High-energy QCD evolution

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dépasser les frontières

Outline

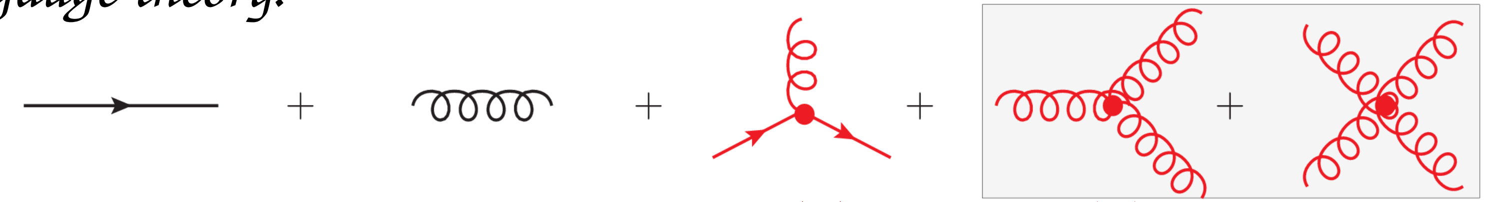
- **Picture of a hadron at high energy**
  - Basic features of quantum chromodynamics
  - Heuristic discussion of quantum fluctuations
  - Evolution of hadronic states towards higher energies: the color dipole model
  - The BFKL equation, the Balitsky-Kovchegov equation and their solutions
- **Probing hadronic wave functions in physical processes**
  - (Mean) parton densities in deep-inelastic scattering
  - Density effects in DIS: How the Balitsky-Kovchegov equation appears
  - How to "see" high-energy evolution in proton-nucleus scattering

Quantum chromodynamics

Theory of the strong interaction

Non-Abelian gauge theory:

$\mathcal{L}_{\text{classical}} =$



Quark propagator | Gluon propagator | $O(g_s)$ | $O(g_s)$ | $O(g_s^2)$

Particles: quarks and gluons; carry *color* charges

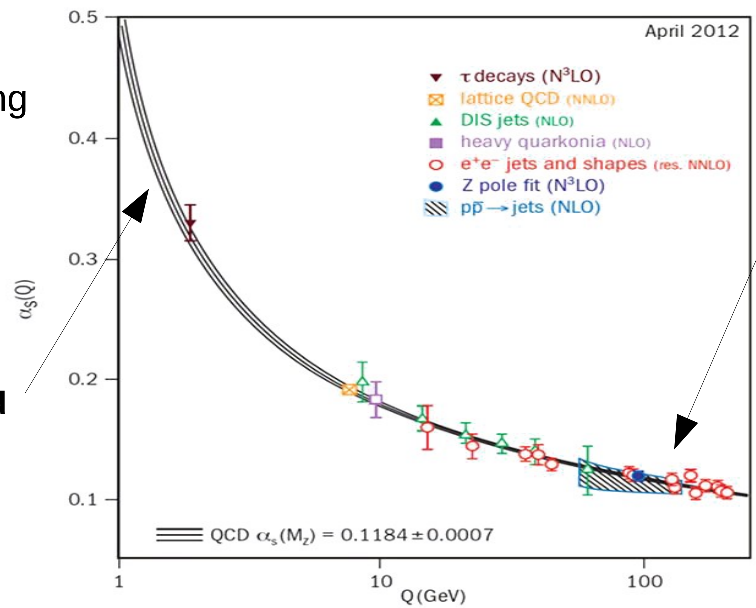
Interactions between color charges

Asymptotically free:

Effective coupling constant:

$$\alpha_s = \frac{g_s^2}{4\pi}$$

Strongly coupled at large distance



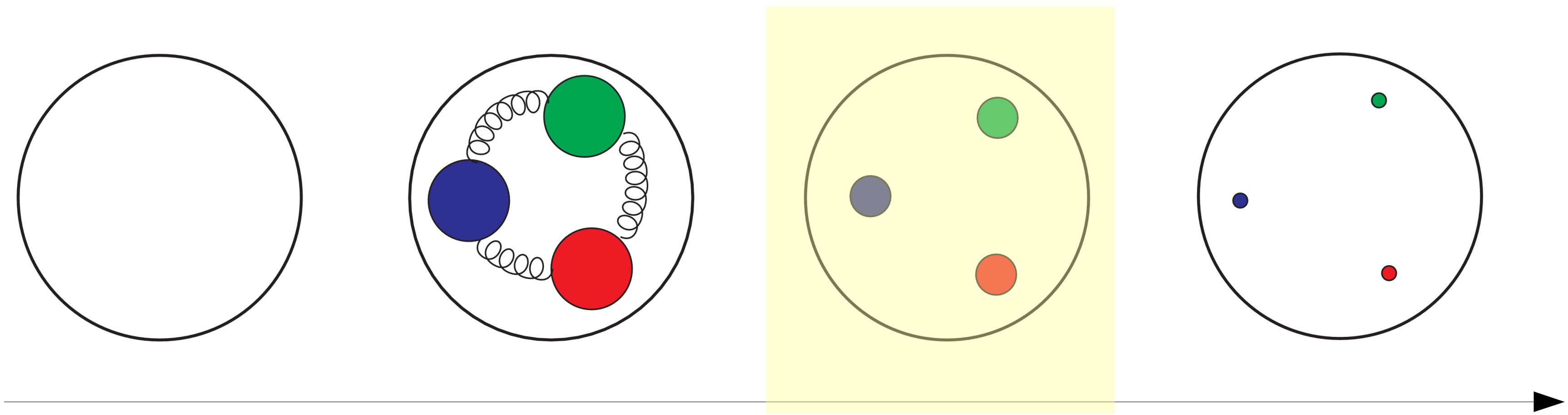
Free theory at small distance = trivial!

[Larger momentum = shorter distance]

Thanks to asymptotic freedom, QCD is a theoretically sound theory, which can also be expanded around the (trivial) free theory at large momenta/small distances!

How a hadron looks at different resolutions

(Over)simplified picture



[Larger momentum = shorter distance = higher "resolution"]

<u>This talk</u>: momenta of a few GeV

Large distances (several fm)	**A fraction of a fm**	**Small distances (much less than 1 fm)**
Colorless extended object (size ~ 1 fm)	"Constituent" quarks	Almost free quarks Pointlike: apparent size given by the spatial resolution

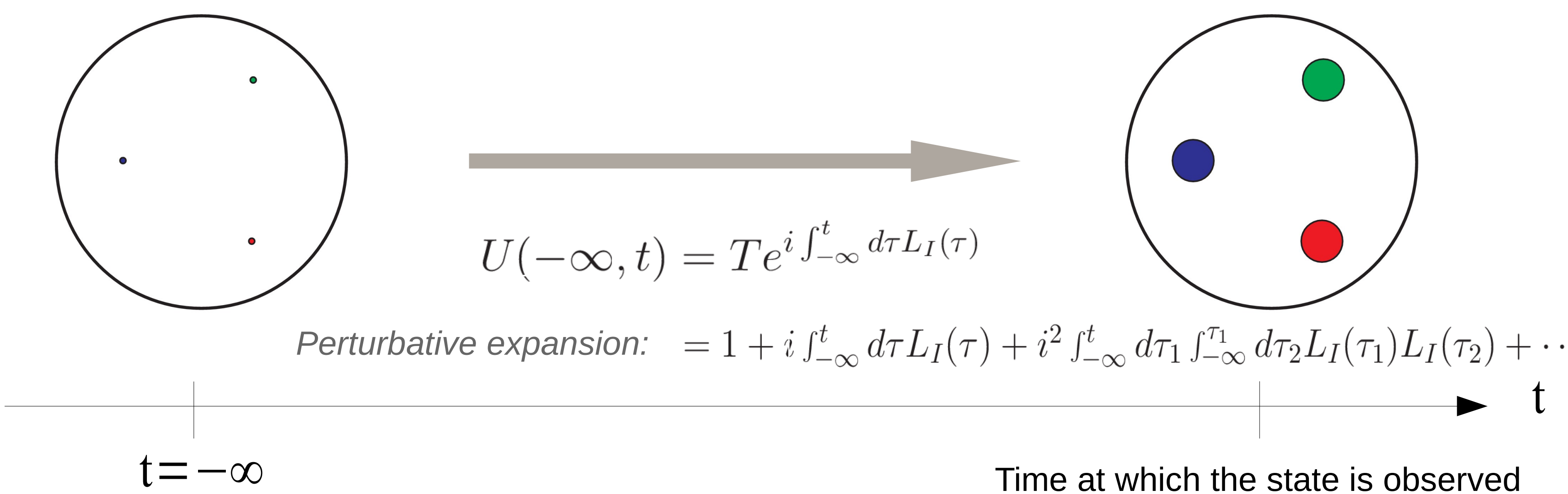
$$\alpha_s = O(1)$$

Strongly coupled quantum field theory
No analytical methods...

$$\alpha_s \ll 1$$

The interactions are small corrections
Perturbation theory applies!

Quantum dynamics



$$U(-\infty, t) = T e^{i \int_{-\infty}^{t} d\tau L_I(\tau)}$$

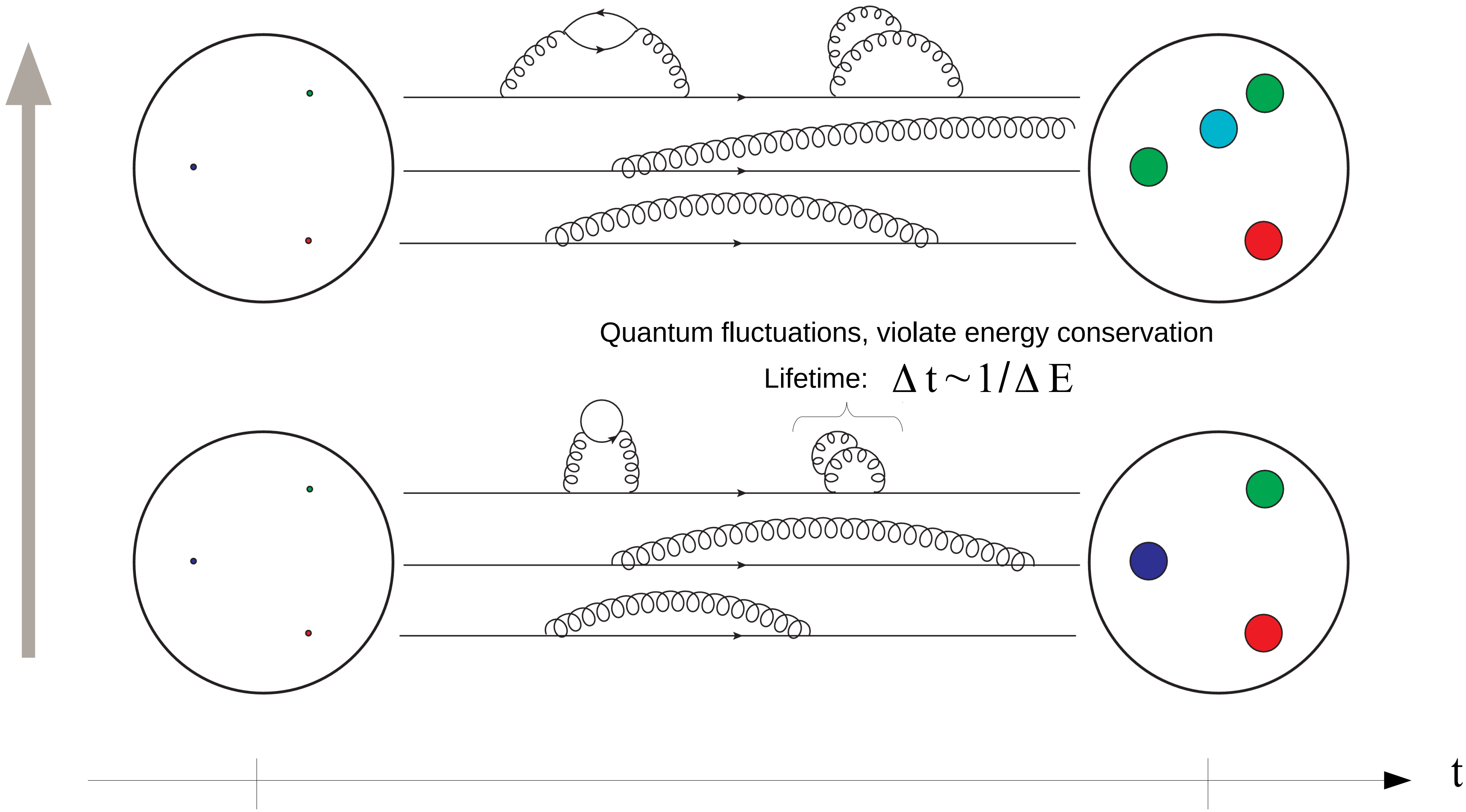
Perturbative expansion: $= 1 + i \int_{-\infty}^{t} d\tau L_I(\tau) + i^2 \int_{-\infty}^{t} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 L_I(\tau_1) L_I(\tau_2) + \cdots$

Proton in its ground state
= 3 pointlike quarks

Quantum dynamics

Faster hadron in the frame
of the observer:

Fluctuations are longer-lived due to Lorentz time dilation!

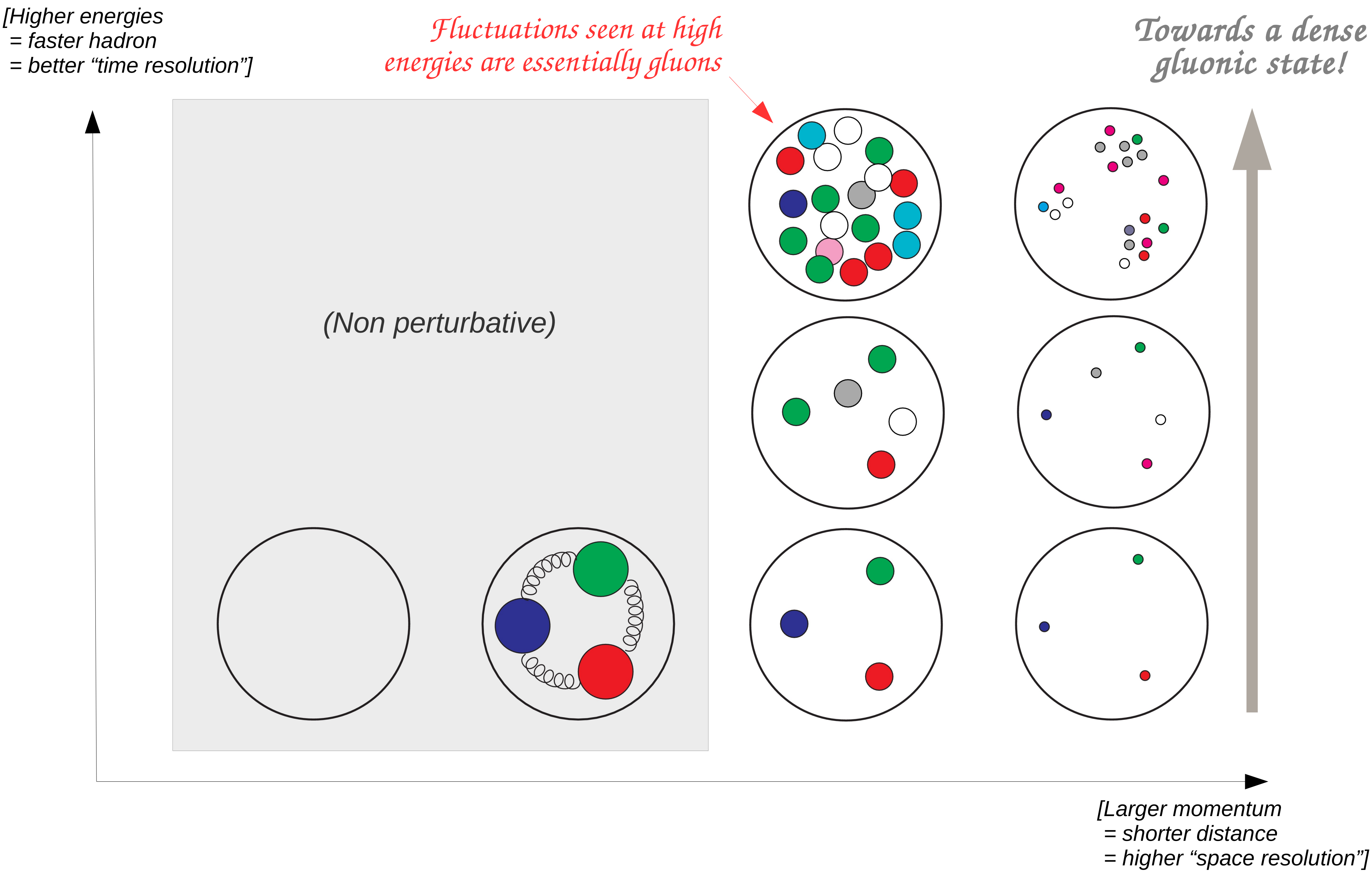


$t = -\infty$

Time at which the state is observed

Proton in its ground state
= 3 pointlike quarks

Picture of a hadron

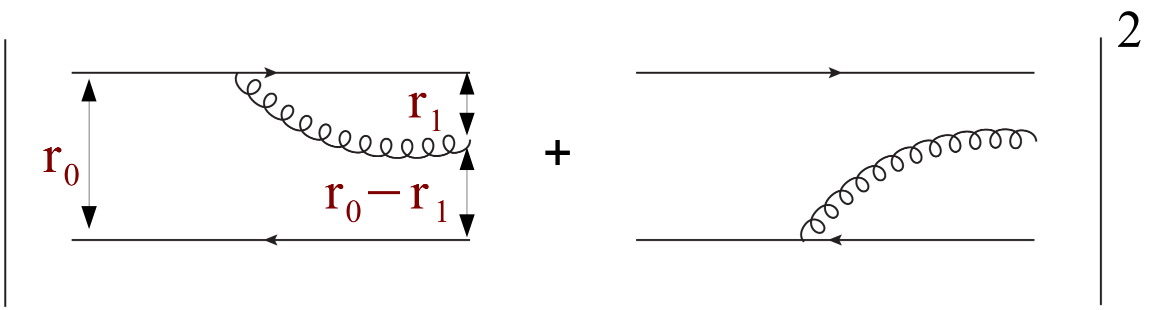


QCD calculation: the color dipole model

*To simplify, we start with a **color neutral quark-antiquark pair** (=meson) of given transverse size.*

*Probability of observing 1 gluon fluctuation when one increases the rapidity **y** by **dy**:*

dP =



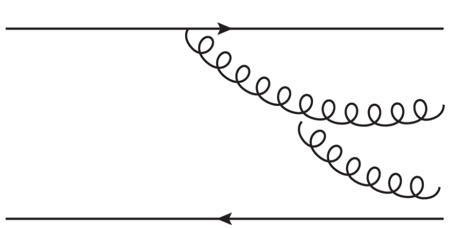
$$= dy\, \frac{\alpha_s (N_c^2 - 1)}{\pi N_c} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} \frac{d^2 r_1}{2\pi}$$

$r_1 =$ position of the gluon with respect to the quark

Singular when the gluon is close to the quark or to the antiquark
Collinear singularity

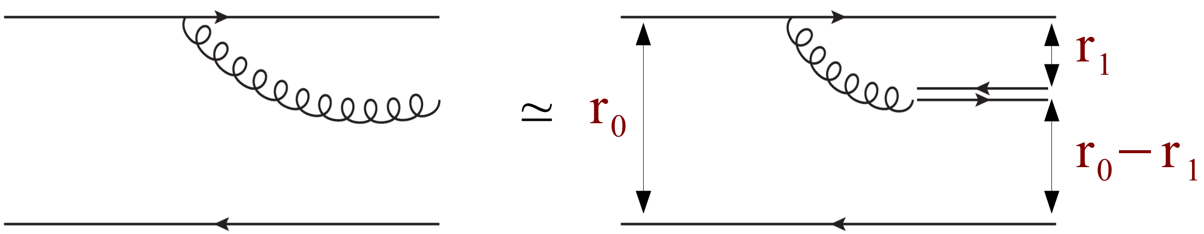
When $y \sim \frac{1}{\alpha_s (N_c^2 - 1)/N_c}$ *, the probability to observe a fluctuation with* $|r_1| \sim |r_0|$ *is O(1)*

One needs to consider higher-order fluctuations:



QCD calculation: the color dipole model

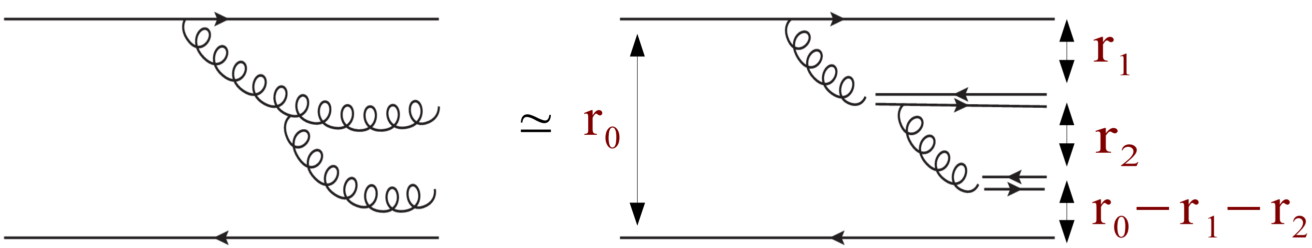
Trick: ***large number-of-color limit!***



Gluon emission is interpreted as a color-dipole splitting, with proba.

$$dP = dy\,\frac{\alpha_s N_c}{\pi}\,\frac{r_0^2}{r_1^2 (r_0 - r_1)^2}\,\frac{d^2 r_1}{2\pi}$$

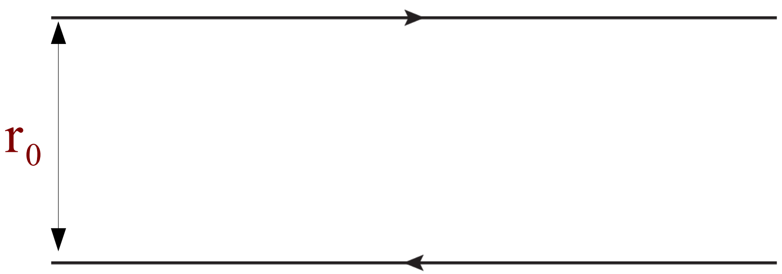
Higher-order fluctuations are generated by a branching process:

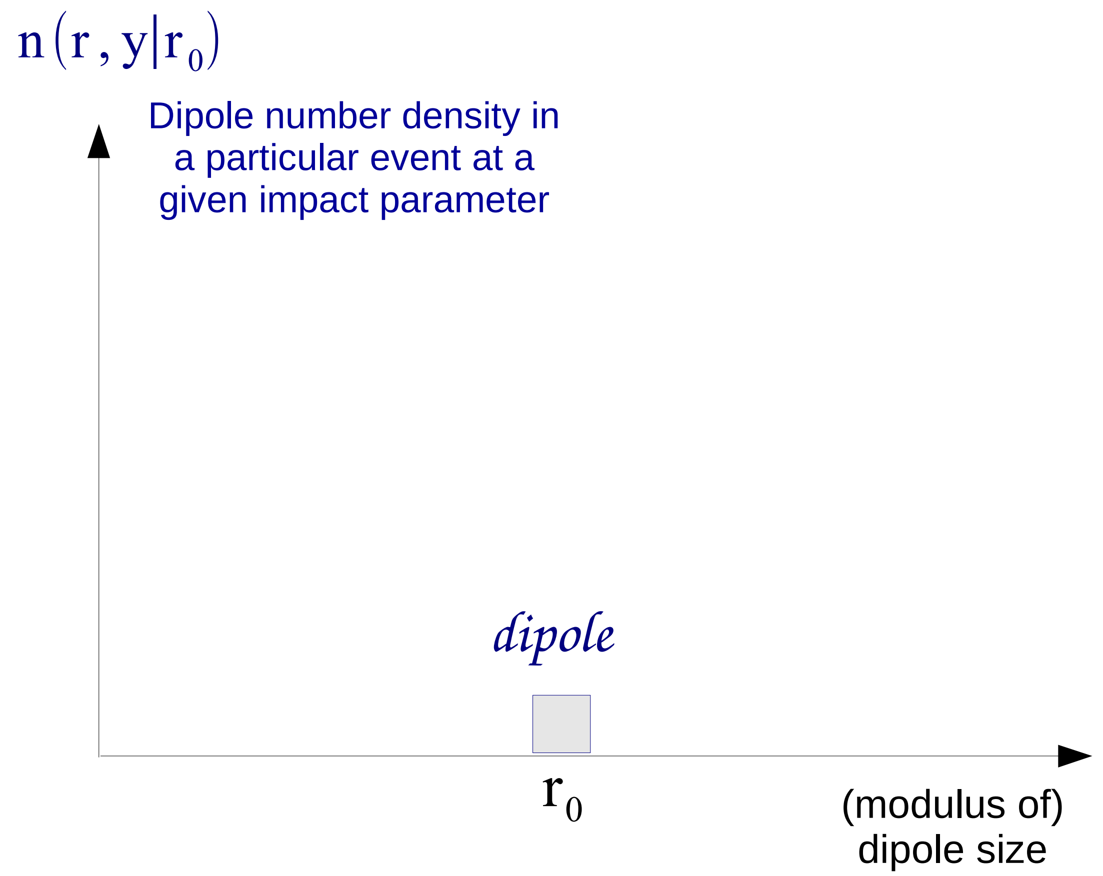


Two successive dipole branchings

Gluons density of momentum k ~ mean density of dipoles of size 1/k

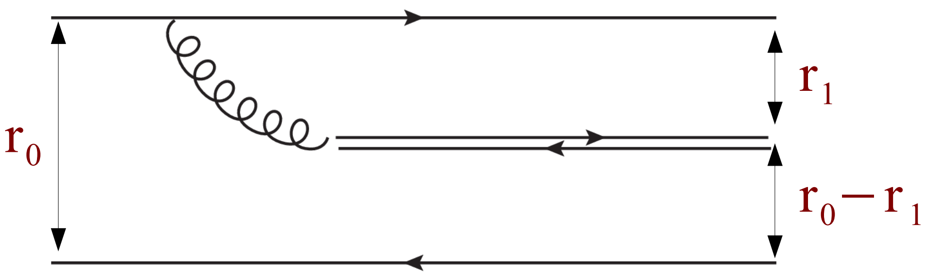
How the dipole model works

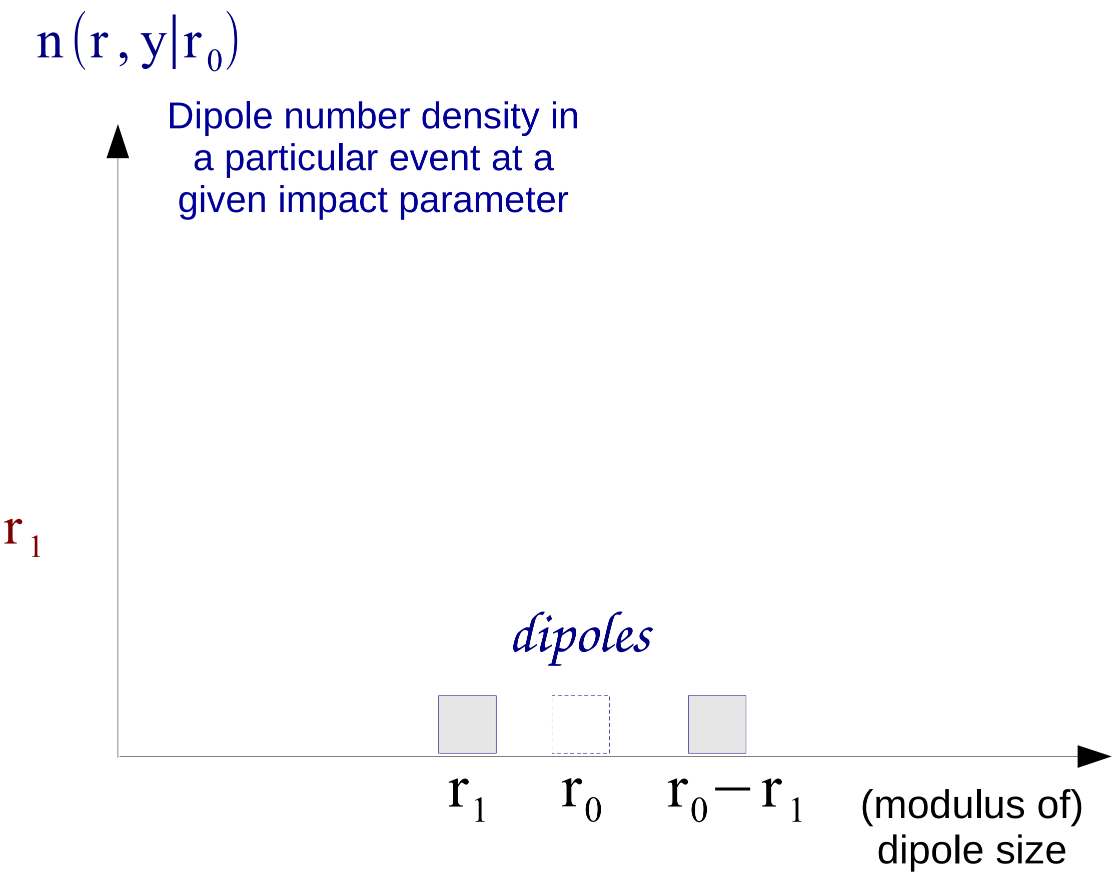




$y = 0$

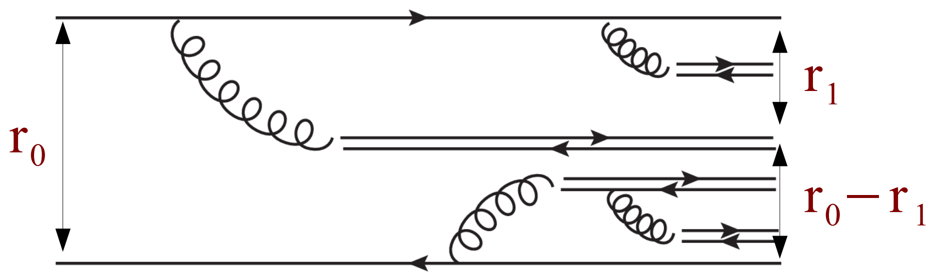
How the dipole model works

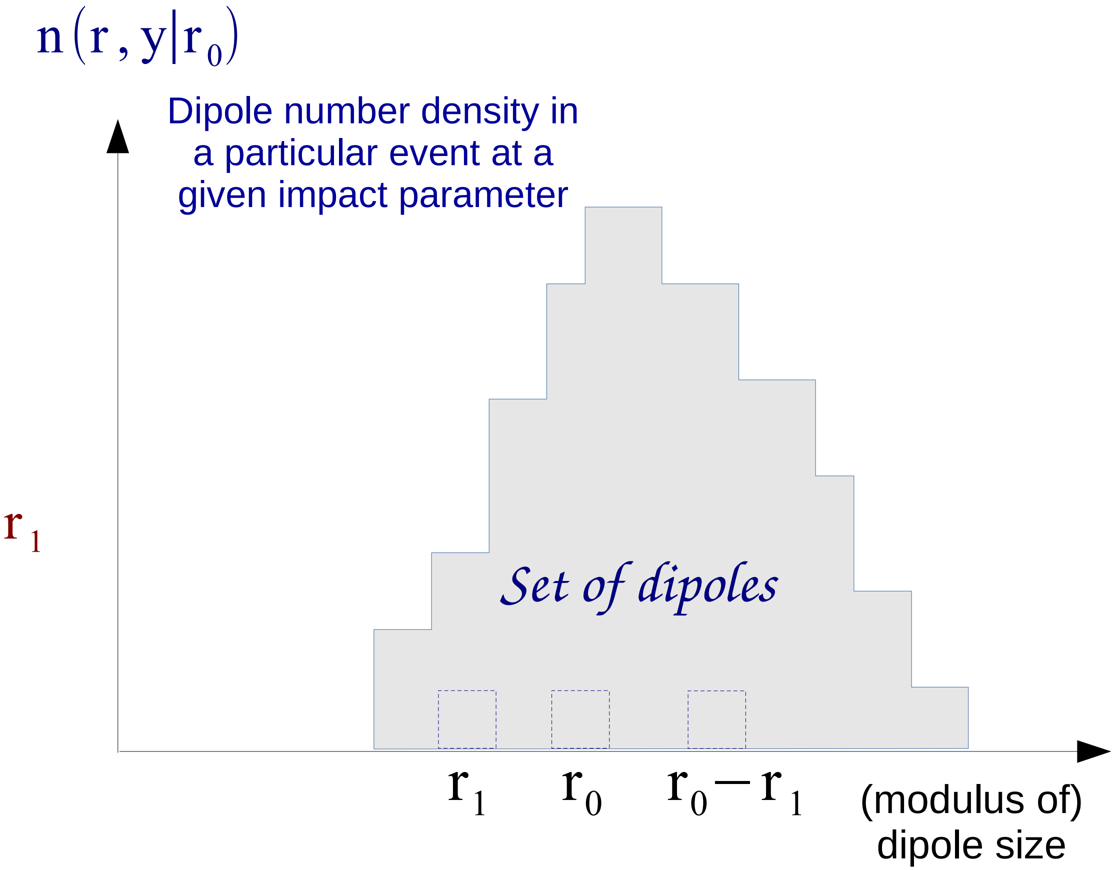




$$y \sim \frac{1}{\alpha_s N_c}$$

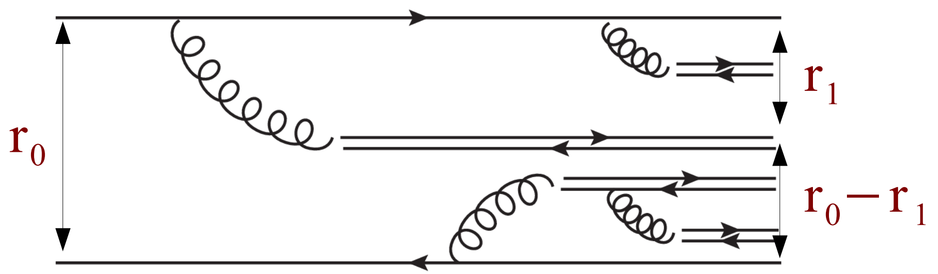
How the dipole model works

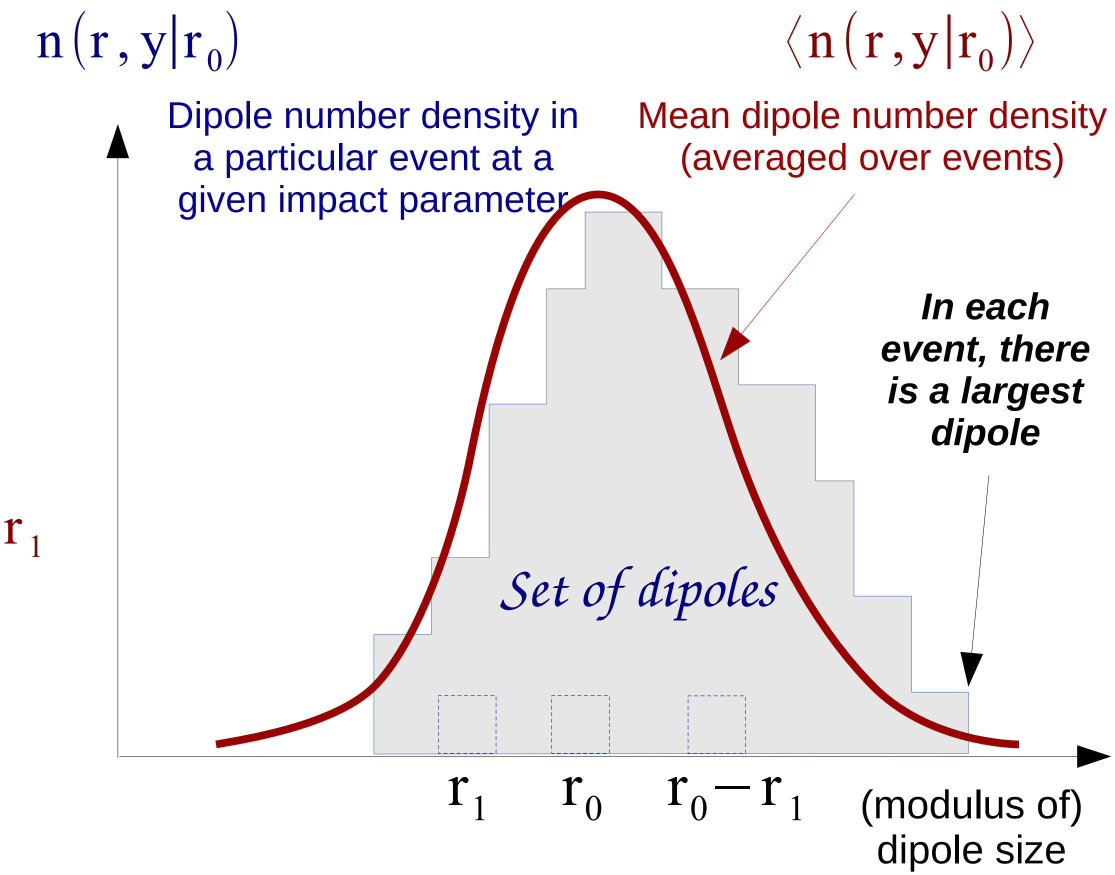




$$y \gg \frac{1}{\alpha_s N_c}$$

How the dipole model works



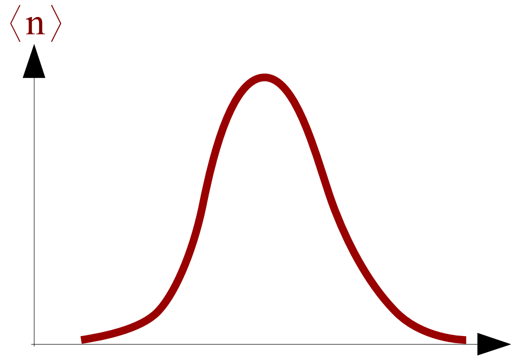


$$y \gg \frac{1}{\alpha_s N_c}$$

Now, establish and solve equations for the mean dipole number density, and for the probability distribution of the size of the largest dipole!

Mean dipole number density

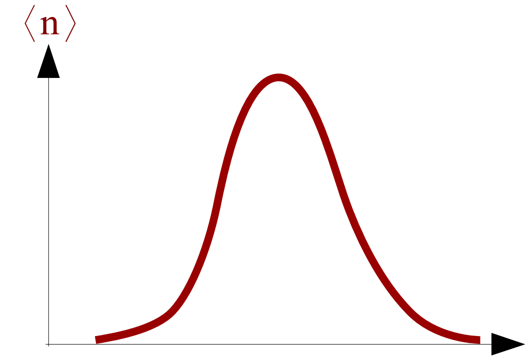
$\langle n(r,y|r_0)\rangle$ **=mean density of dipoles of size r at rapidity y**

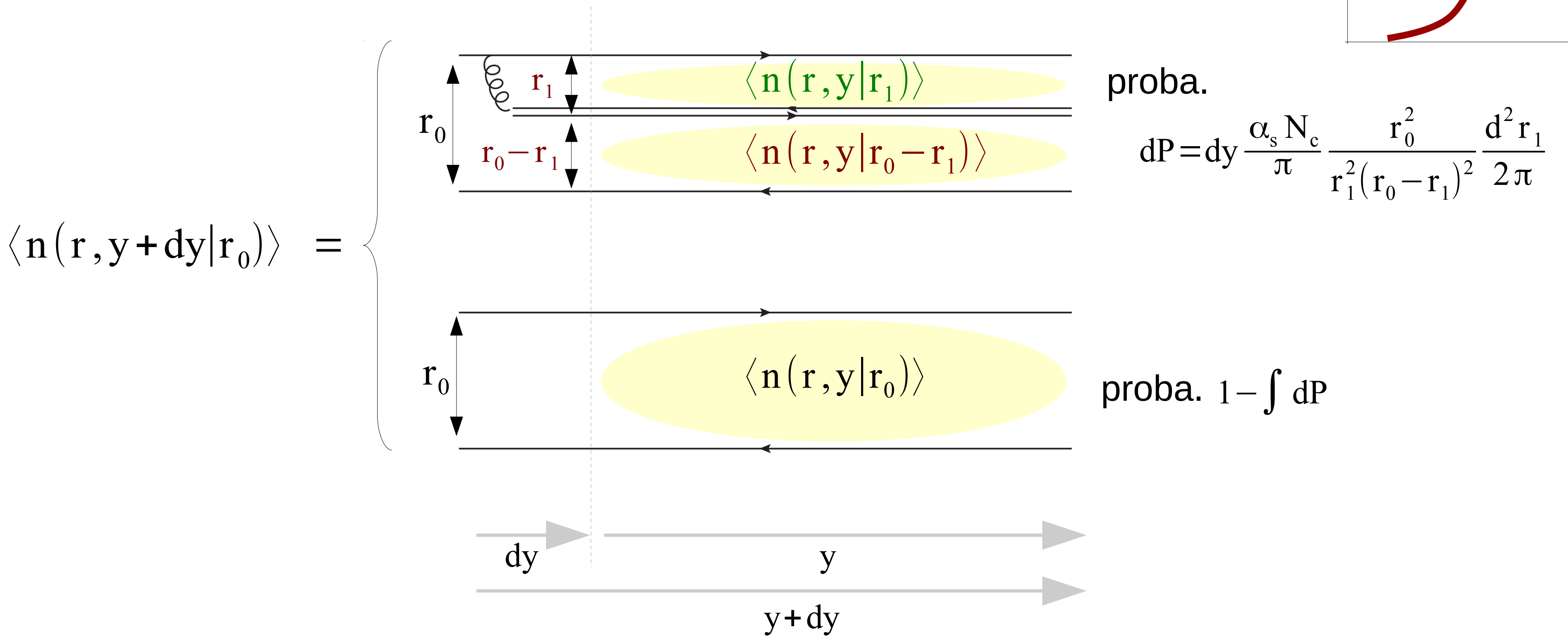


$$\langle n(r,y+dy|r_0)\rangle \quad = \quad \text{functional of } \langle n(r,y|r_0)\rangle$$

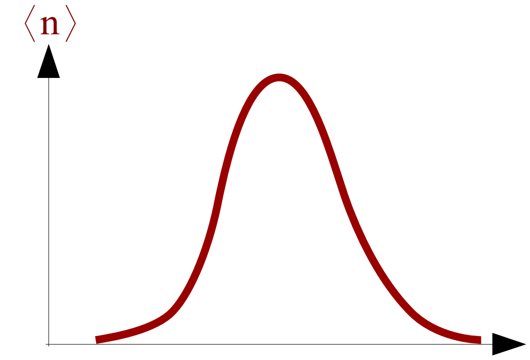
Mean dipole number density

$\langle n(r,y|r_0)\rangle$ **=mean density of dipoles of size r at rapidity y**



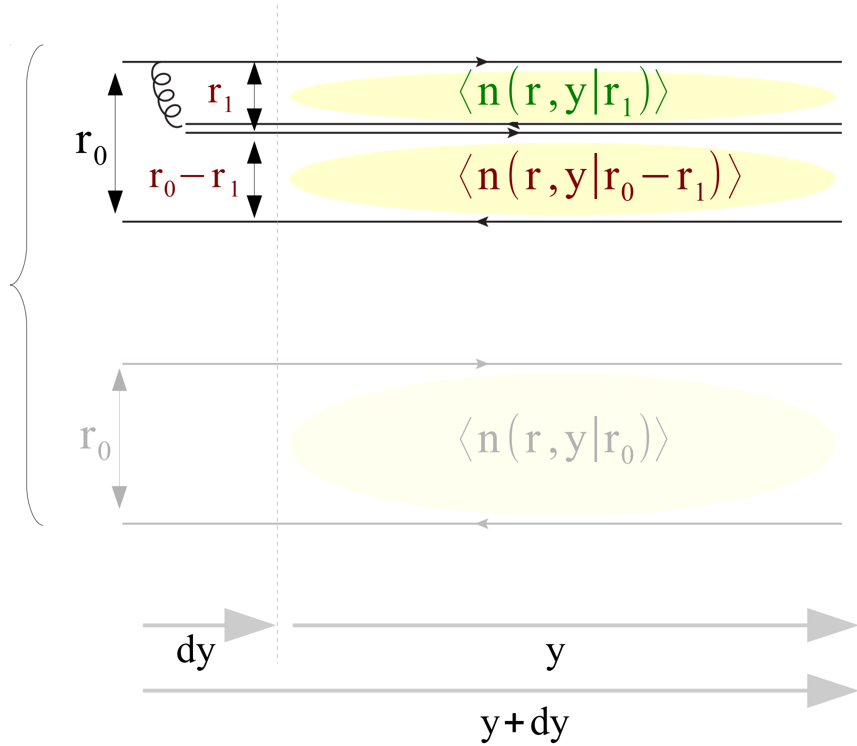


Mean dipole number density



$\langle n(r,y|r_0)\rangle$ **=mean density of dipoles of size r at rapidity y**

$$\langle n(r,y+dy|r_0)\rangle =$$



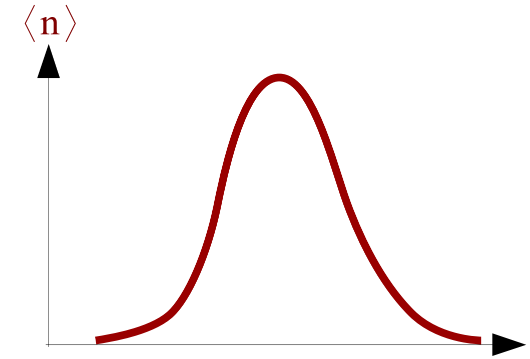
proba.

$$dP = dy\,\frac{\alpha_s N_c}{\pi}\,\frac{r_0^2}{r_1^2 (r_0 - r_1)^2}\,\frac{d^2 r_1}{2\pi}$$

proba. $1 - \int dP$

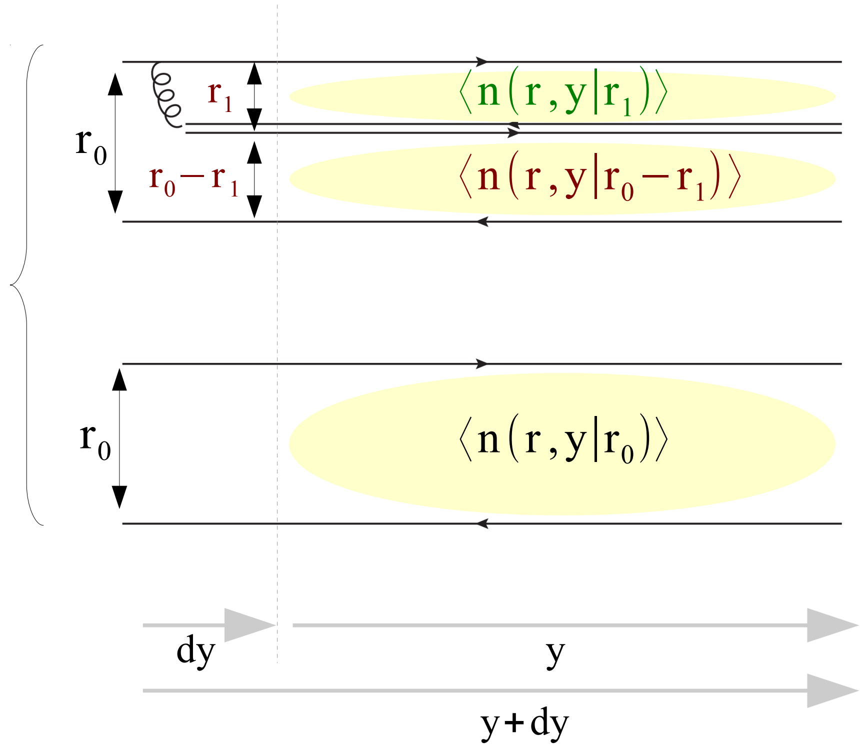
$$\langle n(r,y+dy|r_0)\rangle = \int dP\left[\langle n(r,y|r_1)\rangle + \langle n(r,y|r_0 - r_1)\rangle\right]$$

Mean dipole number density



$\langle n(r,y|r_0)\rangle$ **=mean density of dipoles of size r at rapidity y**

$\langle n(r,y+dy|r_0)\rangle \;=$



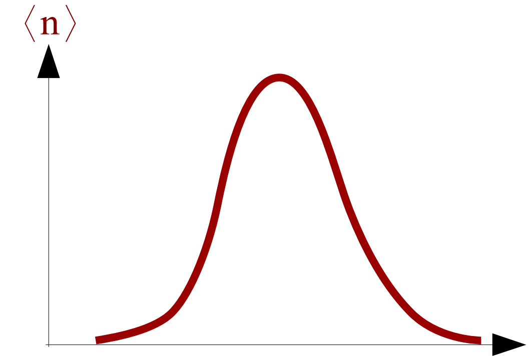
proba. $dP = dy\,\frac{\alpha_s N_c}{\pi}\,\frac{r_0^2}{r_1^2(r_0-r_1)^2}\,\frac{d^2 r_1}{2\pi}$

proba. $1-\int dP$

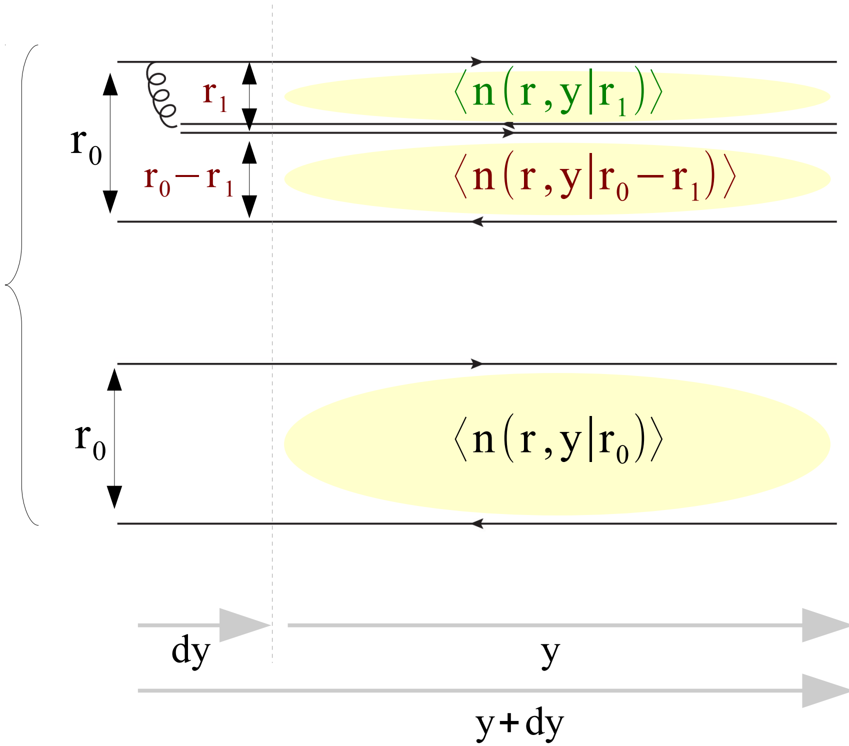
$$\langle n(r,y+dy|r_0)\rangle = \int dP\left[\langle n(r,y|r_1)\rangle + \langle n(r,y|r_0-r_1)\rangle\right] + \left(1-\int dP\right)\langle n(r,y|r_0)\rangle$$

Mean dipole number density

$\langle n(r,y|r_0)\rangle$ **=mean density of dipoles of size r at rapidity y**



$\langle n(r,y+dy|r_0)\rangle \; =$



proba. $dP = dy \frac{\alpha_s N_c}{\pi} \frac{r_0^2}{r_1^2 (r_0-r_1)^2} \frac{d^2 r_1}{2\pi}$

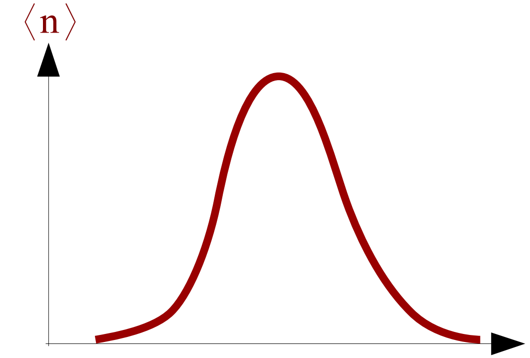
proba. $1-\int dP$

$$\langle n(r,y+dy|r_0)\rangle = \int dP \left[\langle n(r,y|r_1)\rangle + \langle n(r,y|r_0-r_1)\rangle\right] + \left(1-\int dP\right)\langle n(r,y|r_0)\rangle$$

$$\frac{\partial}{\partial y}\langle n(r,y|r_0)\rangle = \frac{\alpha_s N_c}{\pi}\int \frac{d^2 r_1}{2\pi}\frac{r_0^2}{r_1^2 (r_0-r_1)^2}\left[\langle n(r,y|r_1)\rangle + \langle n(r,y|r_0-r_1)\rangle - \langle n(r,y|r_0)\rangle\right]$$

BFKL (linear) equation

The BFKL equation and its solution



$\langle n(r,y|r_0)\rangle$ **=mean density of dipoles of size r at rapidity y**

$$\frac{\partial}{\partial y}\langle n(r,y|r_0)\rangle = \frac{\alpha_s N_c}{\pi}\int \frac{d^2 r_1}{2\pi}\frac{r_0^2}{r_1^2 (r_0-r_1)^2}\left[\langle n(r,y|r_1)\rangle + \langle n(r,y|r_0-r_1)\rangle - \langle n(r,y|r_0)\rangle\right]$$

Linear integro-differential equation

Standard method: look for functions which diagonalize of the kernel

eigenfunctions $\langle n(r|r_0)\rangle \sim (r^2/r_0^2)^{\gamma}$

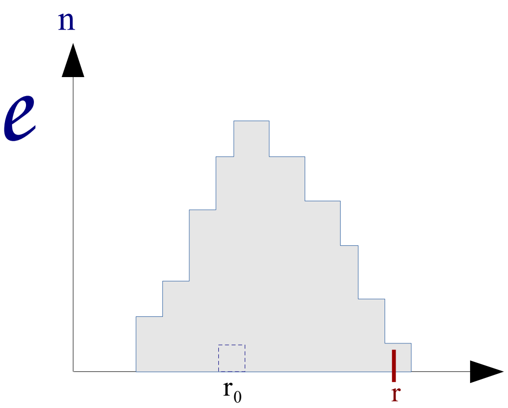
eigenvalues $\frac{\alpha_s N_c}{\pi}\chi(\gamma)$, $\chi(\gamma)\equiv 2\psi(1)-\psi(\gamma)-\psi(1-\gamma)$

Solution: $\langle n\rangle = \frac{1}{r^2}\int \frac{d\gamma}{2i\pi}\left(\frac{r^2}{r_0^2}\right)^{\gamma} e^{\frac{\alpha_s N_c}{\pi}\chi(\gamma)\times y}$

Main feature at large rapidity: exponential growth of the mean dipole density:

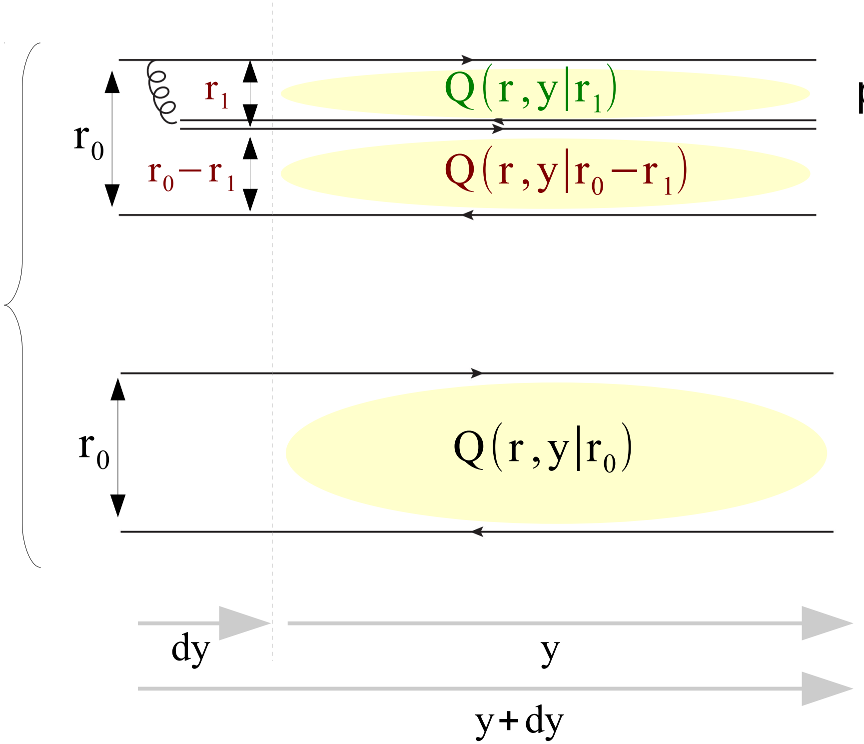
$$\langle n\rangle \sim \exp\left(\frac{\alpha_s N_c}{\pi} 4\ln 2\times y\right)$$

Probability distribution of the largest size



$Q(r,y|r_0)$ =**probability that all dipoles have a size smaller than r**

$Q(r,y+dy|r_0) \;=$



proba

$$dP = dy\,\frac{\alpha_s N_c}{\pi}\,\frac{r_0^2}{r_1^2(r_0-r_1)^2}\,\frac{d^2 r_1}{2\pi}$$

proba. $1-\int dP$

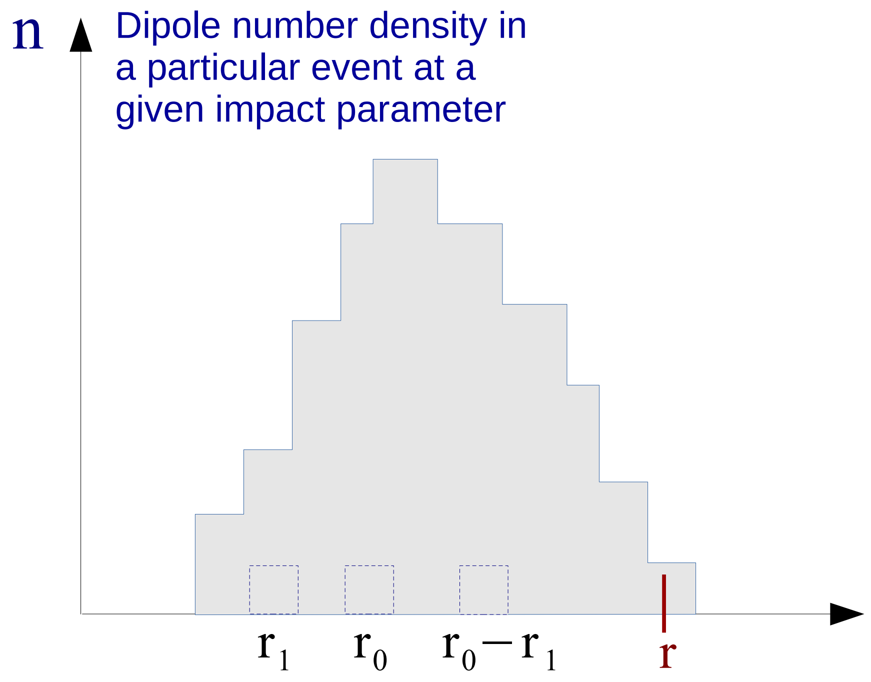
$$Q(r,y+dy|r_0)=\int dP\left[Q(r,y|r_1)\times Q(r,y|r_0-r_1)\right]+\left(1-\int dP\right)Q(r,y|r_0)$$

$$\frac{\partial}{\partial y}Q(r,y|r_0)=\frac{\alpha_s N_c}{\pi}\int\frac{d^2r_1}{2\pi}\,\frac{r_0^2}{r_1^2(r_0-r_1)^2}\left[Q(r,y|r_1)\times Q(r,y|r_0-r_1)-Q(r,y|r_0)\right]$$

Balitsky-Kovchegov (nonlinear) equation!

The BK equation and its solution

$T = 1 - Q =$ **probability that at least one dipole has a size larger than r**

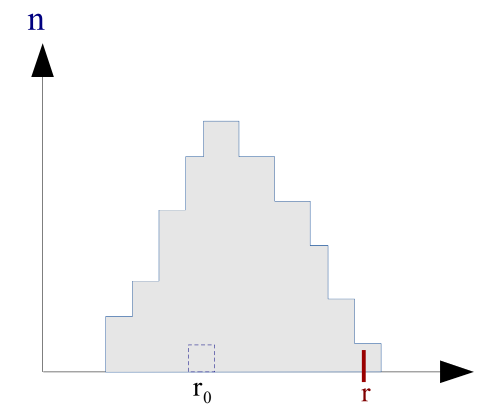


$$\partial_y T(r, y|r_0) = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 r_1}{2\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} \left[\underbrace{T(r, y|r_1) + T(r, y|r_0 - r_1) - T(r, y|r_0)}_{\text{Linear part: BFKL equation}} - T(r, y|r_1) T(r, y|r_0 - r_1) \right]$$

Nonlinear integro-differential equation

The BK equation and its solution

$T = 1 - Q =$ **probability that at least one dipole has a size larger than r**



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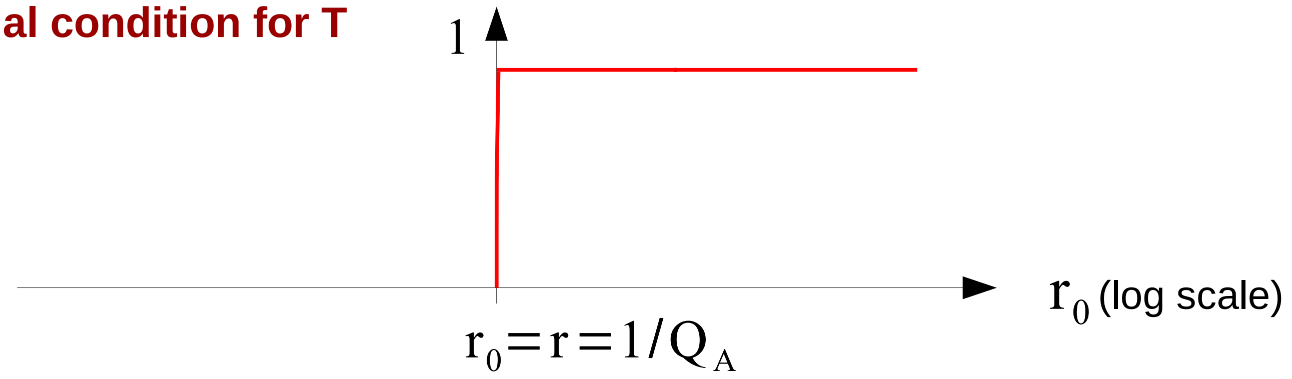
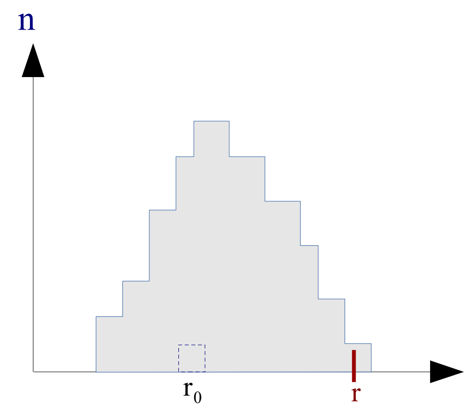
Linear part: BFKL equation

Initial condition for T



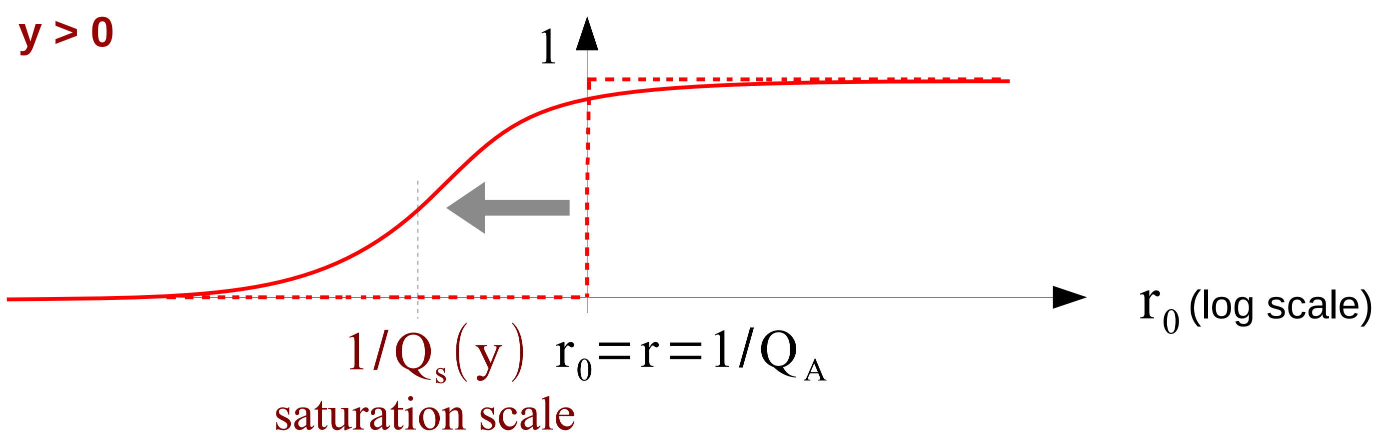
The BK equation and its solution

$T = 1 - Q =$ **probability that at least one dipole has a size larger than r**



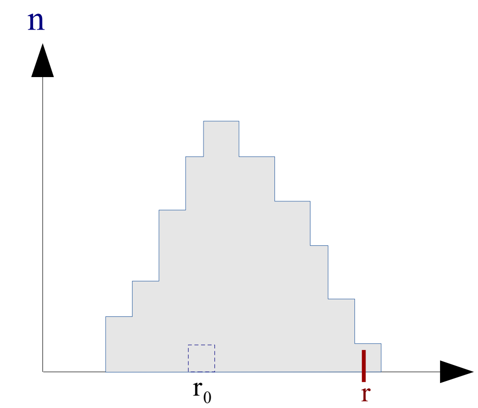
$$\partial_y T(r,y|r_0) = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 r_1}{2\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} \left[T(r,y|r_1) + T(r,y|r_0 - r_1) - T(r,y|r_0) - T(r,y|r_1) T(r,y|r_0 - r_1) \right]$$

Linear part: BFKL equation

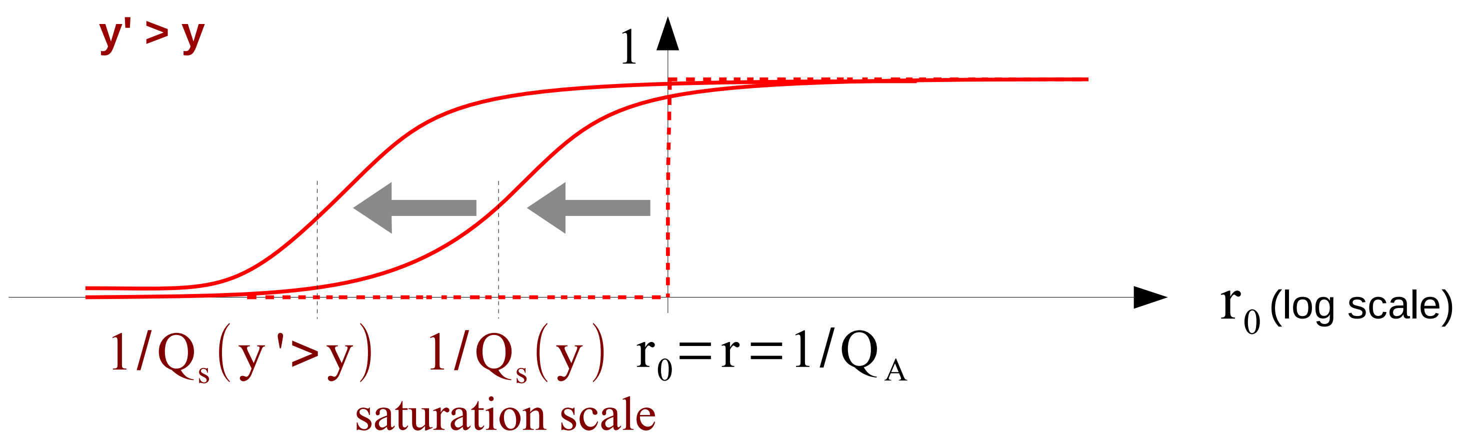


The BK equation and its solution

$T = 1 - Q =$ **probability that at least one dipole has a size larger than r**

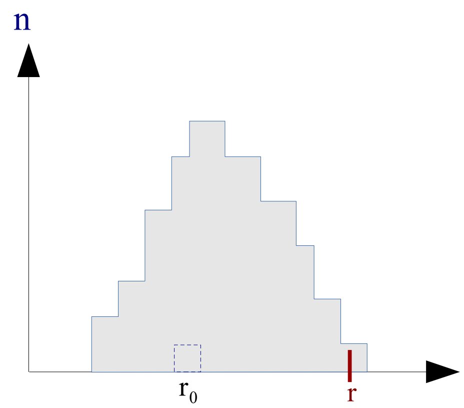


$$\partial_y T(r,y|r_0) = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 r_1}{2\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} \left[\underbrace{T(r,y|r_1) + T(r,y|r_0 - r_1) - T(r,y|r_0)}_{\text{Linear part: BFKL equation}} - T(r,y|r_1) T(r,y|r_0 - r_1) \right]$$



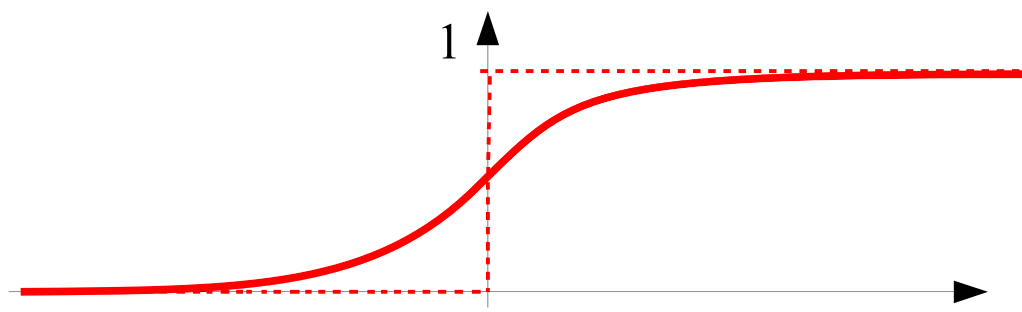
The BK equation and its solution

$T = 1 - Q =$ **probability that at least one dipole has a size larger than r**



$$\partial_y T(r,y|r_0) = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 r_1}{2\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} \left[\underbrace{T(r,y|r_1) + T(r,y|r_0 - r_1) - T(r,y|r_0)}_{\text{Linear part: BFKL equation}} - T(r,y|r_1) T(r,y|r_0 - r_1) \right]$$

Linear part: BFKL equation

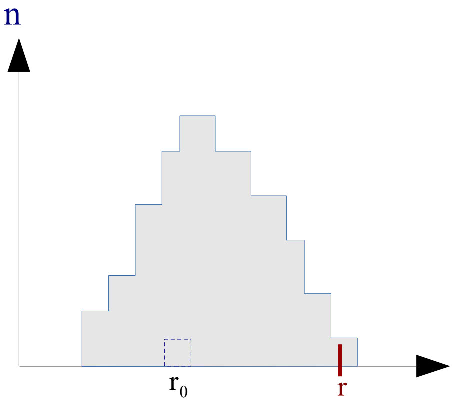


$$T(r,y|r_0) \underset{y \gg \frac{1}{\alpha_s N_c}}{\sim} \text{function of } (r_0 Q_s(y))$$

Traveling wave property

The BK equation and its solution

$T = 1 - Q =$ **probability that at least one dipole has a size larger than r**



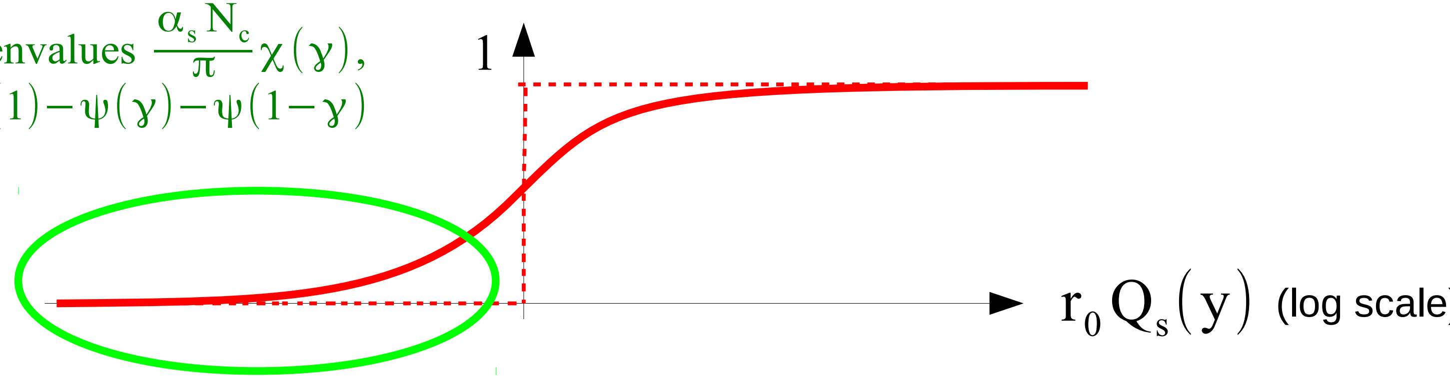
$$\partial_y T(r,y|r_0) = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 r_1}{2\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} \left[T(r,y|r_1) + T(r,y|r_0 - r_1) - T(r,y|r_0) - T(r,y|r_1) T(r,y|r_0 - r_1) \right]$$

Linear part: BFKL equation

eigenfunctions $T(r|r_0) \sim (r_0^2 / r^2)^\gamma$,

eigenvalues $\frac{\alpha_s N_c}{\pi} \chi(\gamma)$,

$\chi(\gamma) \equiv 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$



$$T(r,y|r_0) \underset{y \gg \frac{1}{\alpha_s N_c}}{\sim} \text{function of } (r_0 Q_s(y))$$

Traveling wave property

$$T \underset{r_0 Q_s(y) \ll 1}{\sim} \ln \frac{1}{r_0^2 Q_s^2(y)} \left(r_0^2 Q_s^2(y) \right)^{\gamma_0}$$

$$Q_s^2(y) \simeq Q_A^2 e^{\frac{\alpha_s N_c}{\pi} \chi'(\gamma_0) y}$$

γ_0 solves $\gamma_0 \chi'(\gamma_0) = \chi(\gamma_0)$

Intermediate recap

- A fast (="rapid") hadron appears as a **dense state of gluons**.
- The building up of these quantum states as rapidity increases can be represented by the **color dipole model**, which provides a **systematic perturbative QCD calculation** in the limit of high rapidity and of large number of colors.
- The dipole model reduces QCD evolution to a **statistical branching process**. Such processes are quite general, and appear in many different fields of science.
- The **mean dipole density** obeys the **linear BFKL evolution equation**, while the **event-by-event distribution of the size of the largest dipole** (= statistics of extremes) obeys the **nonlinear BK evolution equation**.

NB: This particular interpretation of the BK equation is not the only possible one!

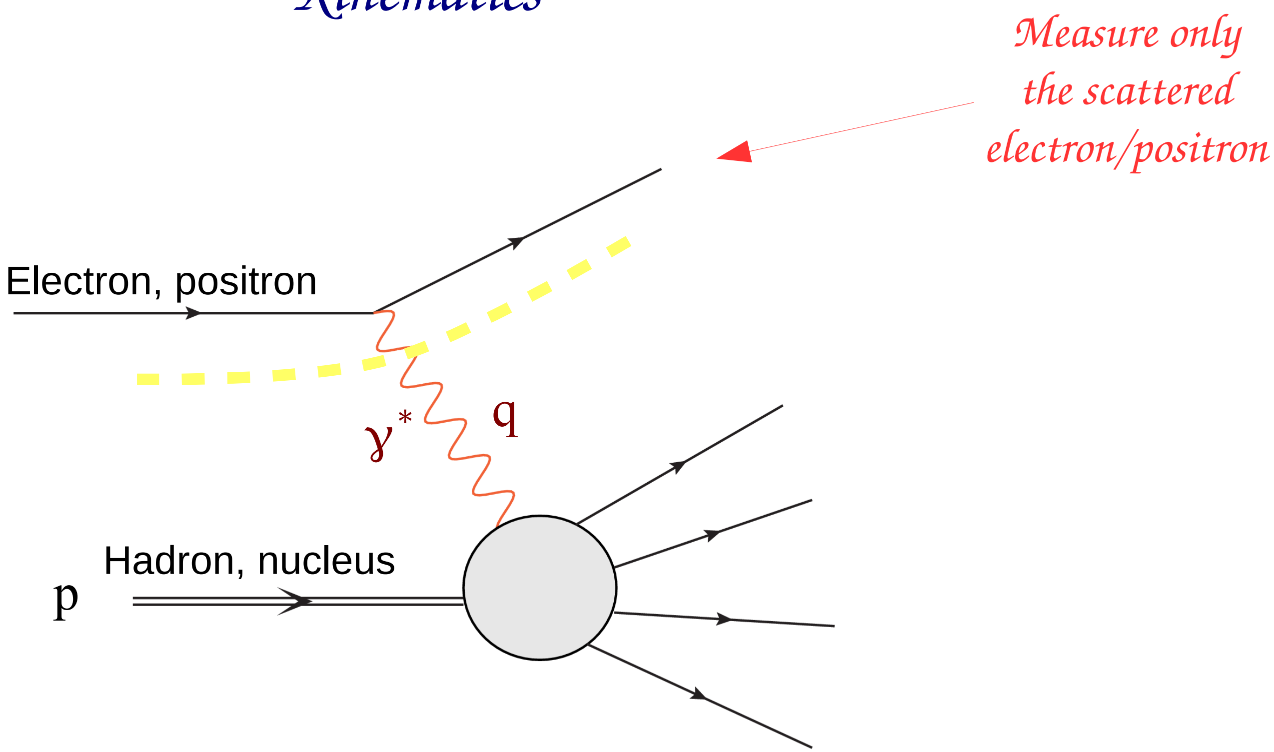
Can one measure the properties of these quantum states generated by high-energy QCD evolution in actual experiments?

Outline

- **Picture of a hadron at high energy**
  - Basic features of quantum chromodynamics
  - Heuristic discussion of quantum fluctuations
  - Evolution of hadronic states towards higher energies: the color dipole model
  - The BFKL equation, the Balitsky-Kovchegov equation and their solutions
- **Probing hadronic wave functions in physical processes**
  - (Mean) parton densities in deep-inelastic scattering
  - Density effects in DIS: How the Balitsky-Kovchegov equation appears
  - How to "see" high-energy evolution in proton-nucleus scattering

Deep-inelastic scattering

Kinematics



Variables: $p, q \Rightarrow Q^2 \equiv -q^2, x_{Bj} \equiv \frac{Q^2}{2\, p \cdot q}$

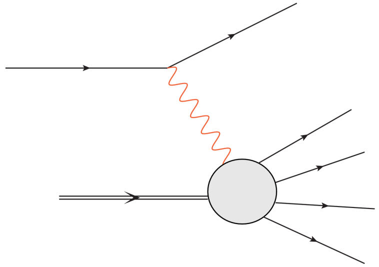
Rapidity: $y = \ln \frac{1}{x_{Bj}} \simeq \ln \frac{(p+q)^2}{Q^2}$

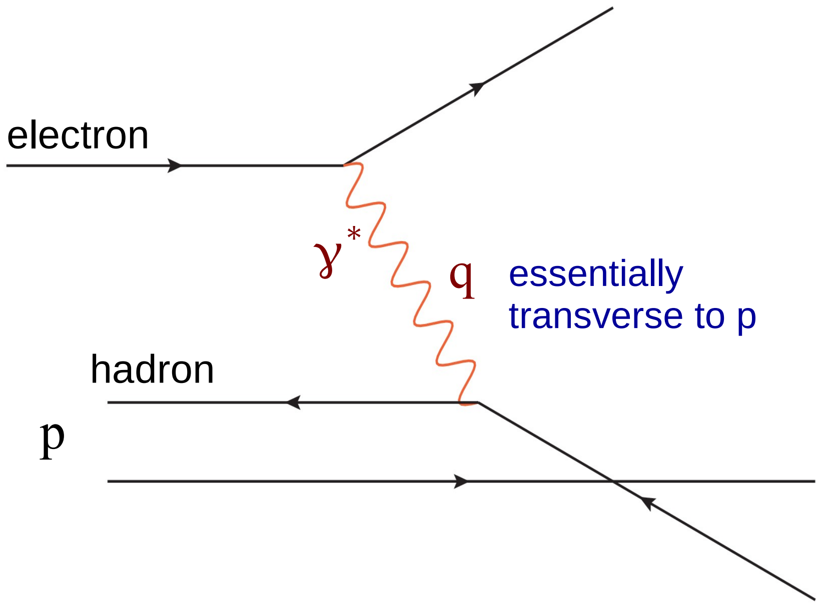
Large y = small x = high energy

Observable: $\sigma^{\gamma^* h}(Q^2, x_{Bj})$

Deep-inelastic scattering

Picture in the Bjorken frame





Low rapidity y

$$y = \ln \frac{1}{x_{Bj}}$$

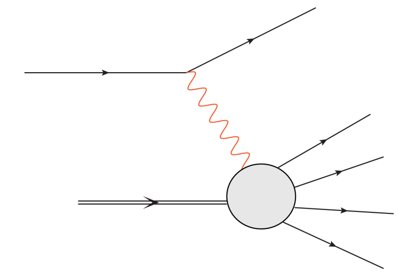
Parton model formula:

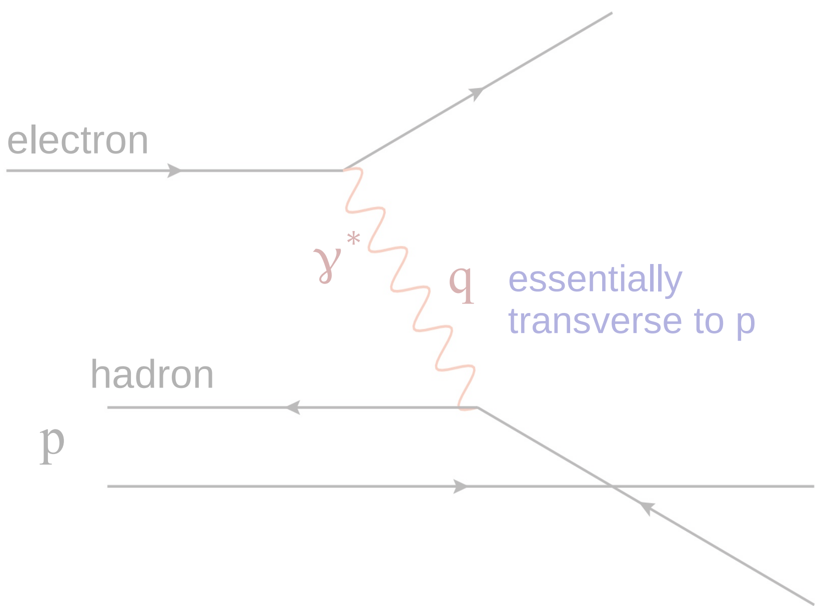
$$\sigma^{\gamma^* h}\left(Q^2, x_{Bj}\right) = \frac{4\pi^2 \alpha_{em}}{Q^2} \sum_q e_q^2 \left[\underbrace{x_{Bj}\, q\left(x_{Bj}\right)}_{\text{(Mean) integrated (valence) quark density}} + x_{Bj}\, \bar{q}\left(x_{Bj}\right) \right]$$

~ Bjorken scaling
(pointlike quarks)

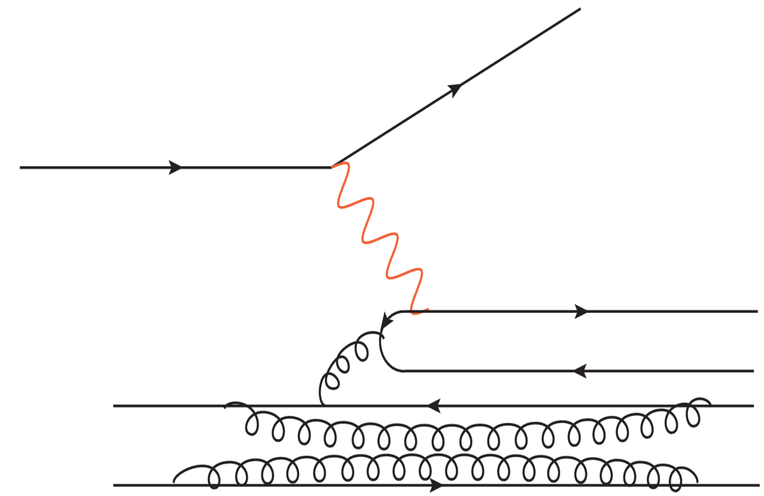
Deep-inelastic scattering

Picture in the Bjorken frame





Low rapidity y



Higher rapidity y

$$y = \ln \frac{1}{x_{Bj}}$$

Parton model formula "improved":

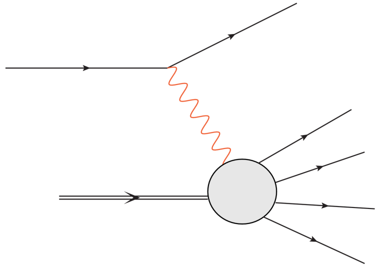
$$\sigma^{\gamma^* h}\left(Q^2, x_{Bj}\right) = \frac{4\pi^2 \alpha_{em}}{Q^2} \sum_q e_q^2 \left[x_{Bj}\, q\left(x_{Bj}, Q^2\right) + x_{Bj}\, \bar{q}\left(x_{Bj}, Q^2\right) \right]$$

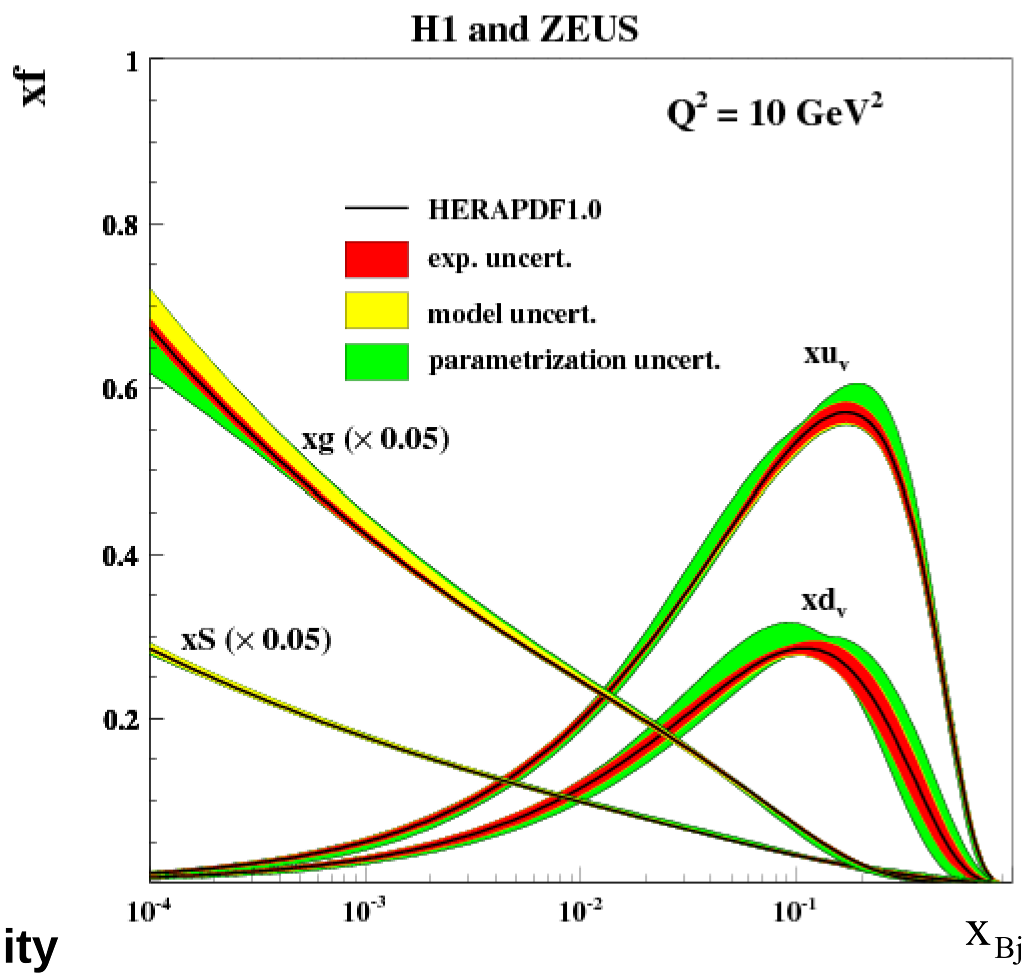
~~~ Bjorken scaling (pointlike quarks)~~

(Mean) integrated ~~(valence)~~ quark density
~~~

Deep-inelastic scattering

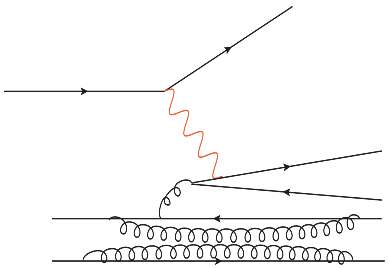
Picture in the Bjorken frame

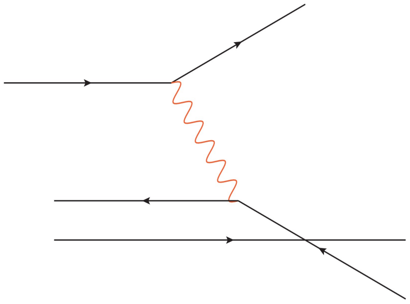




High rapidity

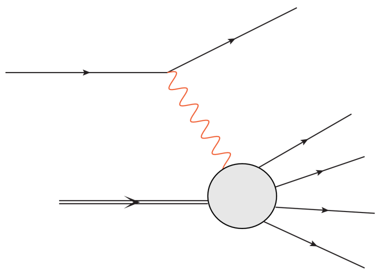
Low rapidity

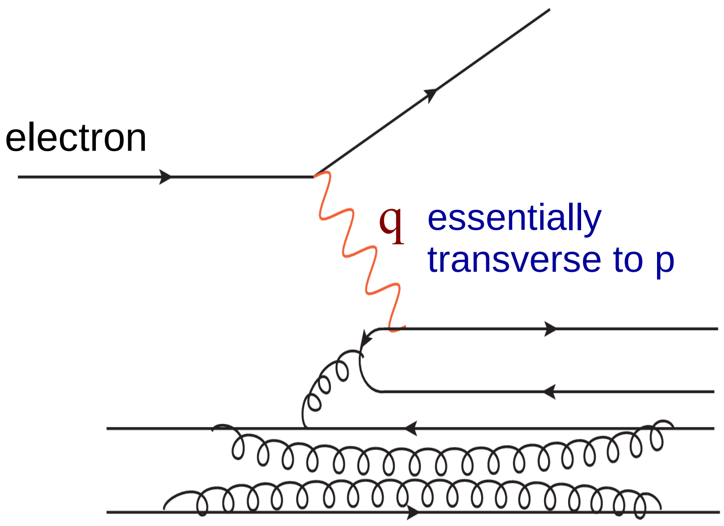




Deep-inelastic scattering at high energy

Picture in the dipole frame

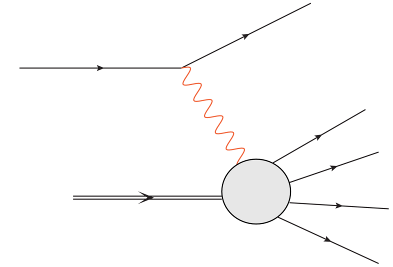


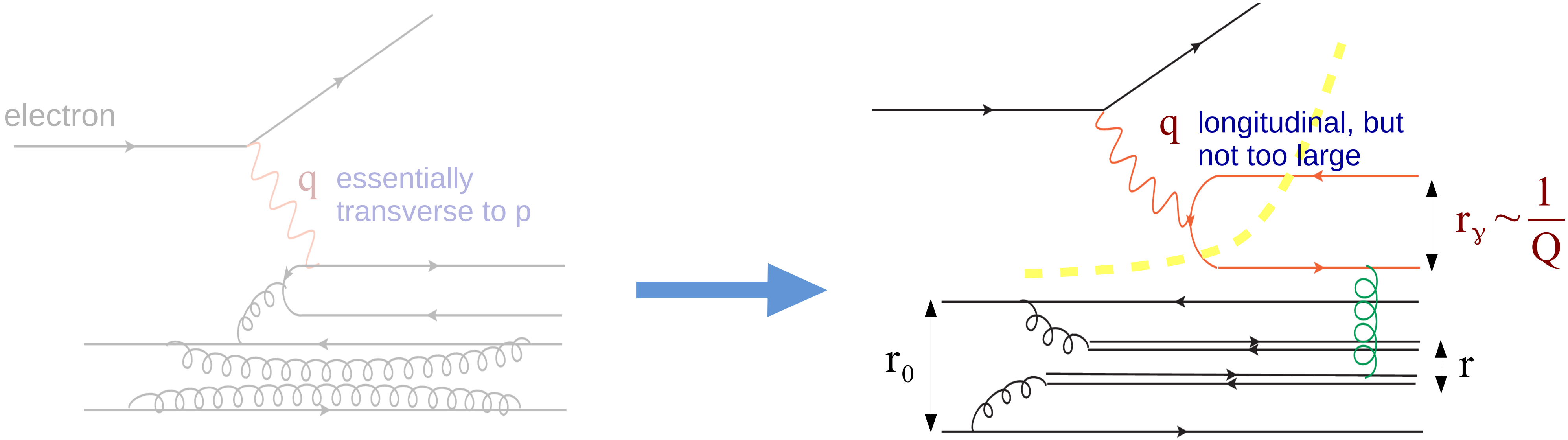


Bjorken frame

Deep-inelastic scattering at high energy

Picture in the dipole frame

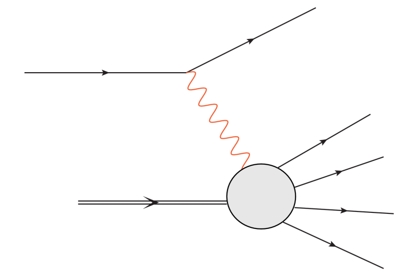


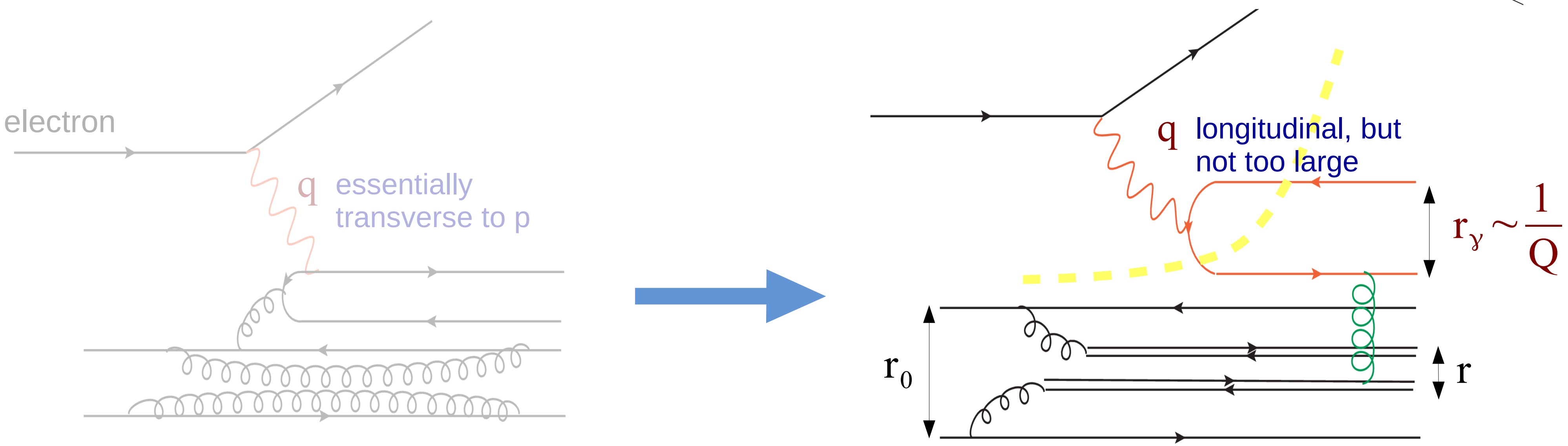


Bjorken frame **Dipole frame**

Deep-inelastic scattering at high energy

Picture in the dipole frame





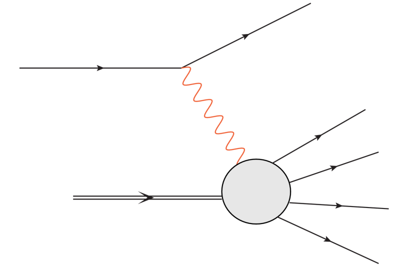
Bjorken frame **Dipole frame**

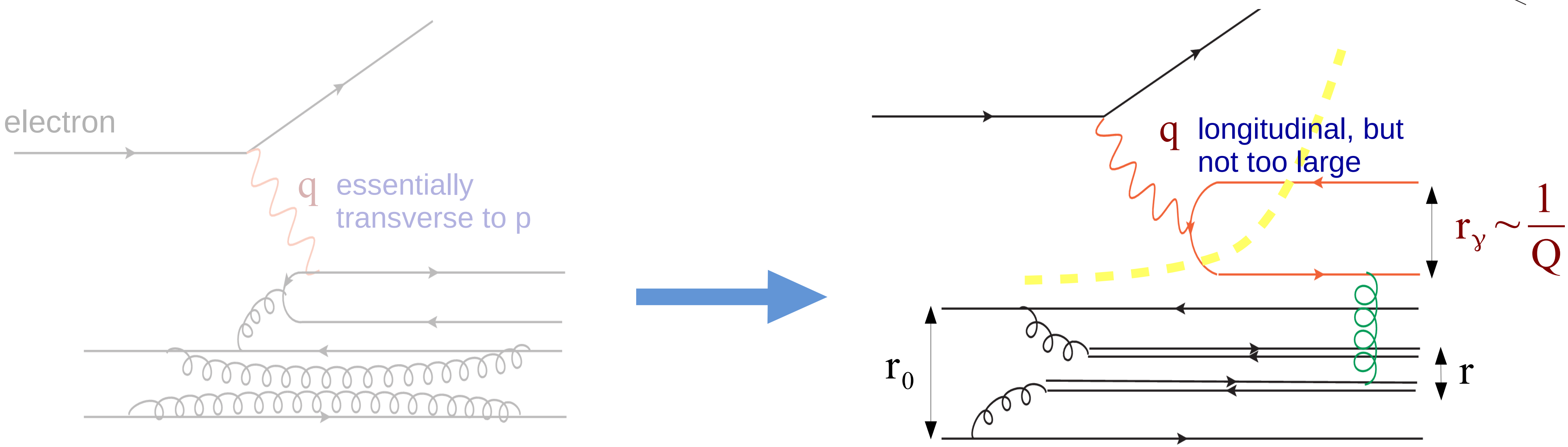
$$\sigma^{\gamma^* h}(Q^2, x_{Bj}) = \underbrace{\text{Proba}\left(\gamma^* \to q\,\bar{q}\,(r_\gamma)\right)}$$

Distribution of quark-antiquark pairs
in the photon (QED)

Deep-inelastic scattering at high energy

Picture in the dipole frame





Bjorken frame **Dipole frame**

$$\sigma^{\gamma^* h}(Q^2, x_{Bj}) = \int d^2 r_\gamma \, \text{Proba}(\gamma^* \rightarrow q\,\bar{q}(r_\gamma)) \times \sigma^d(r_\gamma, y = \ln 1/x_{Bj})$$

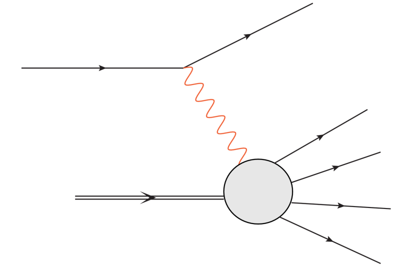
Distribution of quark-antiquark pairs in the photon (QED)

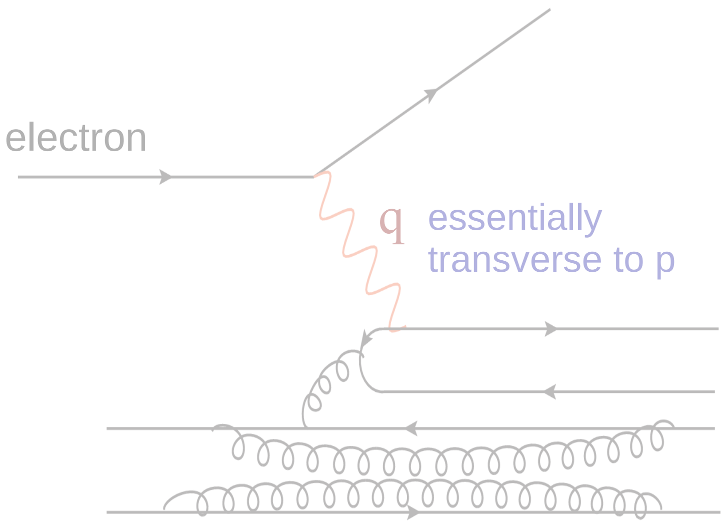
Dipole-target cross section

DIS can be interpreted as dipole scattering off a given target

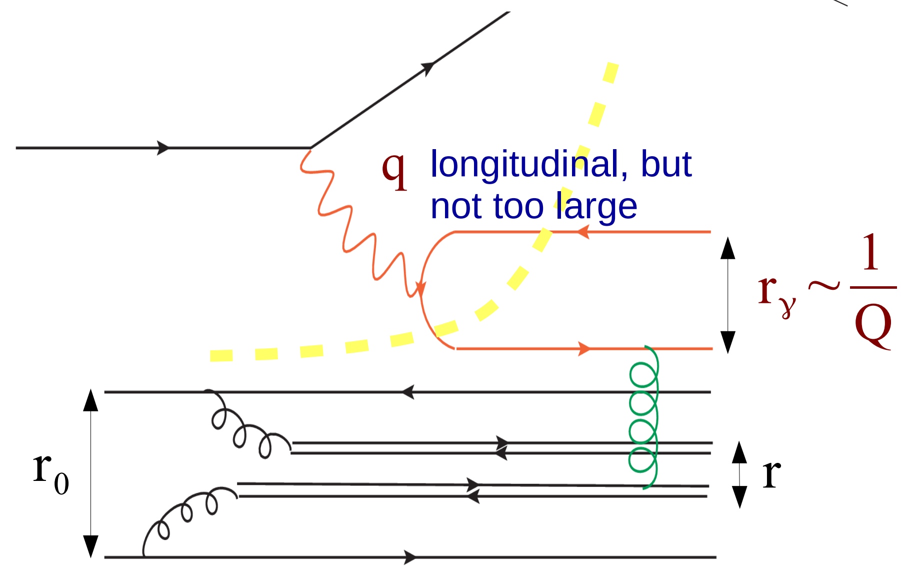
Deep-inelastic scattering at high energy

Picture in the dipole frame





Bjorken frame



Dipole frame

$$\sigma^{\gamma^* h}(Q^2, x_{Bj}) = \int d^2 r_\gamma \underbrace{\text{Proba}\left(\gamma^* \to q\,\bar{q}(r_\gamma)\right)}_{\text{Distribution of quark-antiquark pairs in the photon (QED)}} \times \overbrace{\int d^2 r\, \underbrace{\sigma^{dd}(r_\gamma, r)}_{\text{Elementary dipole-dipole cross section}} \times \underbrace{\langle n(r, y | r_0)\rangle}_{\text{(Mean) dipole number density}}}^{\sigma^d(r_\gamma, y)}$$

Distribution of quark-antiquark pairs in the photon (QED)

Elementary dipole-dipole cross section
Exchange of one gluon

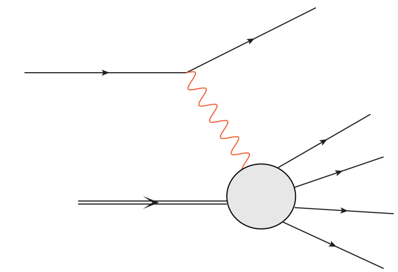
(Mean) dipole number density
y-dependence from BFKL

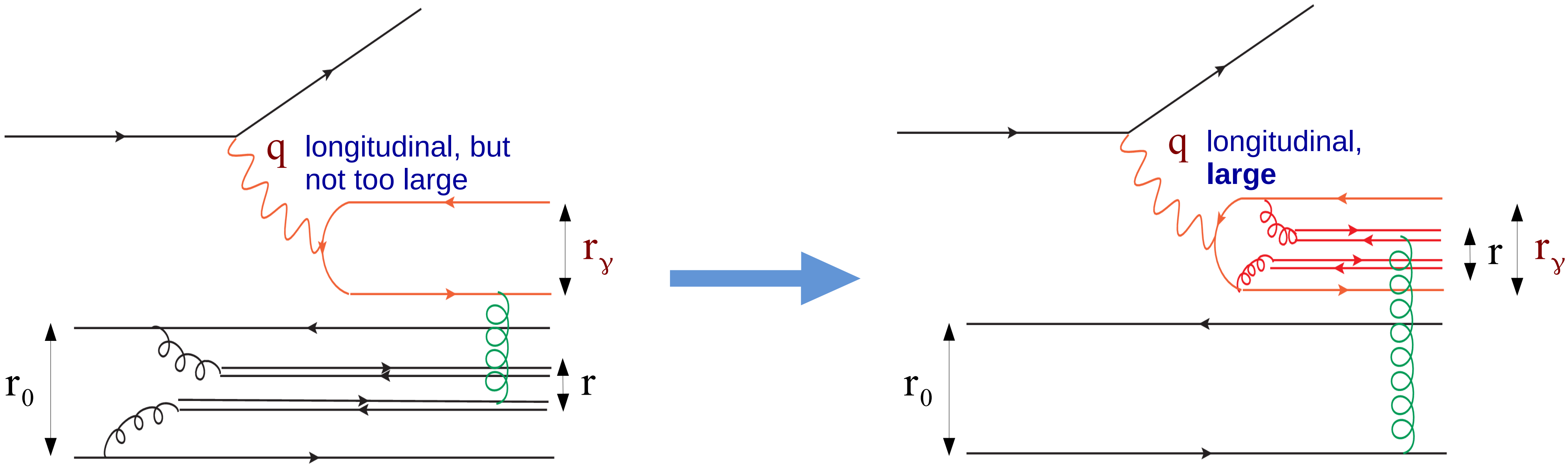
DIS can be interpreted as dipole scattering off a given target

It “measures” the mean dipole (=gluon) density in the target, of size ~ 1/Q

Deep-inelastic scattering at high energy

Picture in the target restframe





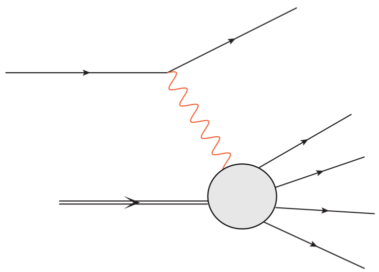
Dipole frame **Target restframe**

$$\sigma^{\gamma^* h}\left(Q^2, x_{Bj}\right) = \int d^2 r_\gamma \, \mathrm{Proba}\left(\gamma^* \to q\,\bar{q}\,(r_\gamma)\right) \times \begin{cases} \int d^2 r\, \sigma^{dd}(r_\gamma, r) \times \langle n(r, y | r_0) \rangle & \text{(dipole frame)} \\ \quad \| & \\ \int d^2 r\, \langle n(r, y | r_\gamma) \rangle \times \sigma^{dd}(r, r_0) & \text{(target restframe)} \end{cases}$$

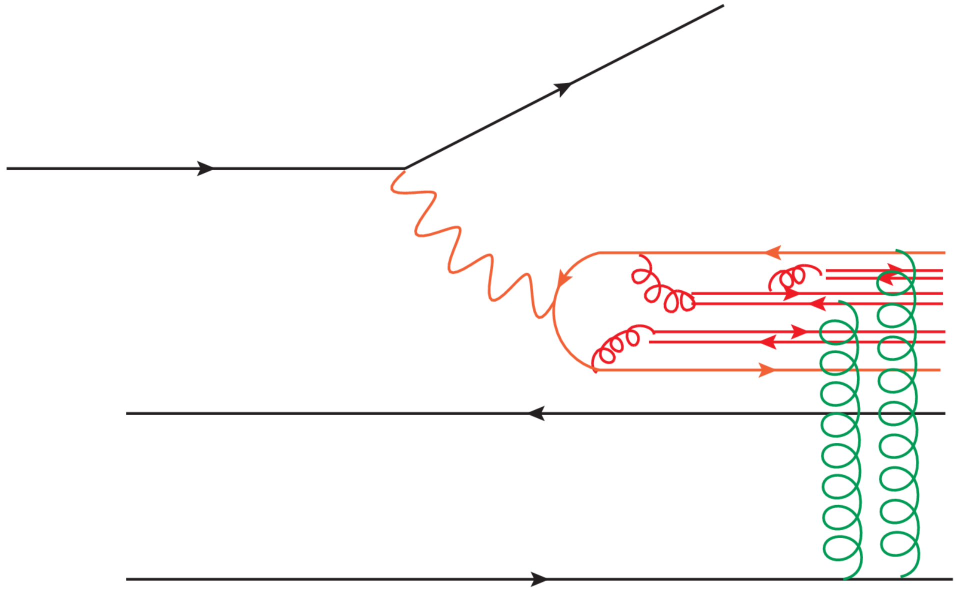
In the target frame, DIS "measures" the mean dipole density in the photon

Deep-inelastic scattering at high energy

Picture in the target restframe



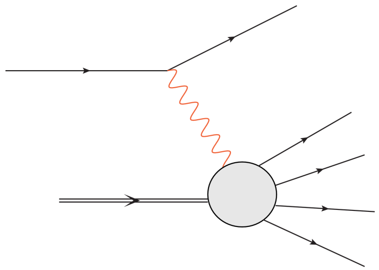
When the rapidity becomes even higher, then the probability that ***multiple scatterings*** *occur becomes significant:*



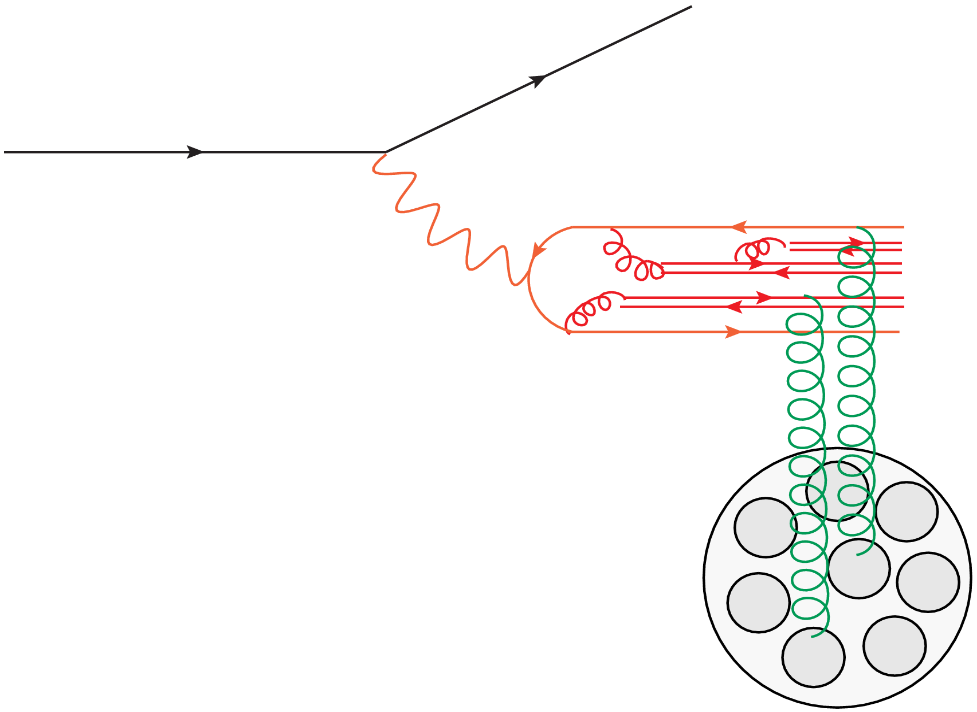
Such processes are very difficult to formulate if the target is a dilute object (meson, proton, single dipole).

Deep-inelastic scattering at high energy

Picture in the target restframe



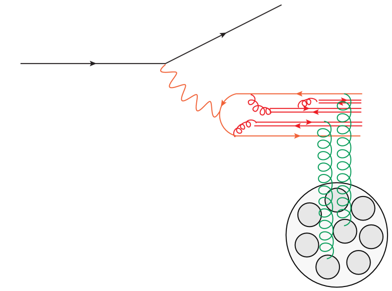
However, if the target is a ***large nucleus*** *(= dense system of independent nucleons)...*



...then the formulation turns out to simplify, since one may assume that all exchanged gluons scatter with different nucleons.

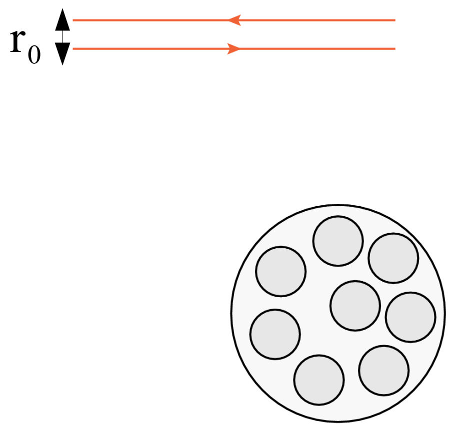
Deep-inelastic scattering off a nucleus

Low rapidity

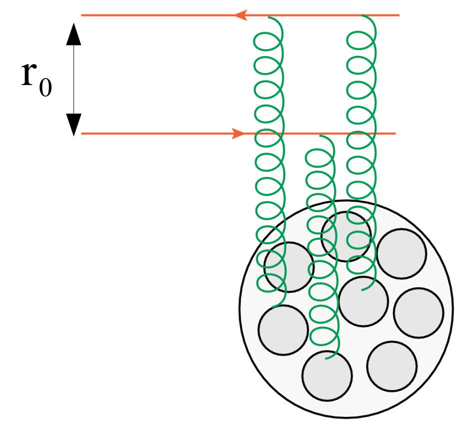


A nucleus is a dense object, characterized by a scale Q_A *, which is essentially* ***transparent*** *to dipoles of size smaller than* $1/Q_A$ *and fully* ***absorptive*** *to larger dipoles:*

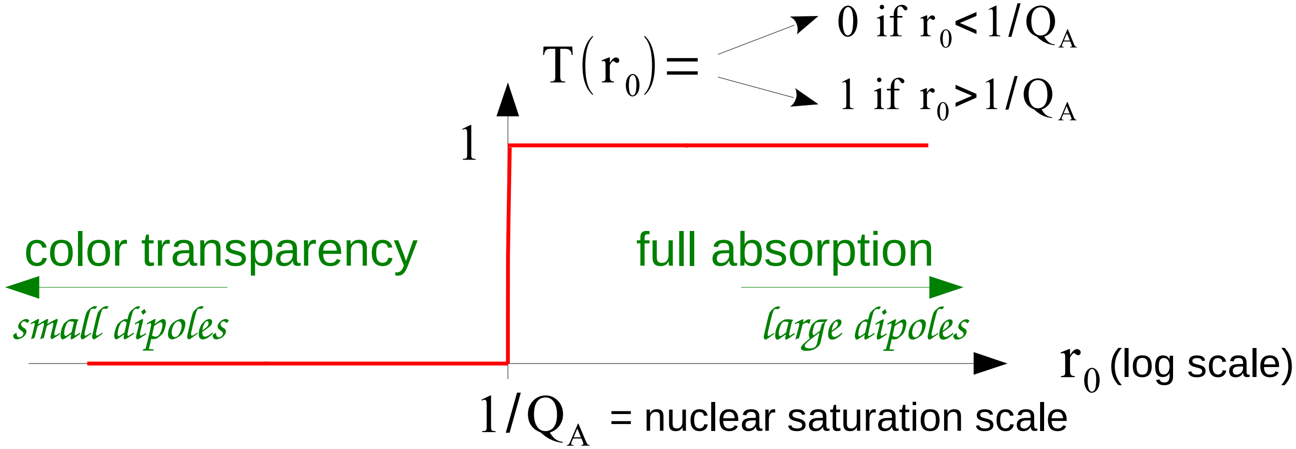
$r_0 \ll \frac{1}{Q_A}$



$r_0 \gg \frac{1}{Q_A}$

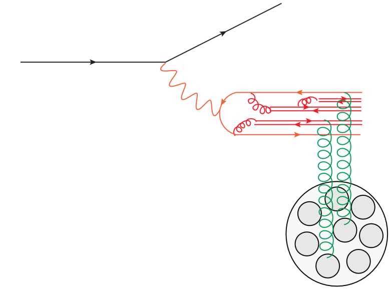


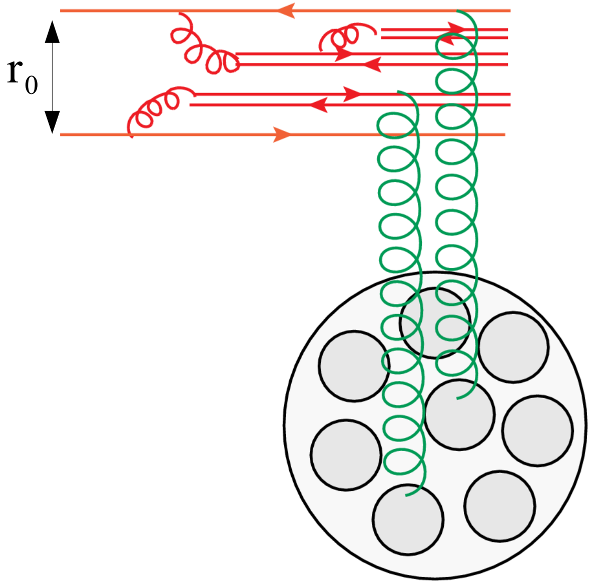
Dipole-nucleus cross section (fixed impact parameter) = scattering probability:

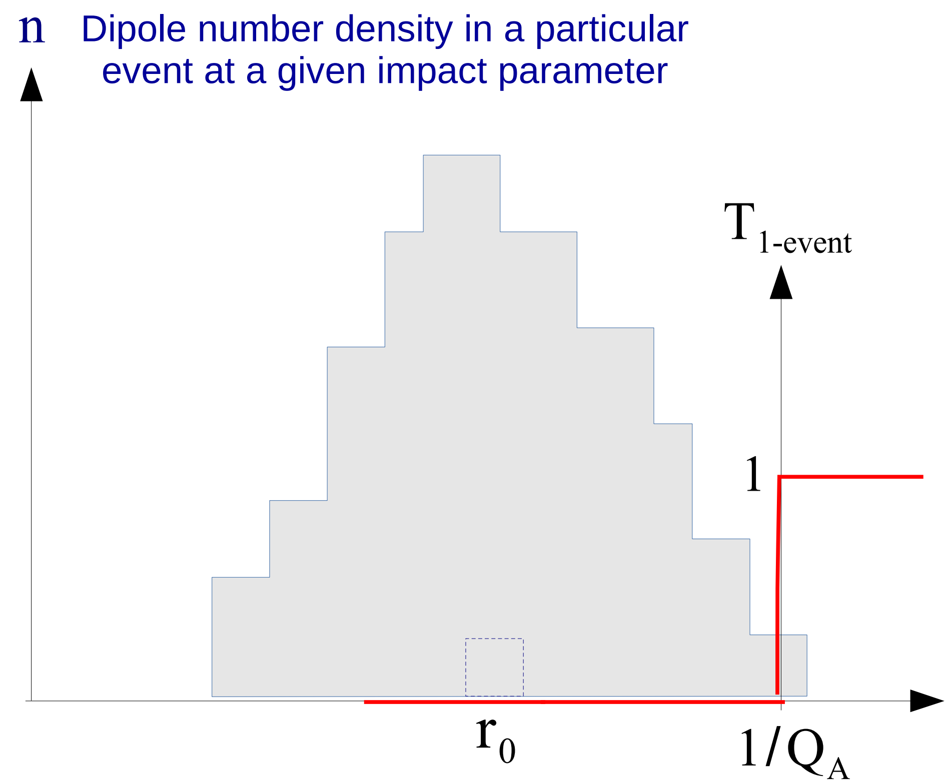


Deep-inelastic scattering off a nucleus

High rapidity







This particular evolved quantum state scatters if and only if at least one dipole at the time of the interaction is larger than the inverse nuclear saturation momentum.

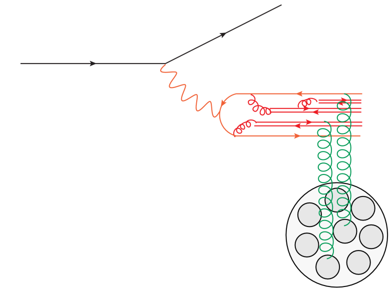
The measured amplitude is the average over events: $T = \langle T_{1\text{-event}} \rangle_{\text{events}}$

In this case, DIS amplitude ~ probability distribution of the largest dipole size.

Solves the BK equation!

Deep-inelastic scattering off a nucleus

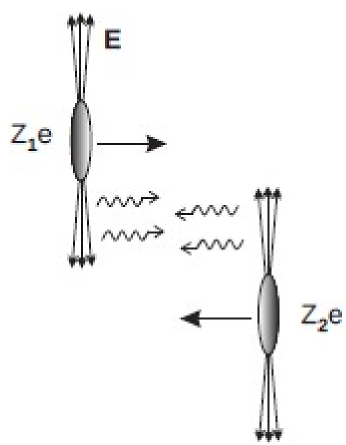
Experimentally?



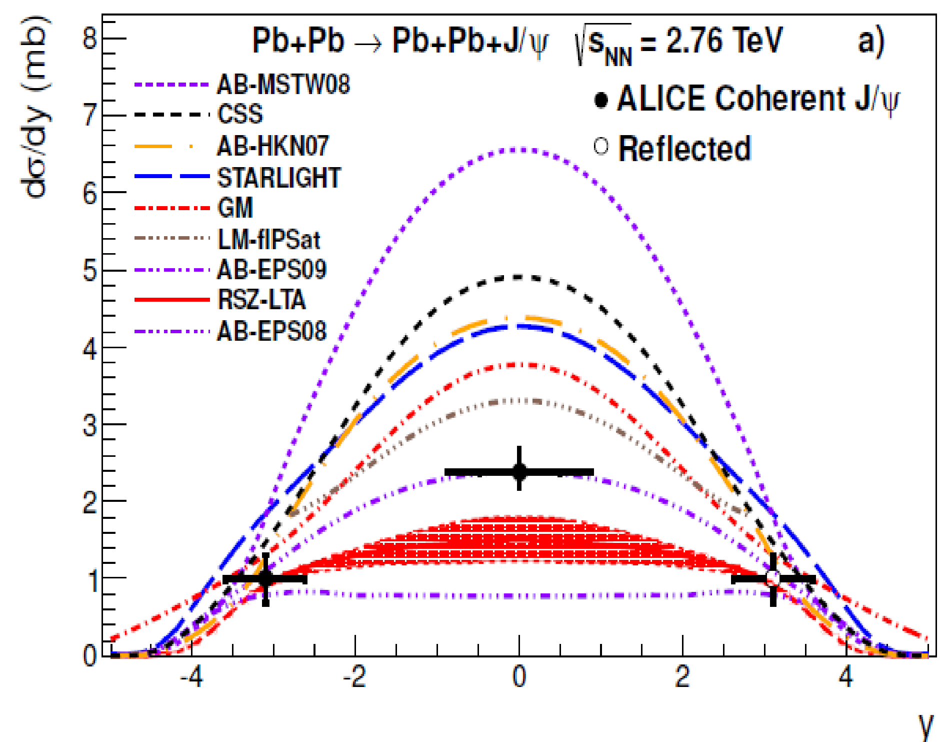
- *At HERA, no nuclei, only protons… but one may extrapolate the theoretical results obtained for the nucleus (solution of the BK equation) to protons!*

See dipole models: Golec-Biernat and Wüsthoff, Kowalski et al.

- *At the LHC, "kind of" DIS off nuclei: ultraperipheral AA collisions!*



$\gamma A \rightarrow J/\psi A'$



Intermediate recap

- The interpretation of deep-inelastic scattering depends on the frame one considers.

- In the **Bjorken frame**, the parton model formula relating the DIS cross section to the "usual" quark + antiquark densities is manifest.

- In the **dipole frame**, the DIS cross section is manifestly related to the **dipole-hadron cross section**, which in turn, for moderate energies, probes the **mean dipole density**, whose high-energy evolution is given by the **(linear) BFKL equation**.

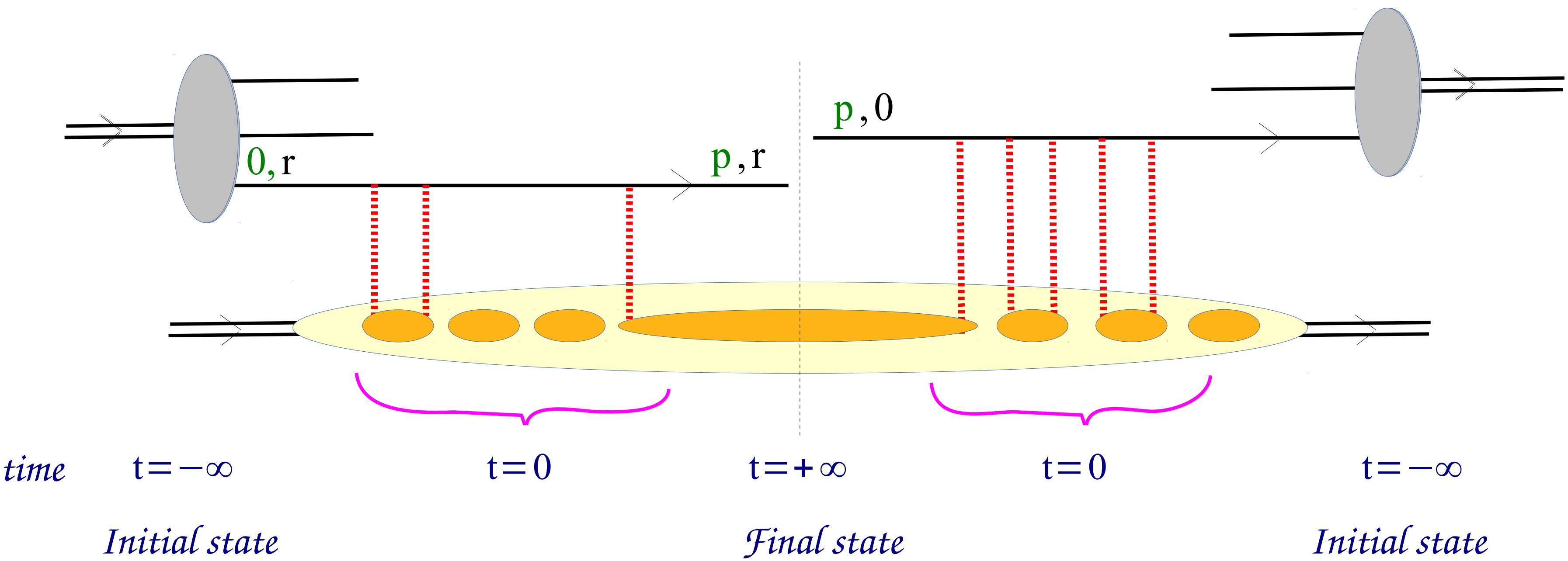
- In the target restframe, the quantum evolution is transferred to the virtual photon.

- For higher rapidities or for a dense target like a nucleus, the quantum evolution becomes **nonlinear,** and the **BK equation** becomes the relevant evolution equation. The DIS cross section can be related to the distribution of the **fluctuations of the size of the largest dipole** in the quantum evolution of the virtual photon.

Proton-nucleus scattering

Example: Transverse momentum broadening

Measure a jet in proton-nucleus collisions



Distribution of the transverse momentum of this jet:

$$\frac{dN}{d^2 p} = \int \frac{d^2 x}{(2\pi)^2} e^{-i\,p r} \left[\text{cross section for a dipole of size r}\right]$$

*This formula is also true **when quantum evolution is included**, and the latter is given by the BK equation for the dipole cross section! (NB: up to next-to-leading order)*

Summary

*At high energies, **hadrons look like dense states of gluons** (sometimes called "color glass condensates"), very far from the valence picture. This is a property of QCD.*

*The evolution of hadronic wave functions towards high energy can be computed in QCD. It is quite **universal**. The **color dipole model** is a convenient implementation of this evolution.*

---

*Different properties of the evolution can be investigated experimentally, most easily in **DIS**, but also in **pA** collisions in which processes like **broadening** may formally be related to DIS.*

*DIS off a hadron at moderate rapidity probes the evolution of the **mean dipole number density,** given by the BFKL equation.*

*DIS off a nucleus, or broadening, probe instead the **fluctuations in the tail of the dipole size distribution**.*

Outlook: The event-by-event fluctuations of the total multiplicity in pA scattering may be related to the fluctuations of the total integrated dipole number!

Recent textbook on high-energy QCD: Kovchegov and Levin, CUP, 2012