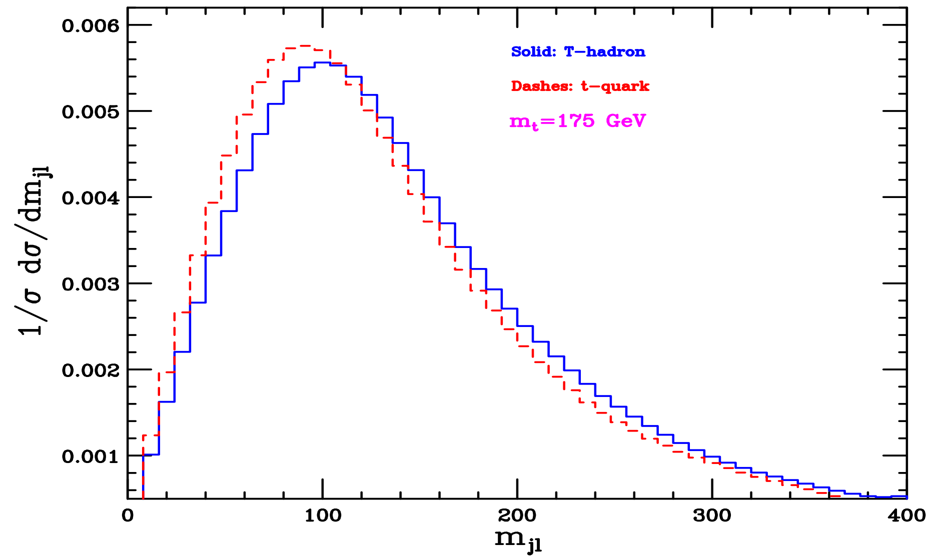


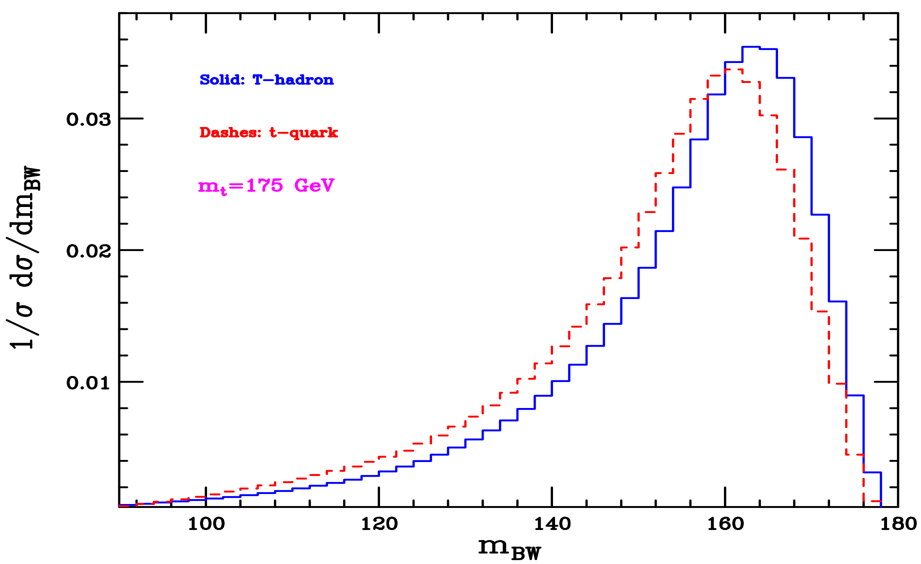
Spectator does not radiate, few events where the b quark does not emit

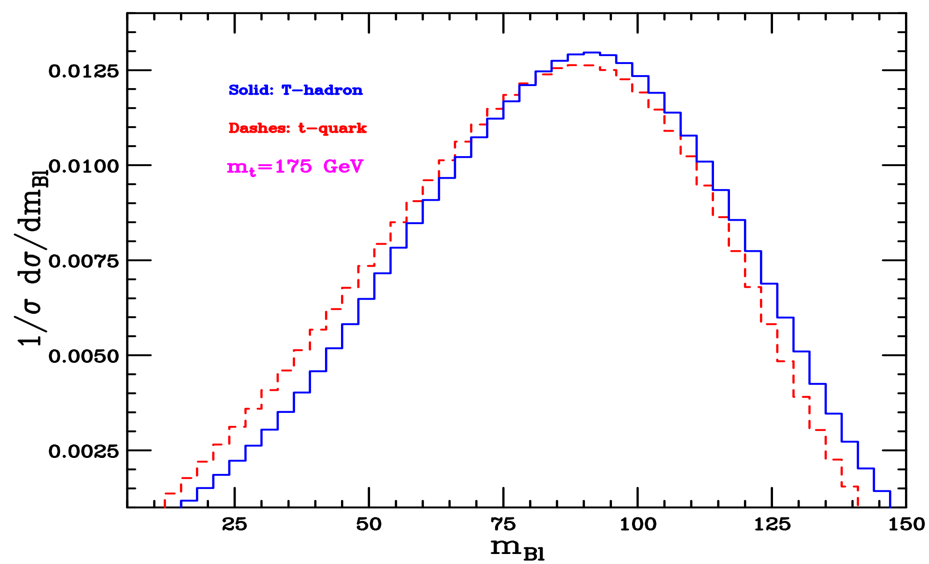
$\bar{b}$ tends to form clusters with spectator quarks with invariant mass closer to $(p_T - p_W)^2$ with respect to standard top decays

pp collisions at $\sqrt{s} = 8$ TeV, dilepton channel, k_T algorithm, $R = 0.7$, $p_{T,j} > 30$ GeV, $p_{T,\ell} > 20$ GeV, MET> 20 GeV, $|\eta_{j,\ell,\nu}| < 2.5$ (HERWIG 6.510, preliminary)









In progress: relate $\langle \Delta m_{jl,Bl} \rangle$ with $\Delta m_{t,T}$ and using PYTHIA

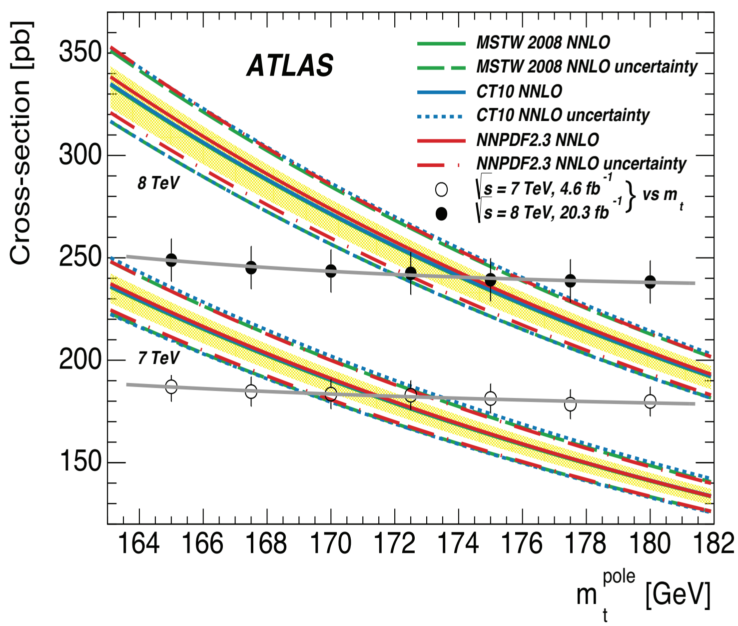
Alternative methods: m_t from NNLO+NNLL $\sigma(pp \to t\bar{t})$ (Czakon, Fielder and Mitov, '13):

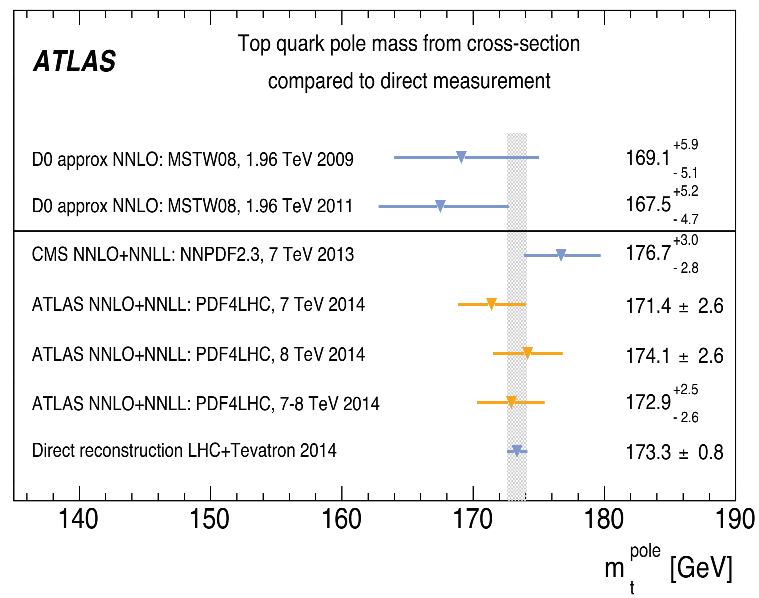
$$\sigma_{\rm tot} = \sum_{i,j} \int d\beta \; \Phi_{ij}(\beta, \mu_F^2) \; \hat{\sigma}_{ij} \quad , \quad \beta = \sqrt{1 - 4m^2/\hat{s}} \quad , \quad \Phi_{ij} = \frac{2\beta}{1-\beta^2} \, x \, (f_i \otimes f_j)$$

At NNLO, $\mu = \mu_F = \mu_R$ and $L = \ln(m_t^2/\mu^2)$, using the pole mass:

$$\hat{\sigma}_{ij} = \frac{\alpha_S^2}{m_t^2} \left\{ \sigma^{(0)} + \alpha_S \left[\sigma_{ij}^{(1)} + L \sigma_{ij}^{(1,1)} \right] + \alpha_S^2 \left[\sigma_{ij}^{(2)} + L \sigma_{ij}^{(2,1)} + L^2 \sigma_{ij}^{(2,2)} \right] \right\}$$

Threshold logs: $\alpha_S^n [\ln^m (1-z)/(1-z)]_+ \; , \; z = m_t^2/(x_i x_j s) \to 1 \; , \; m \leq 2n-1$



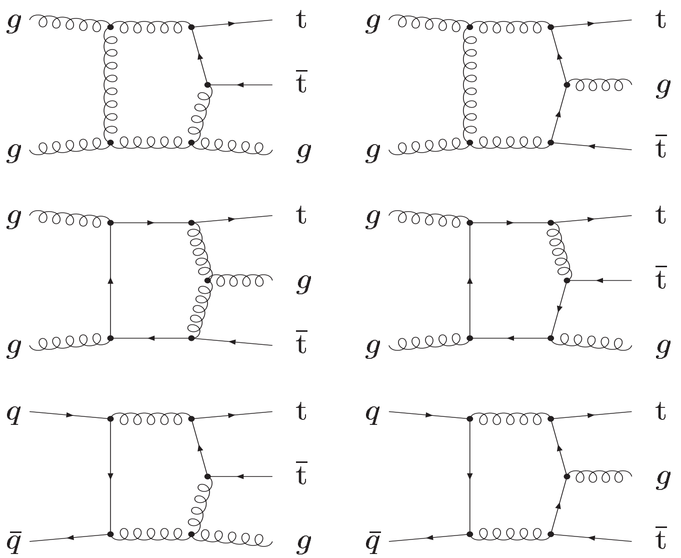


Larger errors than standard methods, but larger statistics are expected in Run 2

Mild dependence on MC m_t; $\Delta(\alpha_S + {\rm PDF}) \simeq 1.7$ GeV, $\Delta(\mu) \simeq^{+0.9}_{-1.3}$ GeV

NLO calculation of $t\bar{t}$+jet cross section with the pole mass (S.Dittmaier et al.,'07)

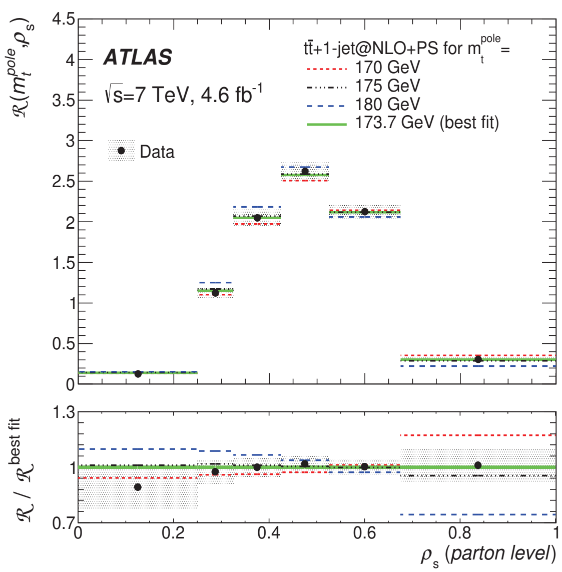
Matching with shower and hadronization through POWHEG+PYTHIA (S.Alioli et al,'13)

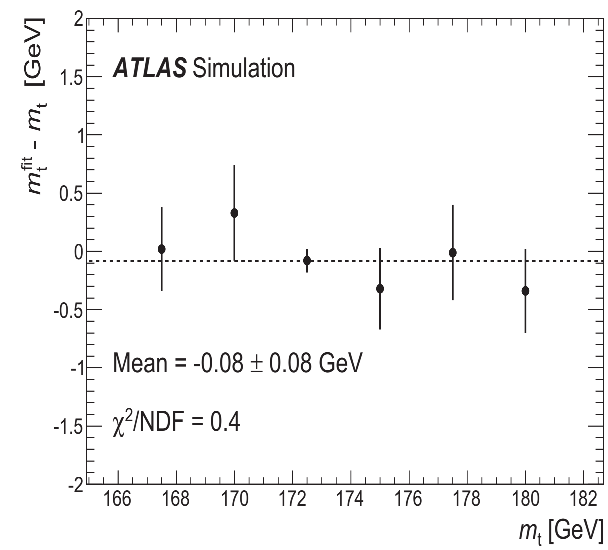


$$\mathcal{R} = \frac{1}{\sigma_{t\bar{t}j}} \frac{d\sigma_{t\bar{t}j}(m_t^{\rm pole})}{d\rho_S}$$

$$\rho_S = \frac{2m_0}{\sqrt{s_{t\bar{t}j}}} \quad , \quad m_0 = 170 \text{ GeV}$$

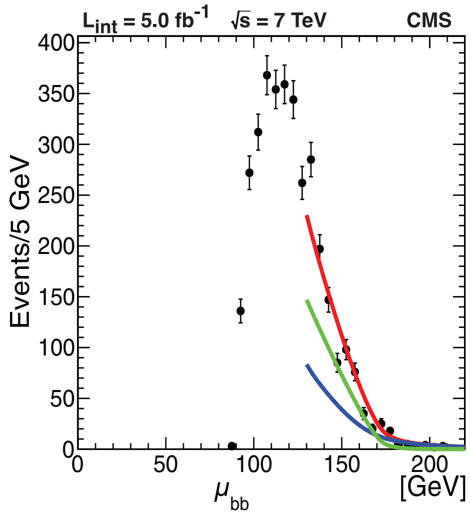
$$t\bar{t} + {\rm jet\ (det.)} \to t\bar{t} + {\rm jet\ (parton)}$$

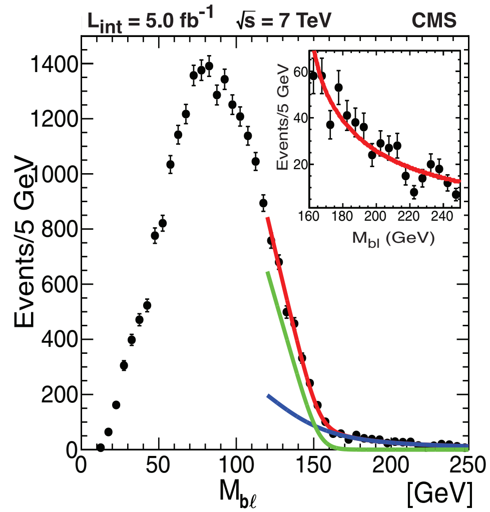




$m_t^{\rm pole} = [173.1 \pm 1.5({\rm stat}) \pm 1.4({\rm syst})^{+1.0}_{-0.5}({\rm theo})]$ GeV

Endpoint method: endpoints of ℓ+b-jet, '$\ell\ell$' or 'bb' invariant masses

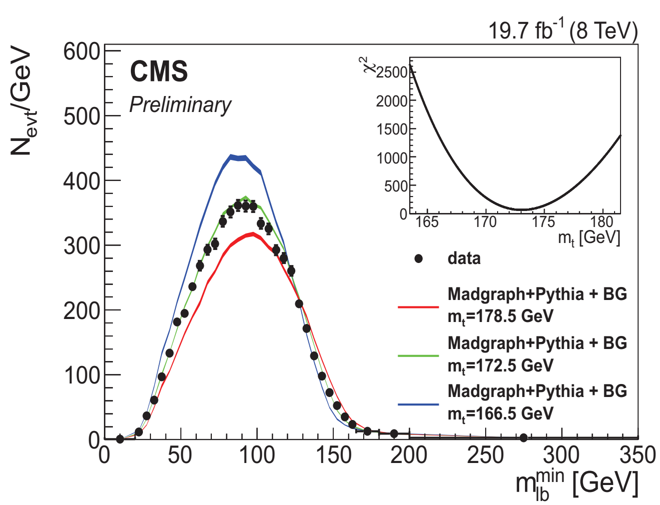


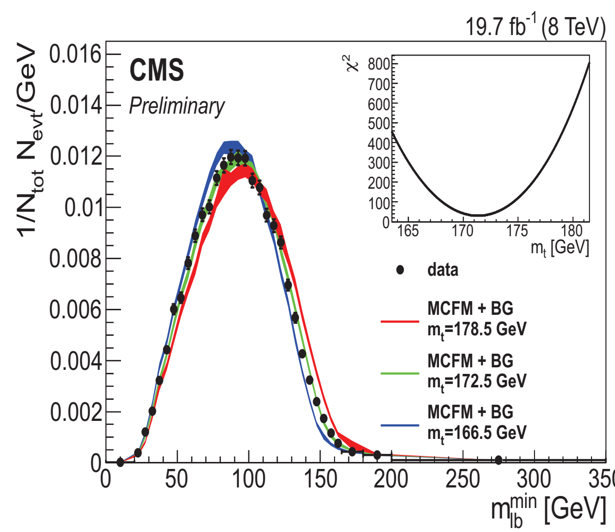


Endpoint relations at LO (no radiative corrections) from energy-momentum conservation

$m_t = 173.9\pm$ 0.9 (stat) $^{+1.7}_{-2.1}$ (syst); $\delta m_t(\mathrm{th}) \simeq \pm 0.6$ GeV

Invariant mass $m_{b\ell}$ vs MadGraph+PYTHIA and MCFM (NLO production, LO decay)





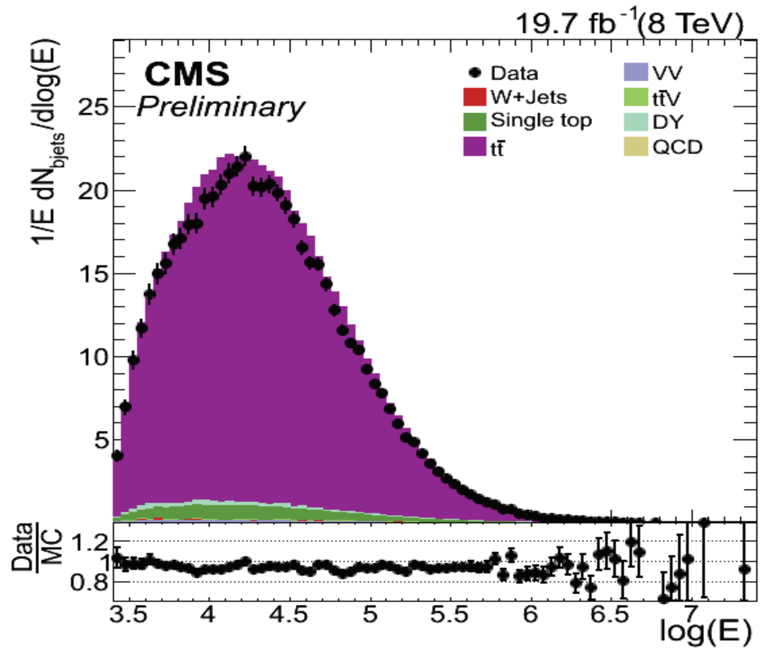
$m_t(\mathrm{PY}) = 172.3^{+1.3}_{-1.3}$ GeV, $m_t(\mathrm{MCFM@LO}) = 171.4^{+1.0}_{-1.1}$ GeV, (NLO in progress)

Top mass measurement from b-jet energy peak (CMS PAS TOP-14-002)

Relation between top mass and b-quark energy in $t \to bW$ in top rest frame:

$$m_t = E_{b,\mathrm{rest}} + \sqrt{m_W^2 - m_b^2 + E_{b,\mathrm{rest}}}$$

Unpolarized top quarks: at LO inclusive E_b-peak roughly independent of the boost and $\ln E_b$ is symmetric around the peak (Agashe, Franceschini, Kim, PRD'13)



CMS investigation: check that boosts change the tails, but peak is roughly invariant

Main systematics: JES (1.2 GeV); MC's (1.5 GeV); top p_T reweighting (1.5 GeV)

Final result: $m_t = [172.3 \pm 1.2(\mathrm{stat}) \pm 2.7(\mathrm{sys})]$ GeV

Perspectives on top mass measurement at the LHC (Snowmass, EPJ'14, to be updated)

Conventional methods:

$\Delta m_t \simeq$ 600-700 MeV (0.3-0.4%)

	Ref.[70]	Projections				
CM Energy	7 TeV	14 TeV				
Cross Section	167 pb	951 pb				
Luminosity	$5 fb^{-1}$	$100 fb^{-1}$		$300 fb^{-1}$		$3000 fb^{-1}$
Pileup	9.3	19	30	19	30	95
Syst. (GeV)	0.95	0.7	0.7	0.6	0.6	0.6
Stat. (GeV)	0.43	0.04	0.04	0.03	0.03	0.01
Total	**1.04**	**0.7**	**0.7**	**0.6**	**0.6**	**0.6**
Total (%)	0.6	0.4	0.4	0.3	0.3	0.3

Endpoint method:

$\Delta m_t \simeq$ 500-1000 MeV (0.3-0.6%)

	Ref.[76]	Projections		
CM Energy	7 TeV	14 TeV		
Cross Section	167 pb	951 pb		
Luminosity	$5 fb^{-1}$	$100 fb^{-1}$	$300 fb^{-1}$	$3000 fb^{-1}$
Syst. (GeV)	1.8	1.0	0.7	0.5
Stat. (GeV)	0.90	0.10	0.05	0.02
Total	**2.0**	**1.0**	**0.7**	**0.5**
Total (%)	1.2	0.6	0.4	0.3

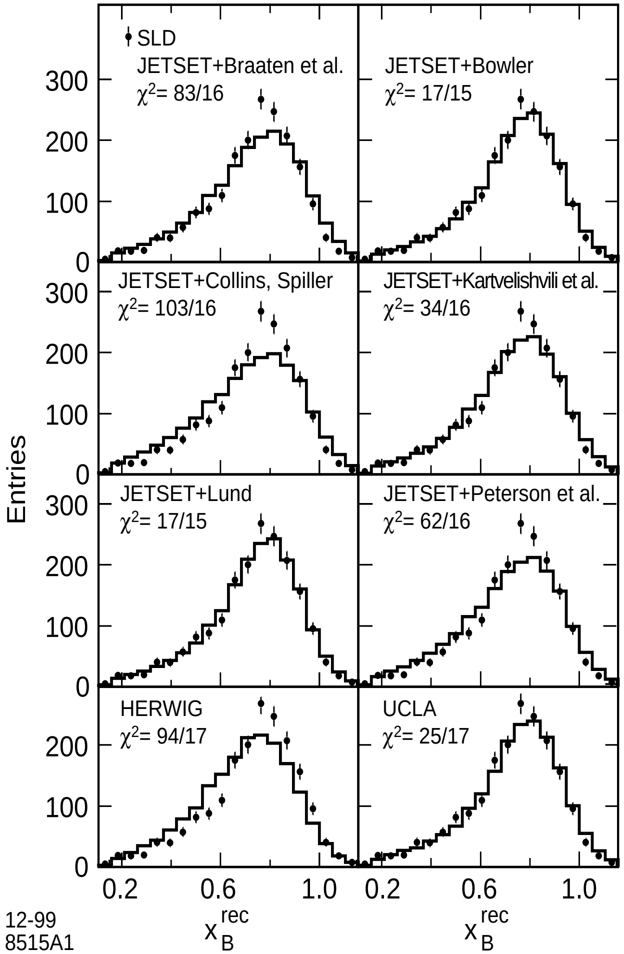
J/ψ method ($m_{J/\psi\ell}$ or $m_{3\ell}$)

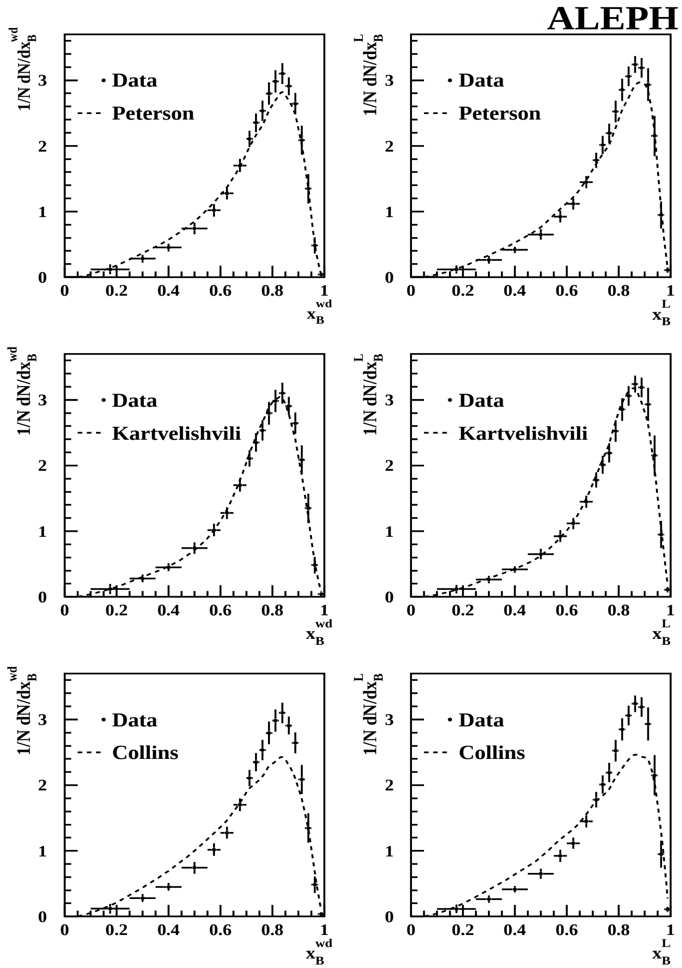
$\Delta m_t \simeq$ 600-2300 MeV (0.4-1.3%)

	Ref. analysis	Projections				
CM Energy	8 TeV	14 TeV			33 TeV	100 TeV
Cross Section	240 pb	951 pb			5522 pb	25562 pb
Luminosity	$20 fb^{-1}$	$100 fb^{-1}$	$300 fb^{-1}$	$3000 fb^{-1}$	$3000 fb^{-1}$	$3000 fb^{-1}$
Theory (GeV)	-	1.5	1.5	1.0	1.0	0.6
Stat. (GeV)	7.00	1.8	1.0	0.3	0.1	0.1
Total	**-**	**2.3**	**1.8**	**1.1**	**1.0**	**0.6**
Total (%)	**-**	**1.3**	**1.0**	**0.6**	**0.6**	**0.4**

Theoretical uncertainties - a case study: b-quark fragmentation

Tune models to $e^+e^- \to b\bar{b}$ data and use them consistently at LHC





LEP tuning of PYTHIA+Peterson used in $J/\psi + \ell$ analysis

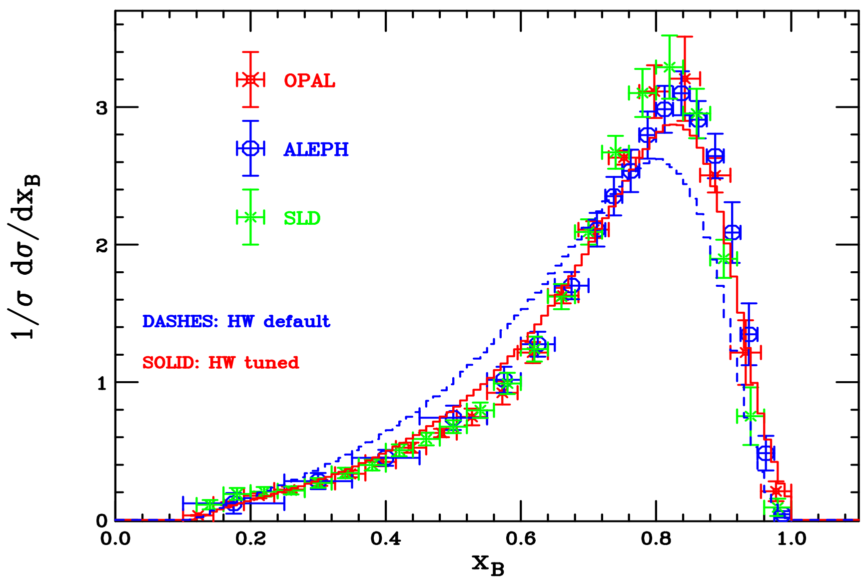
Best-fit parameters not the same, e.g. $\epsilon_b = 0.0033$ (ALEPH), 0.0055 (SLD); $\alpha_K = 11.9$ (OPAL), 13.7 (ALEPH), 10.0 (SLD)

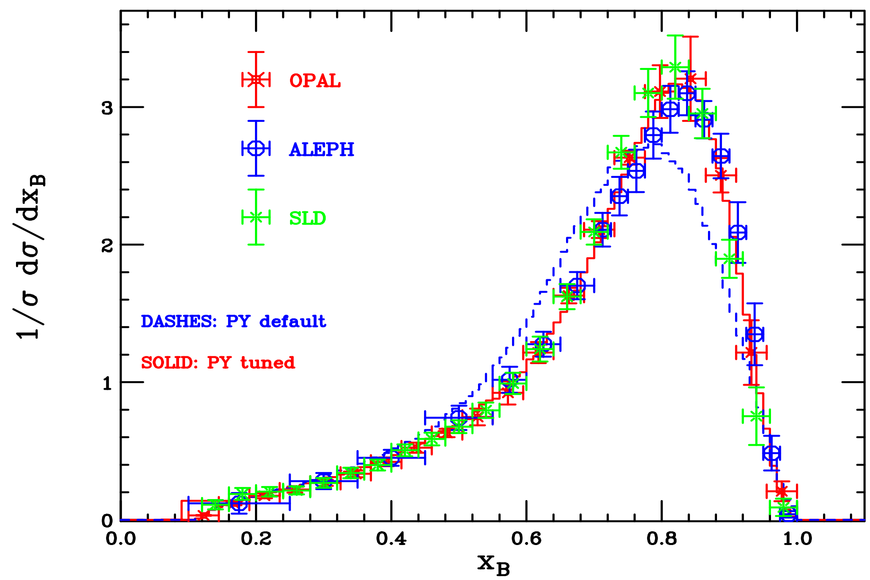
Monte Carlo tuning: $e^+e^- \to Z^0 \to b\bar{b} \to BX_{\bar{b}}$ $\quad x_B = 2E_B/m_Z$

(G. C. and V. Drollinger, NPB 730 (2005) 82)

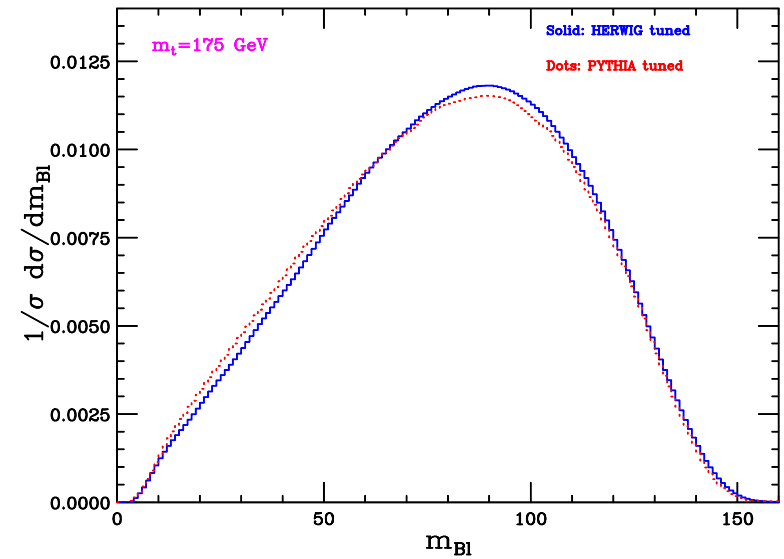
HERWIG	PYTHIA
CLSMR(1) = 0.4 (0.0)	
CLSMR(2) = 0.3 (0.0)	PARJ(41) = 0.85 (0.30)
DECWT = 0.7 (1.0)	PARJ(42) = 1.03 (0.58)
CLPOW = 2.1 (2.0)	PARJ(46) = 0.85 (1.00)
PSPLT(2) = 0.33 (1.00)	
χ^2/dof = 222.4/61 (739.4/61)	χ^2/dof = 45.7/61 (467.9/61)

Lund/Bowler fragmentation function : $f_B(z) \propto \frac{1}{z^{1+brm_b^2}}(1-z)^a \exp(-bm_T^2/z)$

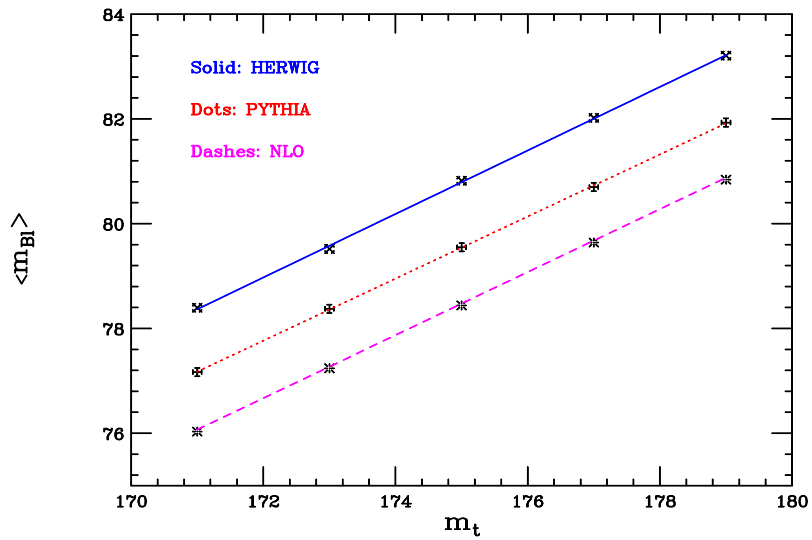




B-lepton invariant mass according to tuned HERWIG and PYTHIA (G.C. and F. Mescia,'10)



Linear fits to extract m_t from $\langle m_{B\ell}\rangle$ using PYTHIA, HERWIG and NLO (S.Biswas et al,'10)



$$\langle m_{B\ell}\rangle_{\rm PY} \simeq -24.11 \text{ GeV} + 0.59\ m_t$$

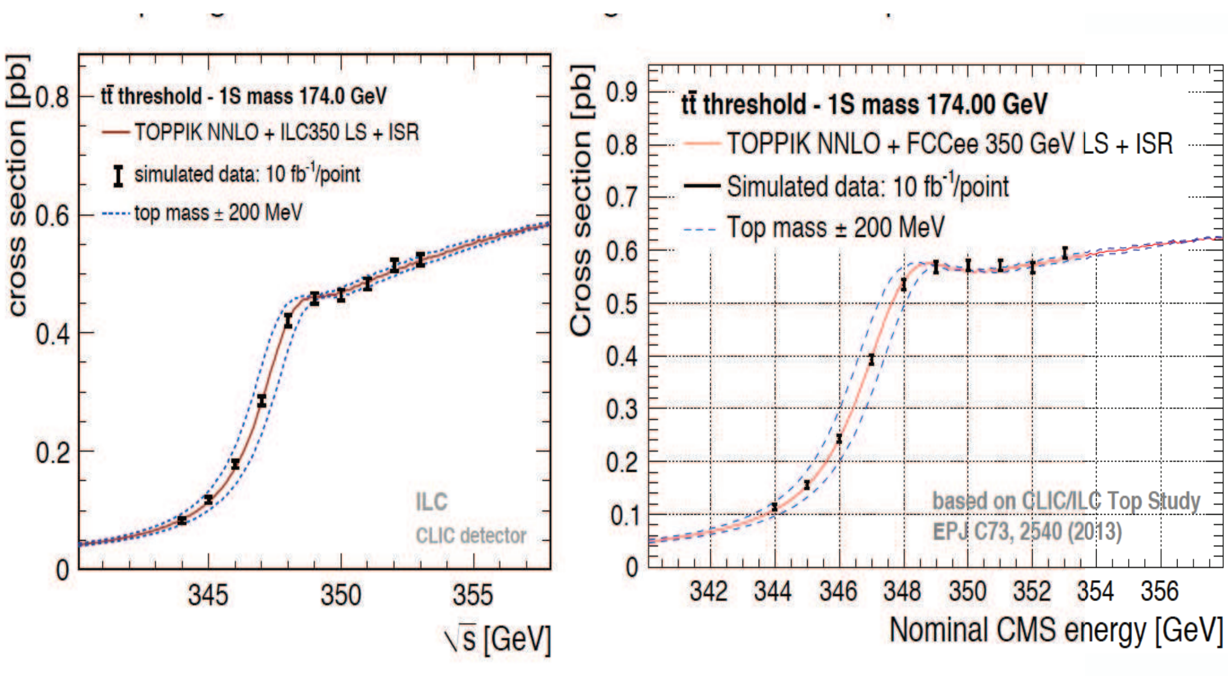
$$\langle m_{B\ell}\rangle_{\rm HW} \simeq -25.31 \text{ GeV} + 0.61\ m_t$$

$$\langle m_{B\ell}\rangle_{\rm NLO} \simeq -26.70 \text{ GeV} + 0.60\ m_t$$

$\Delta\langle m_{B\ell}\rangle_{\rm H,P} \simeq 1.2$ GeV ; $\Delta\langle m_{B\ell}\rangle_{\rm H,NLO} \simeq 2.2$ GeV ; $\Delta\langle m_{B\ell}\rangle_{\rm P,NLO} \simeq 1.1$ GeV

NLO+showers for top decays or C++ codes may shed light on this discrepancy

Perspectives for top mass measurement at lepton colliders (ILC/CLIC, Fcc-ee)



Top mass from threshold scan: potential mass or 1S mass (half the mass of a fictitious $^3S_1$ toponium ground state)

Cross section calculated in NRQCD peaked at $\sqrt{s} \simeq 2m_t$ and strongly dependent on mass, width and coupling constant: $\sigma_{\rm res} \simeq \alpha_S^3/(m_t\Gamma_t)$

Calculations known up to NNNLO (M.Beneke et al,'15)

Total uncertainty at ILC $\mathcal{O}$ (100 MeV), after summing statistics (30 MeV), luminosity (50 MeV), beam energy (35 MeV) and errors on $f(\sqrt{s_{\rm res}}, m_t)$ (80 MeV)

At FCC-ee the uncertainty can go further down to ~ 50 MeV

Conclusions

Top-quark mass is a fundamental SM parameter

Standard reconstruction methods at LHC have reached 0.5% precision

Measured mass close to the pole mass, with renormalon ambiguity even below 100 MeV, after 4-loop calculation of pole vs. $\overline{\rm MS}$ mass relation

Ongoing work using SCET formalism with MSR mass and simulating fictitious T-hadrons may help to shed light on deviations from the pole mass

Higher statistics at Run 2 will reduce the uncertainty on pole mass from $\sigma_{t\bar{t}}$ and $\sigma_{t\bar{t}+j}$ to about 1 GeV

Theoretical uncertainties expected to go down thanks to novel higher order calculations and NLO+shower implementations

Great progress in calculations for $t\bar{t}$ production at threshold at lepton colliders: current theoretical errors $\Delta m_t \leq 50$ MeV

Top phenomenology on the road to become precision physics

Small and understood theory errors and a clear relation between MC and pole masses should be reachable in the near future