Torino, January 28, 2015

SUPERFLUIDTY OF ULTRACOLD ATOMIC GASES

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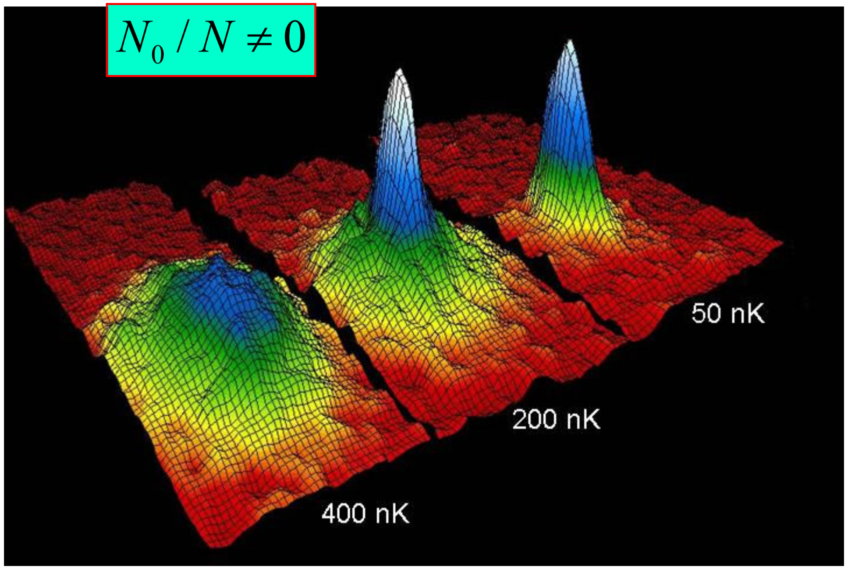
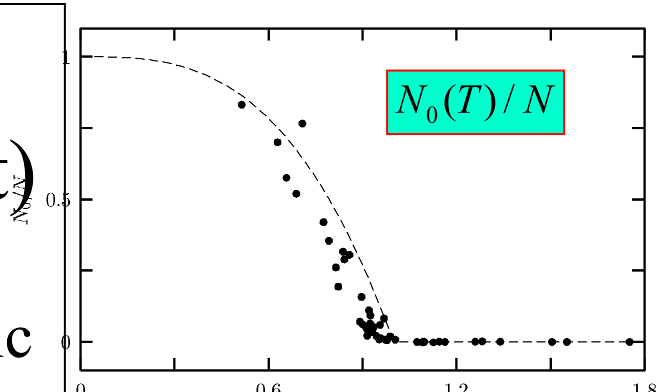
CNR-INO

Bose-Einstein condensation: first experiments



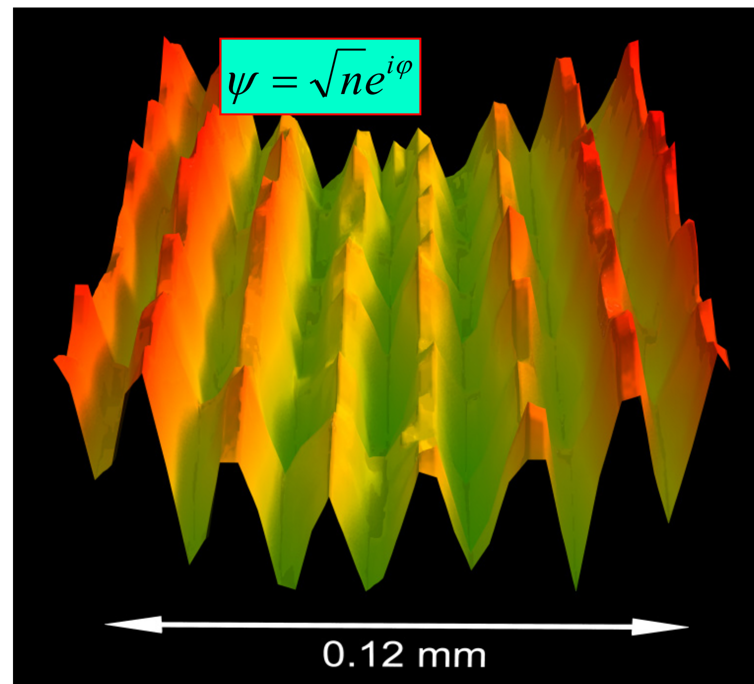
1995 (Jila+Mit)

(Macroscopic occupation of sp state)



Phase transition (Jila 1996)

1996 Mit (coherence + wave nature)



Some important questions

Connections between BEC and superfluidity

- Can the condensate fraction be identified with the superfluid density ?

- Can we measure the superfluid density in ultracold atomic gases ?

- What are the important consequences of superfluidity ?

Some answers

- **Gross-Pitaevskii** equation for the BEC order parameter (non linear Schroedinger eq.)

$$i\partial_t \Psi = \left(-\nabla^2/2 + V_{ext} + g\Psi^*\Psi\right)\Psi$$

predicts important **superfluid** features (quantized vortices, irrotational hydrodynamic flow, quenching of moment of inertia, Josephson oscillation etc..)
Condensate density practically **coincides** with **superfluid** density.

- Relation between BEC and superfluidity much less trivial in strongly interacting fluids (helium, unitary Fermi gas) and in 2D (BKT superfluidity, no BEC in 2D)

- **Superfluid density** recently **measured** in strongly interacting Fermi gas, through observation of second sound

Superfluidity in ultracold atomic gases (measured quantities)

- **Quantized vortices**

- Quenching of **moment of inertia**

- **Josephson** oscillations

- Absence of **viscosity** and **Landau critical velocity**

- **BKT transition** in 2D Bose gases

- **Lambda transition** in resonant Fermi gas

- **First** and **second sound**

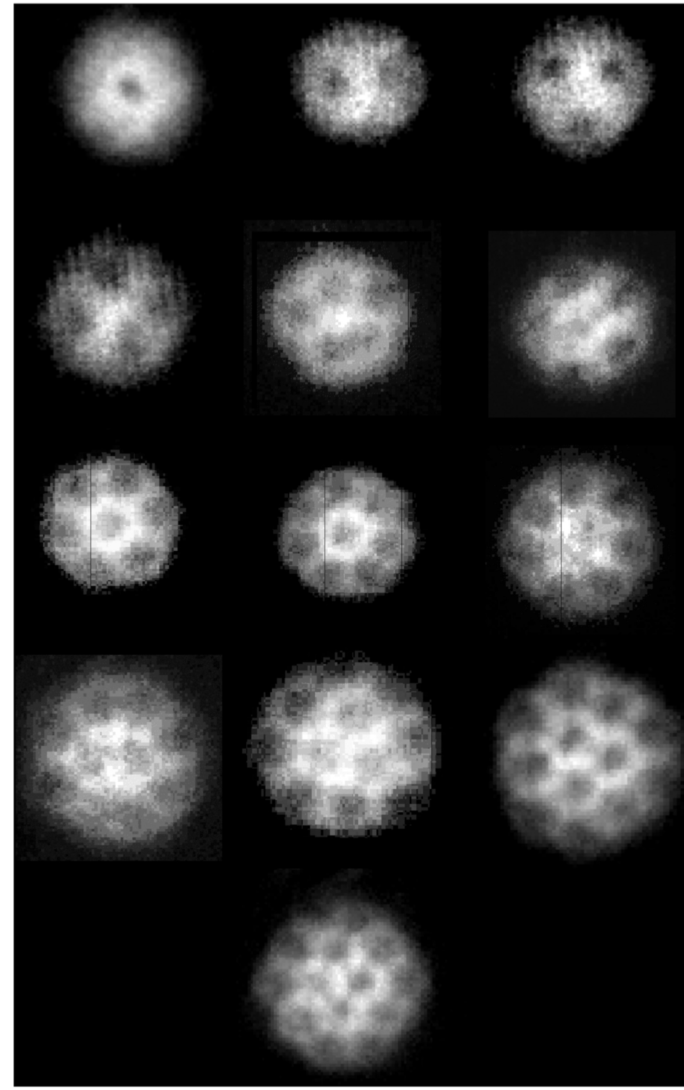
- **Superfluidity in Spin-orbid coupled BEC's**

Quantized vortices in BEC gases

Quantization of vortices
(quantization of circulation and of angular momentum)
follows from **irrotational** constraint of superfluid motion.

In dilute Bose gases vortices were first predicted in original paper by Lev Pitaevskii (1961).

Size of vortex core is of order of healing length (< 1 micron),
Cannot be resolved in situ.
Visibility emerges **after expansion**



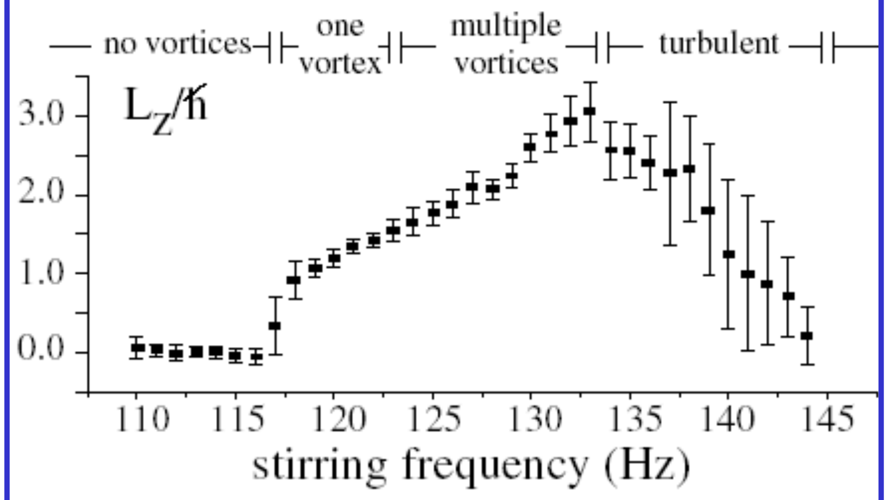
Vortices at ENS
Chevy, 2001

Spectroscopic measurement of angular momentum

Splitting between m=+2 and m=-2 quadrupole frequencies proportional to angular momentum
(Zambelli and Stringari,1999)

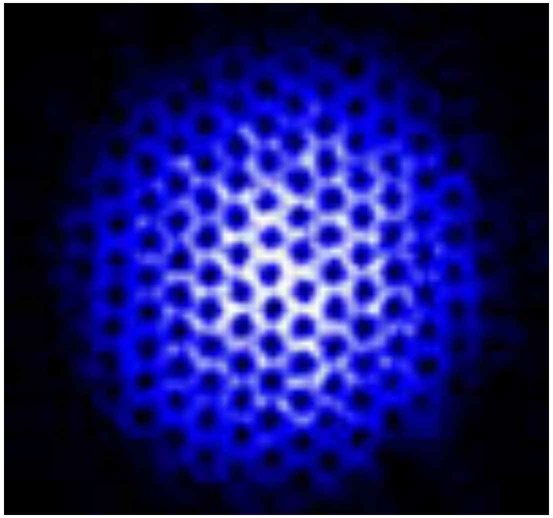
$$\omega_+ - \omega_- = \frac{2}{M}\frac{\langle l_z\rangle}{\langle r_\perp^2\rangle}$$

Measurement of angular momentum in BEC's
(Chevy et al., 2000)

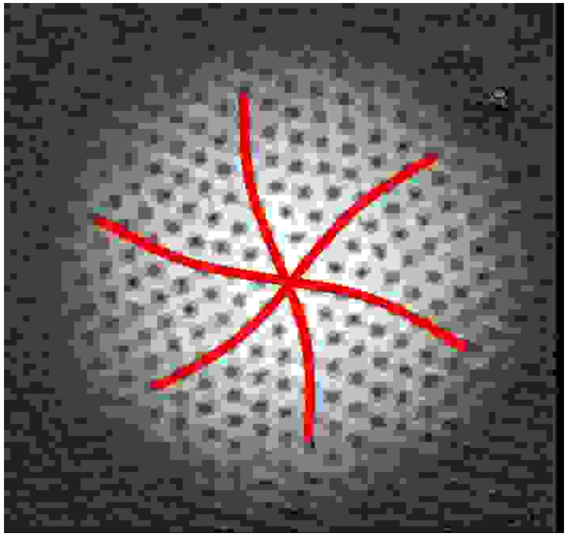


Vortex lattices

By increasing angular velocity one can nucleate more vortices (vortex lattice)



(Jila 2002)



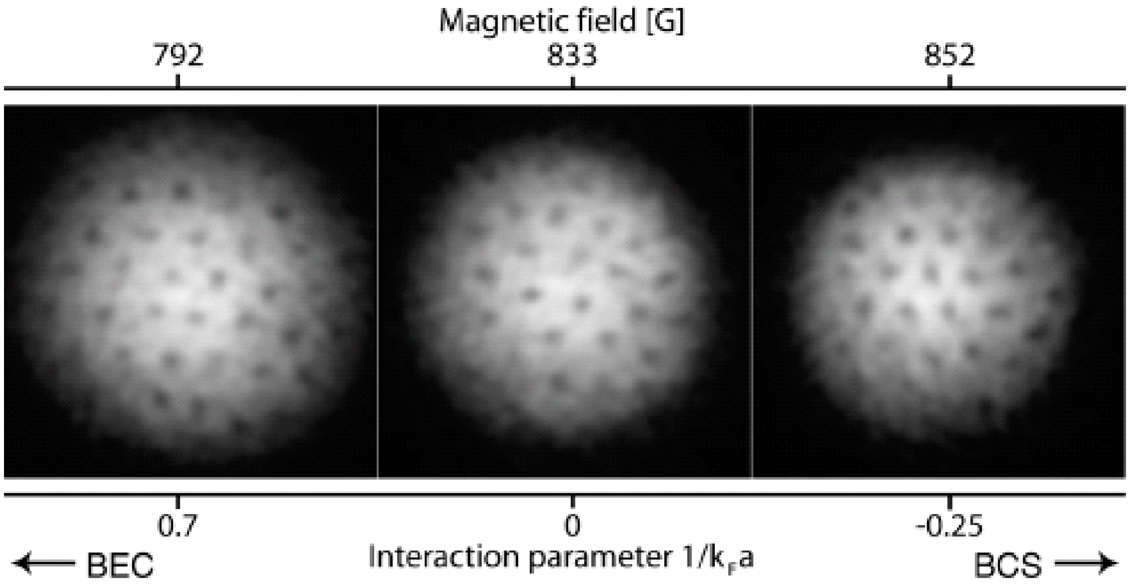
(Jila 2003)

Vortices form a regular **triangular lattice** (cfr Abrikosov lattice In superconductors)

Tkachencko (elastic) waves In a BEC vortex lattice

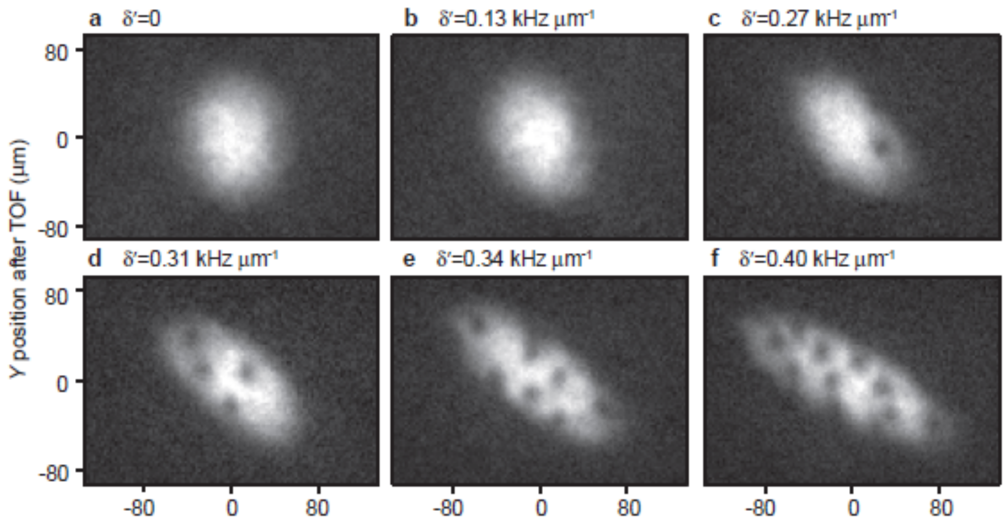
Quantized vortices in Fermi gases observed along the BEC-BCS crossover

(MIT, Nature June 2005, Zwierlein et al.)

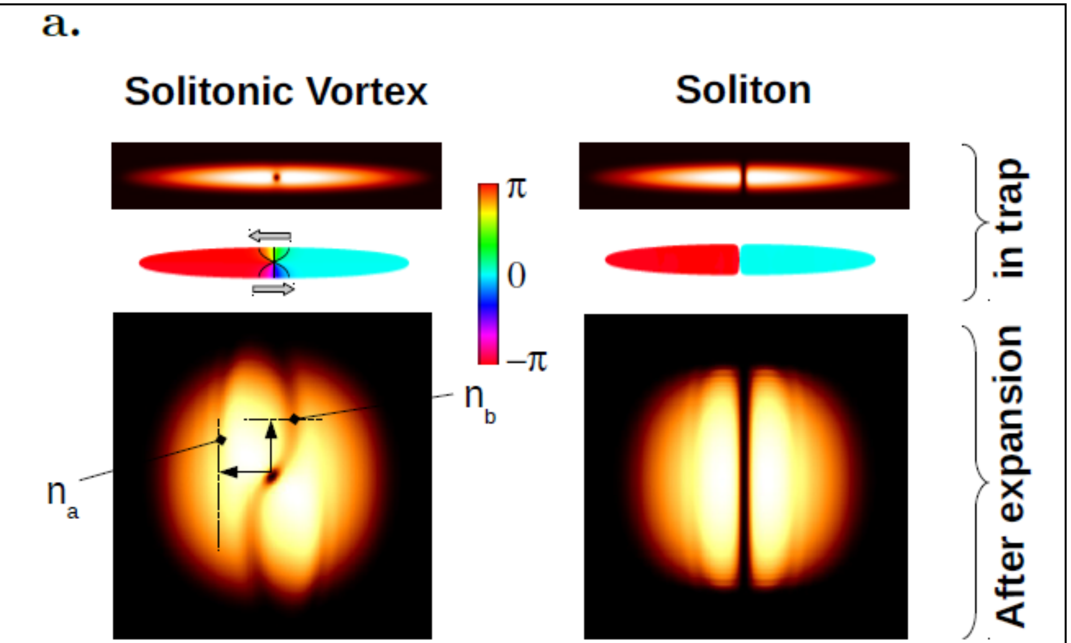


Quantized vortices in BEC gases created with artificial gauge fields

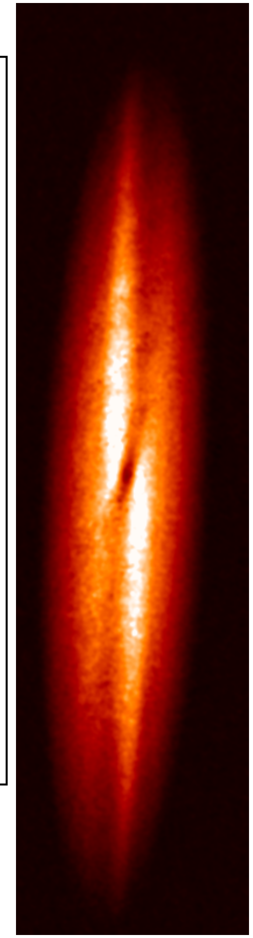
(Lin et al. 2009)



Solitonic vortices observed in BEC's at Trento
Donadello et al. (PRL 2014)



Time dependent GP simulation
Tylutki et al. 2014



Solitonic vortices observed also in Fermi gases at MIT
(Ku et al. PRL 2014)

Quenching of moment of inertia due to irrotationality

Direct measurement of moment of inertia difficult because images of atomic cloud probe **density** distribution (**not angular momentum)**

In deformed traps **rotation** is however **coupled** to **density** oscillations.
Exact relation, holding also in the presence of 2-body forces:

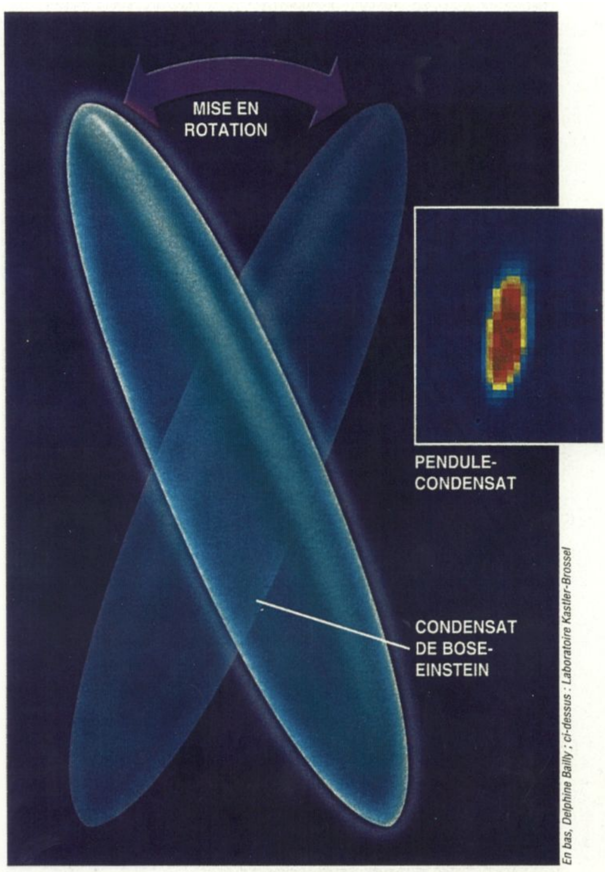
$$[H, L_z] = im(\omega_y^2 - \omega_x^2)\sum_i x_i y_i$$

angular momentum **quadrupole operator**

Response to **transverse** probe measurable thorugh **density** response function !!

Example is provided by **SCISSORS MODE**.
If confining trap is suddenly rotated by angle θ
Behaviour of resulting oscillation depends crucially on value of moment of inertia (**irrotational** vs **rigid**)

Experiments (Oxford 2011) **confirm irrotational nature of moment of inertia**



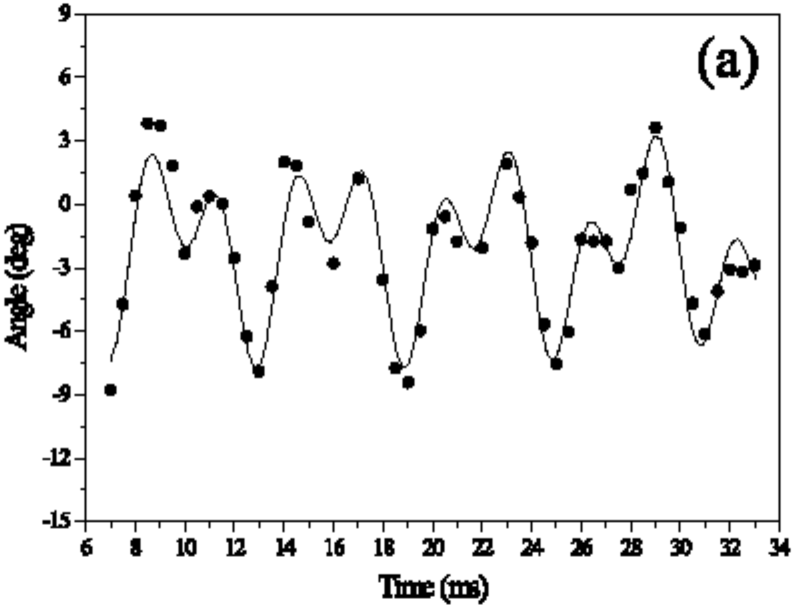
Theory of scissors mode
(Guery-Odelin and S.S., PRL 83 4452 (1999))

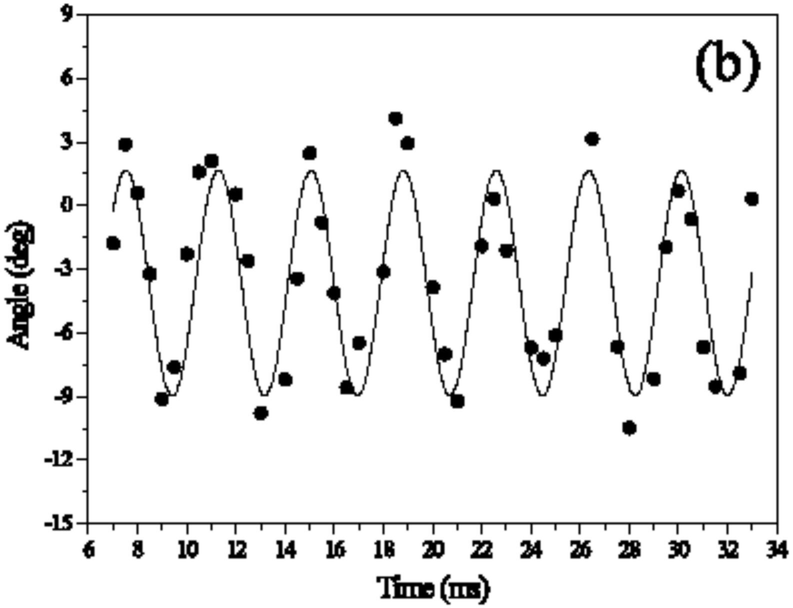
Scissors measured at Oxford in BECs
(Marago'et al, PRL 84, 2056 (2000))

Above T_C (**normal**)

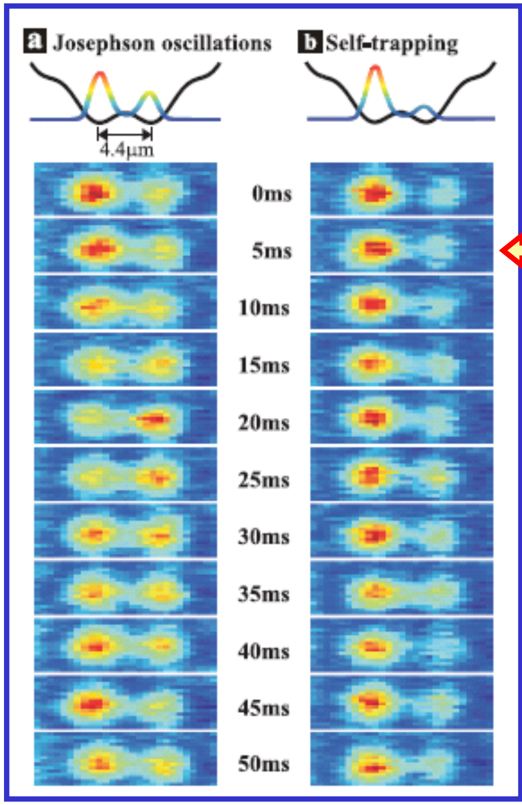
2 modes: $$\omega_{\pm} = \left|\omega_x \pm \omega_y\right|$$

Below T_C (**superfluid**) :
single mode: $$\omega = \sqrt{\omega_x^2 + \omega_y^2}$$

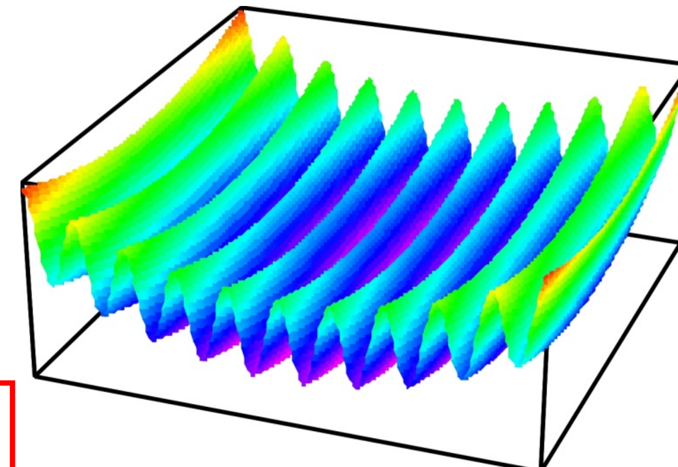




JOSEPHSON OSCILLATIONS

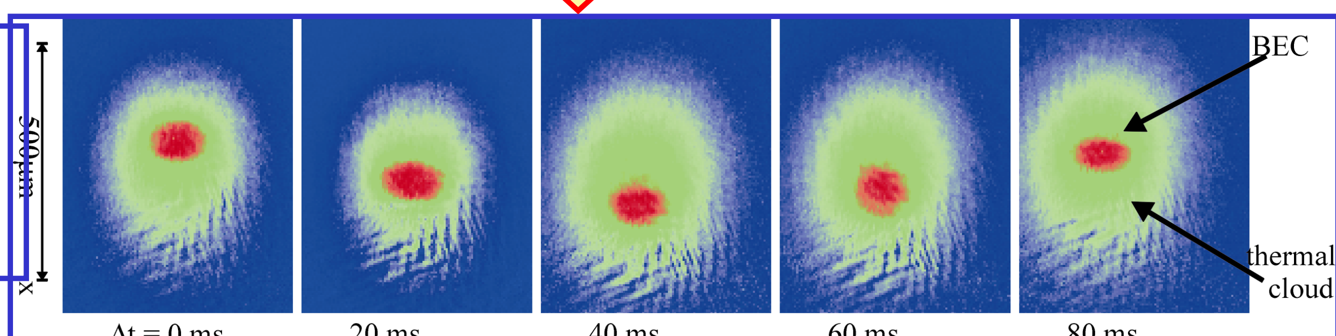


Double well
(Heidelberg 2004)

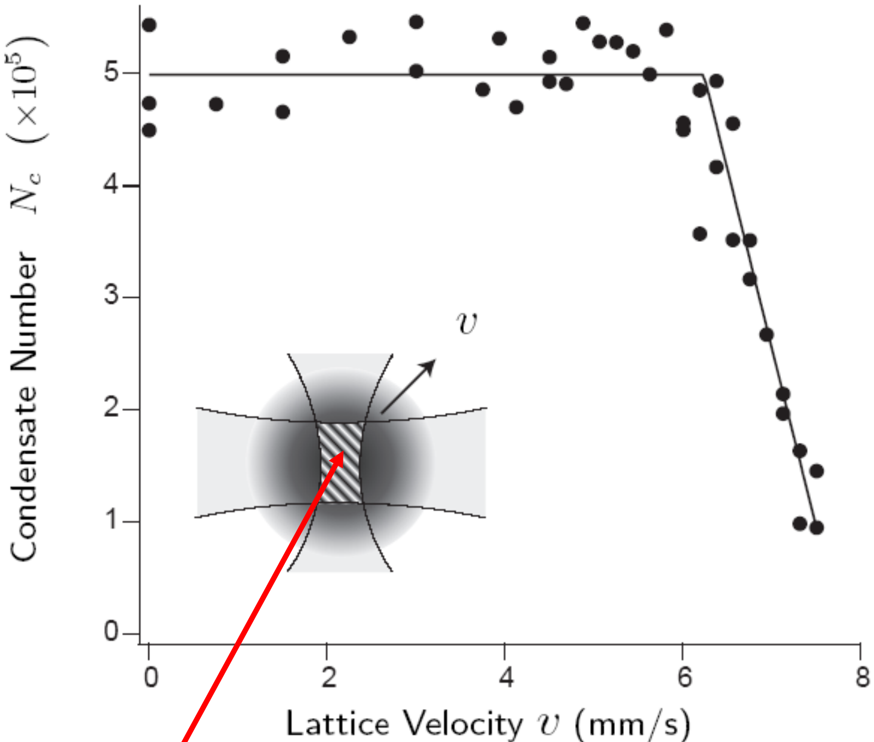


Periodic potential
(Firenze 2001)

Only superfluid can coherently tunnel through the barrier



Absence of viscosity and Landau's critical velocity: Fermi superfluid at unitarity



$$v_c = \min_p \frac{\varepsilon(p)}{p}$$

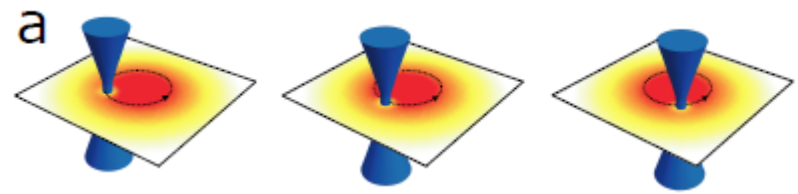
Above critical velocity dissipative effect produced by moving optical lattice is observed

(Mit, Miller et al, 2007)

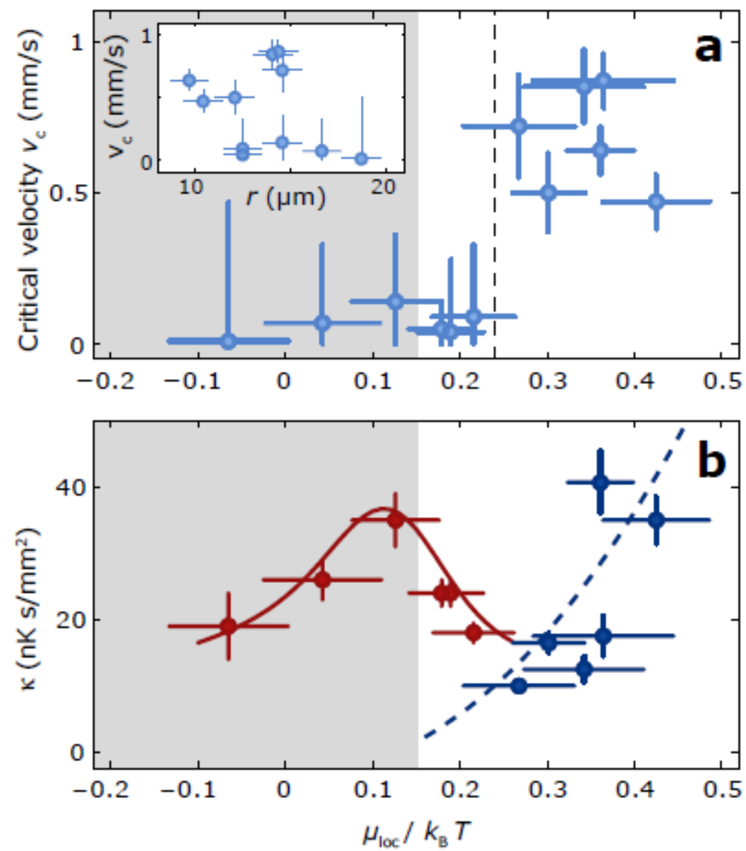
Critical velocity across the BKT transition

Desbuquois et al.

Nature Physics 8, 645 (2012)



While in the normal phase the Landau's critical velocity is practically zero, at some temperature it exhibits a sudden jump to a finite value revealing the occurrence of a phase transition associated with a **jump of the superfluid density**



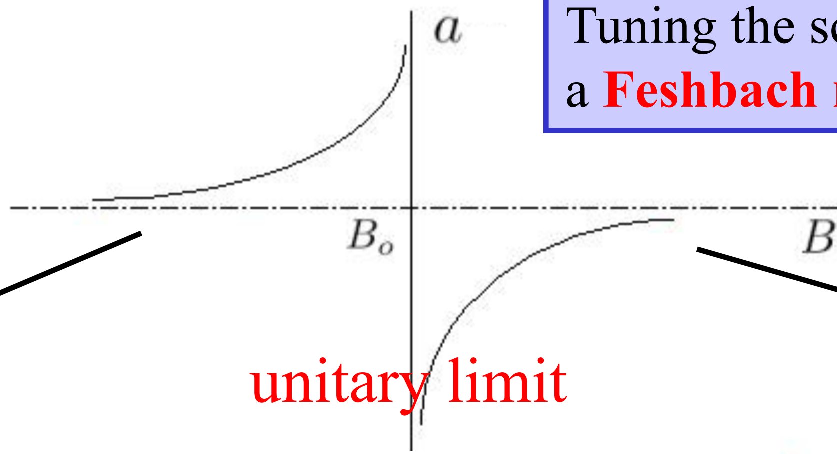
Superfluidity in ultracold atomic gases (measured quantities)

- **Quantized vortices**
- Quenching of **moment of inertia**
- **Josephson** oscillations
- Absence of **viscosity** and **Landau critical velocity**
- **BKT transition** in 2D Bose gases
- **Lambda transition** in resonant Fermi gas
- **First** and **second sound**
- **Superfluidity in Spin-orbit coupled BEC's**

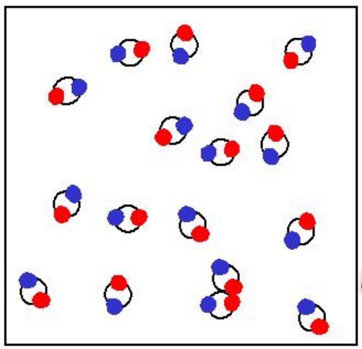
Fermi Superfluidity: the BEC-BCS Crossover

(Eagles, Leggett, Nozieres, Schmitt.Rink, Randeria)

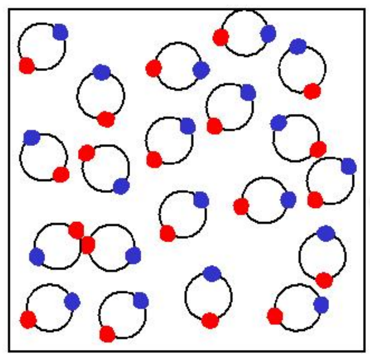
Tuning the scattering length through a **Feshbach resonance**



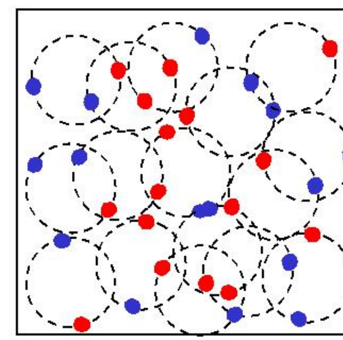
BEC regime
(molecules)



unitary limit



BCS regime
(Cooper pairs)



Dilute Bose gas
(size of molecules much smaller than interparticle distance

At unitarity scattering lenght is much larger than interparticle distance

Unitary Fermi gas (1/a=0): challenging many-body system

- diluteness
(interparticle distance >> range of inetraction)

- strong interactions
(scattering length >> interparticle distance)

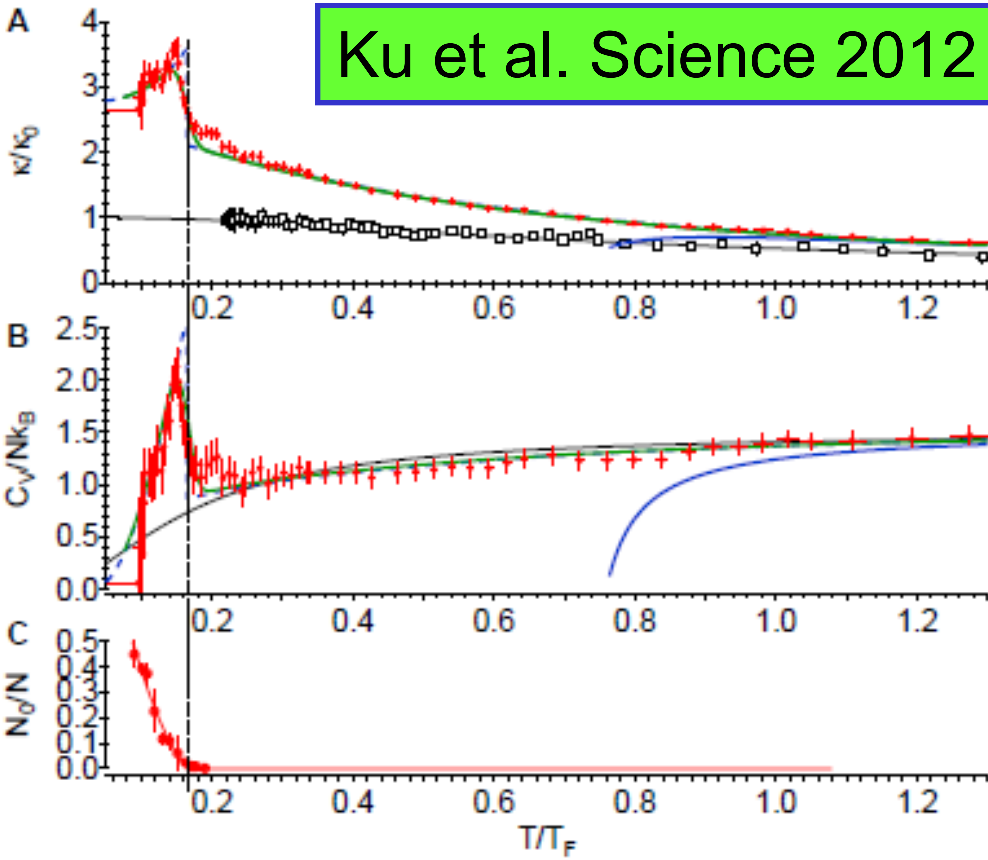
- universality
(no dependence on interaction parameters)

- robust superfluidity (high critical velocity)

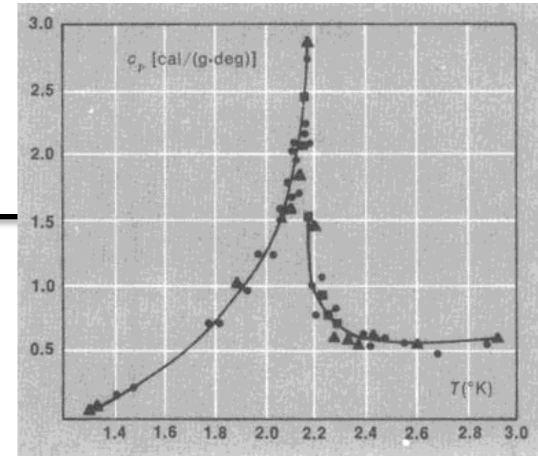
- high Tc
(of the order of
Fermi temperature

Conventional superconductors	10(-5)-10(-4)
Superfluid He3	10(-3)
High-temperature superconductors	10(-2)
Fermi gases with resonant interactions	0.2

Ku et al. Science 2012



Specific heat exhibits characteristic peak at the transition



Superfluid He4

Experimental determination of critical temperature

$$T_C / T_F = 0.167(13)$$

(determined by jump in specific heat and onset of BEC) in agreement with many-body predictions (Burowski et al. 2006; Haussmann et al. (2007); Goulko and Wingate 2010)

Major question: How to **measure** the **superfluid density** ? (not available from equlibrium thermodynamics, needed **transport** phenomena)

Measurement of **second sound** gives access to **superfluid density**

(**Innsbruck-Trento** collaboration)

Second sound and the superfluid fraction in a resonantly interacting Fermi gas

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[1] *Dipartimento di Fisica, Universitá di Trento and INO-CNR BEC Center, I-38123 Povo, Italy and*
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(Dated: February 13, 2013)

Nature 498, 78 (2013)

Superfluidity in ultracold atomic gases (measured quantities)

- **Quantized vortices**
- Quenching of **moment of inertia**
- **Josephson** oscillations
- Absence of **viscosity** and **Landau critical velocity**
- **BKT transition** in 2D Bose gases
- **Lambda transition** in resonant Fermi gas
- **First** and **second sound**
- **Superfluidity in Spin-orbit coupled BEC's**

Dynamic theory for superfluids at finite temperature:
Landau's Two-fluid HD equations

(hold in deep collisional regime $\omega\tau << 1$)

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}(\vec{j}) = 0$$

$$\frac{\partial}{\partial t}s + \vec{\nabla}(s\vec{v}_N) = 0$$

$$m\frac{\partial}{\partial t}\vec{v}_S + \nabla(\mu(n) + V_{ext}) = 0$$

$$\frac{\partial}{\partial t}\vec{j} + \vec{\nabla}P + n\vec{\nabla}V_{ext} = 0$$

Irrotationality of superfluid flow

$$\rho = mn = \rho_S + \rho_N$$

$$\vec{j} = \rho_S\vec{v}_S + \rho_N\vec{v}_N$$

s is entropy density
P is local pressure

Ingredients:
- equation of state
- superfluid density

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}(\vec{j}) = 0$$

$$\cancel{\frac{\partial}{\partial t}s + \vec{\nabla}(s\vec{v}_N) = 0}$$

$$m\frac{\partial}{\partial t}\vec{v}_S + \nabla(\mu(n) + V_{ext}) = 0$$

$$\frac{\partial}{\partial t}\vec{j} + \vec{\nabla}P + n\vec{\nabla}V_{ext} = 0$$

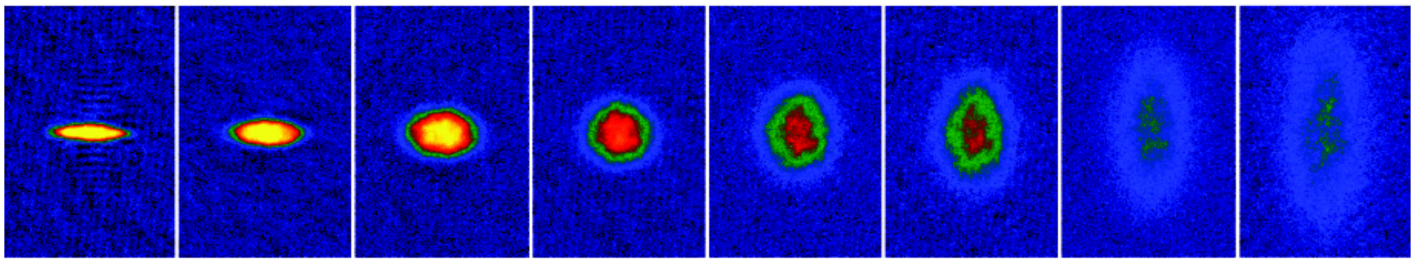
equivalent at T=0

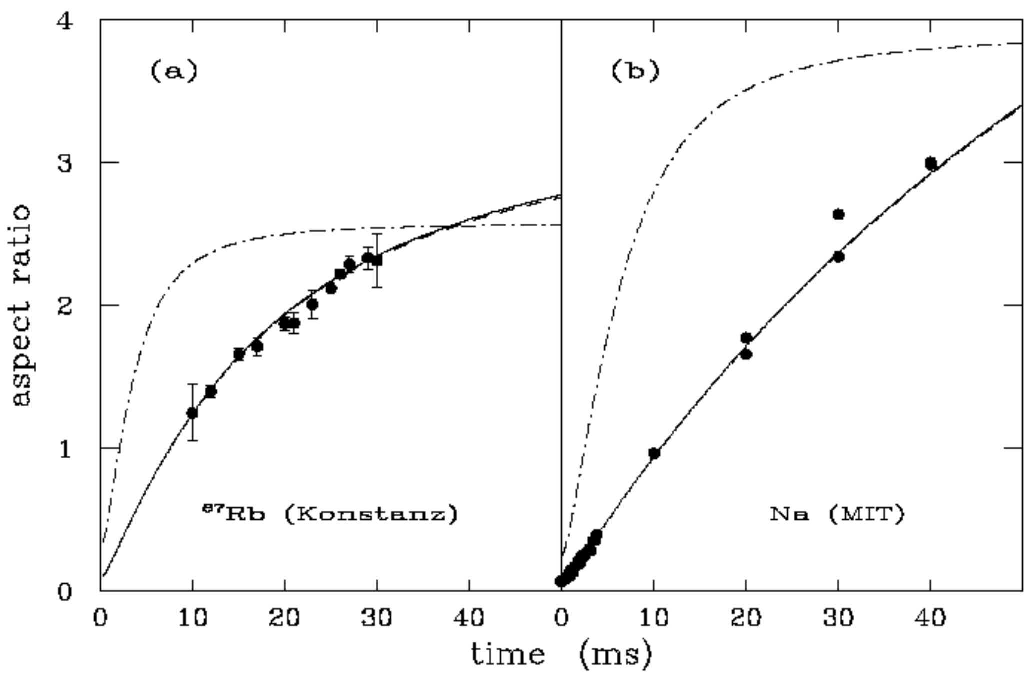
At T=0: $\rho = \rho_S \, ; \vec{j} = \rho\vec{v}_S$
eqs. reduce to
T=0 **irrotational**
superfluid HD equations

*At T=0 irrotational hydrodynamics follows from superfluidity (role of the phase of the order parameter). Quite successful to describe the macroscopic dynamic behavior of trapped atomic gases (Bose and Fermi) (**expansion**, **collective oscillations**)*

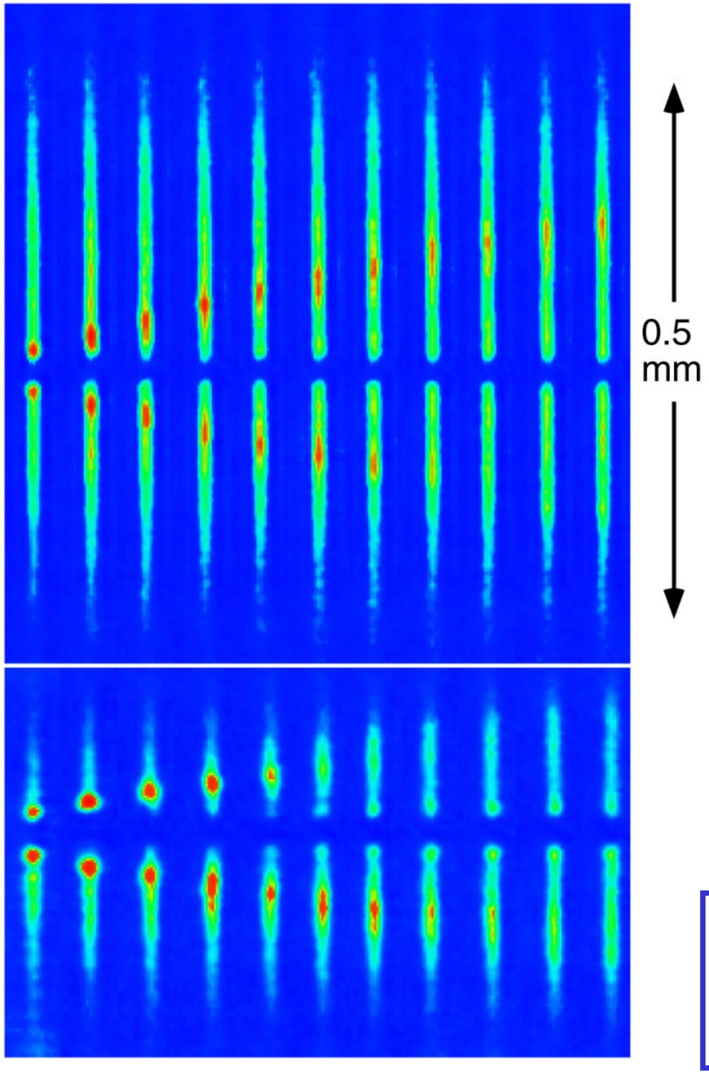
Hydrodynamics predicts anisotropic expansion of the superfluid

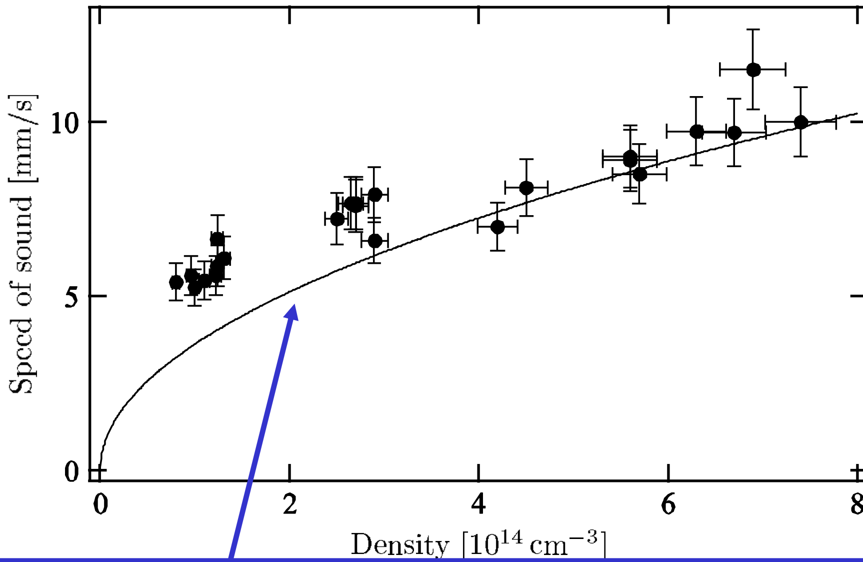
(Kagan, Surkov, Shlyapnikov 1996; Castin, Dum 1996,





T=0 Bogoliubov sound
(wave packet propagating in a dilute BEC, Mit 97)





sound velocity as a function of central density

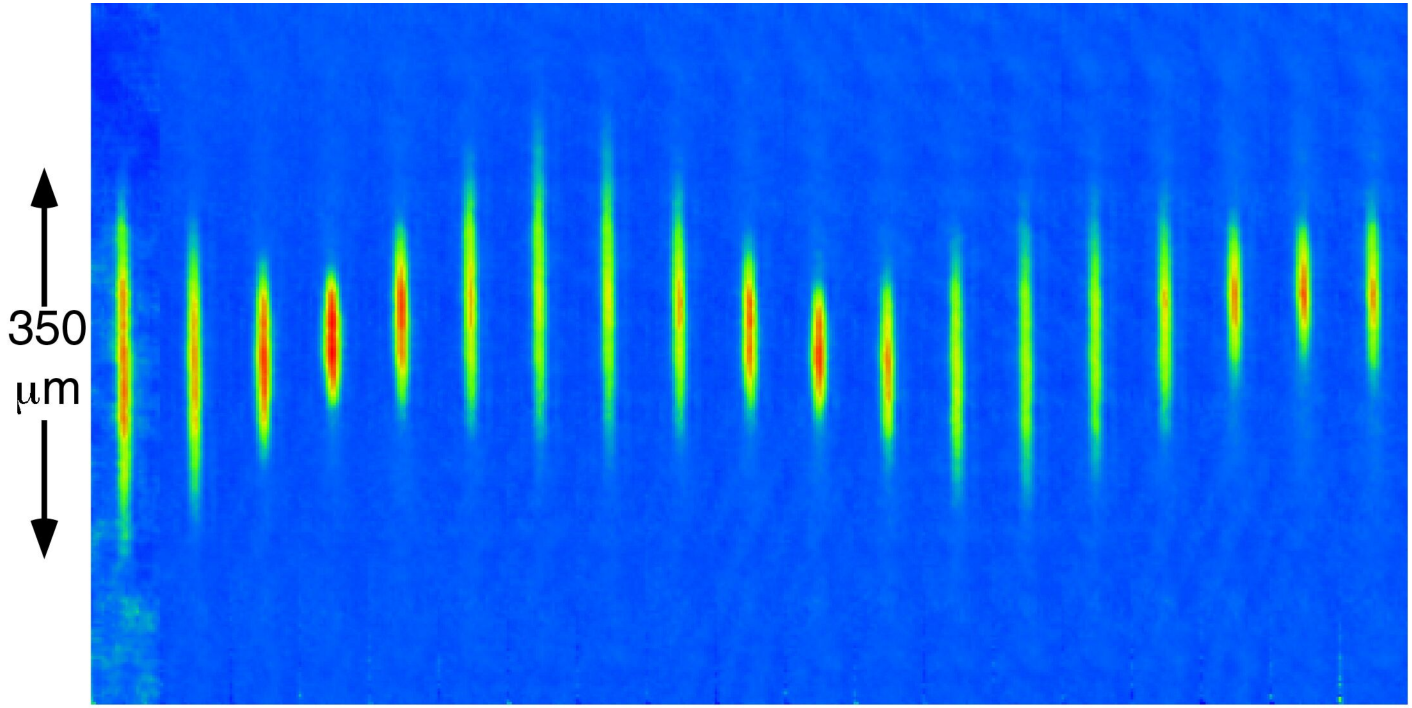
$$c = \sqrt{gn/2m}$$

factor 2 accounts for harmonic radial trapping (Zaremba, 98)

T=0 Collective oscillations in dilute BEC (axial compression mode) : checking validity of **hydrodynamic theory** of superfluids in **trapped gases**

Exp (Mit, 1997) $\omega = 1.57\omega_z$

HD Theory (S.S. 1996): $\omega = \sqrt{5/2}\,\omega_z = 1.58\omega_z$



5 milliseconds per frame

SOLVING THE HYDRODYNAMIC EQUATIONS OF SUPERFLUIDS

AT FINITE TEMPERATURE

In **uniform matter** Landau equations gives rise to two solutions below the critical temperature:

First sound: superfluid and normal fluids move **in phase**

Second sound: superfluid and normal fluids move **in opposite phase**.

If condition $\boxed{\frac{c_2^2}{c_1^2}\frac{C_P - C_V}{C_V} << 1}$ is satisfied (small compressibility and/or small expansion coefficient) well satisfied by unitary Fermi gas)

second sound reduces to Isobaric oscillation (**constant pressure**)

$$c_2^2 = \frac{1}{m}\frac{n_s T s^2}{n_n C_P}$$

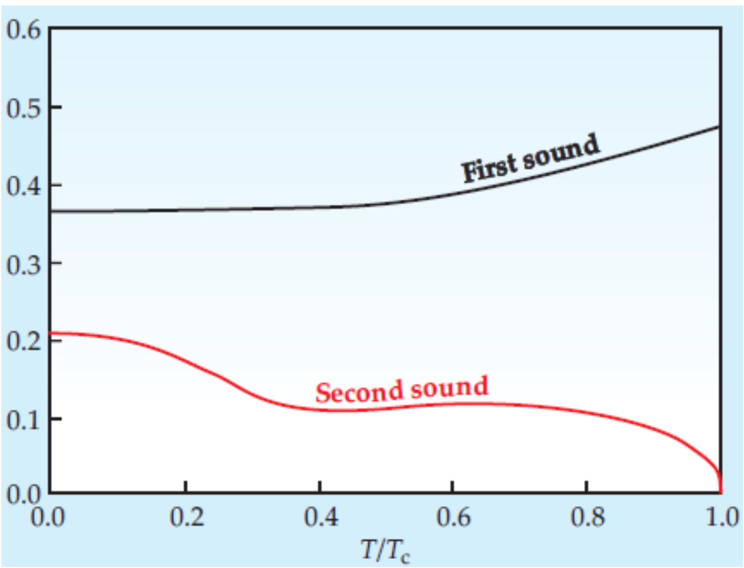
entropy

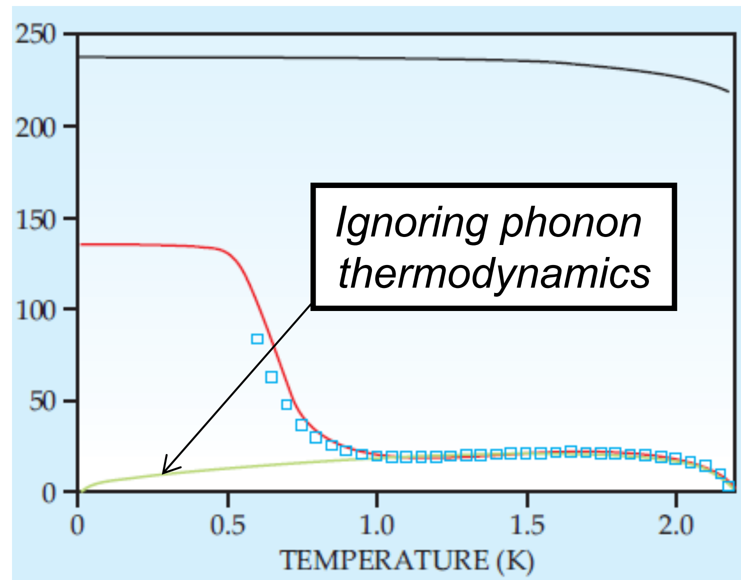
Specific heat

In this regime second sound velocity is fixed by superfluid density

First and second sound velocities in **uniform matter**

$$c_1^2 = \tfrac{1}{m}\left(\frac{\partial P}{\partial n}\right)_S \qquad c_2^2 = \tfrac{1}{m}\frac{n_s T s^2}{n_n C_P}$$



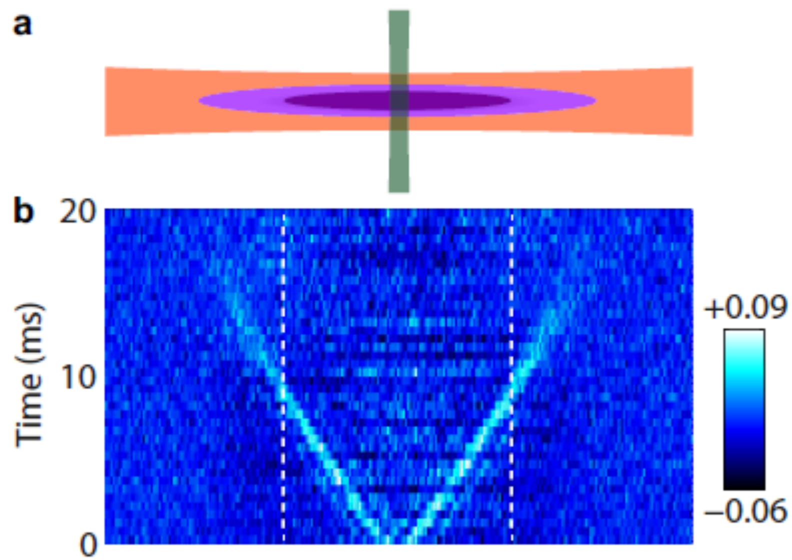


Unitary Fermi gas
Hu et al. , NJP et al. 2010

Liquid He
(experiment, Peshkov 1946)

In recent IBK experiment both **first** and **second** sound waves have been investigated in **cigar**-like traps

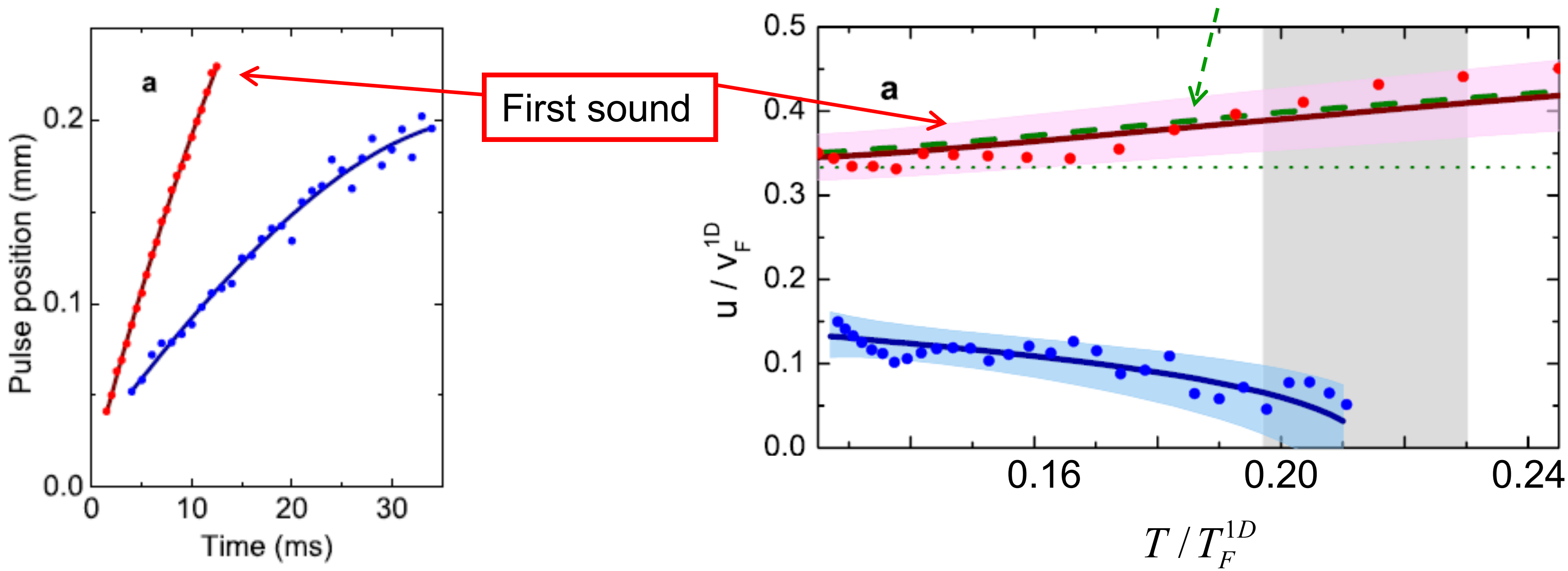
To excite **first sound** one suddenly turns on a repulsive (green) laser beam in the center of the trap [similar tecnhnique used at Mit (1998) and Utrecht (2009) to generate Bogoliubov sound in dilute BEC and at Duke (2011) to excite sound in a Fermi gas along the BEC-BCS crossover at T=0]



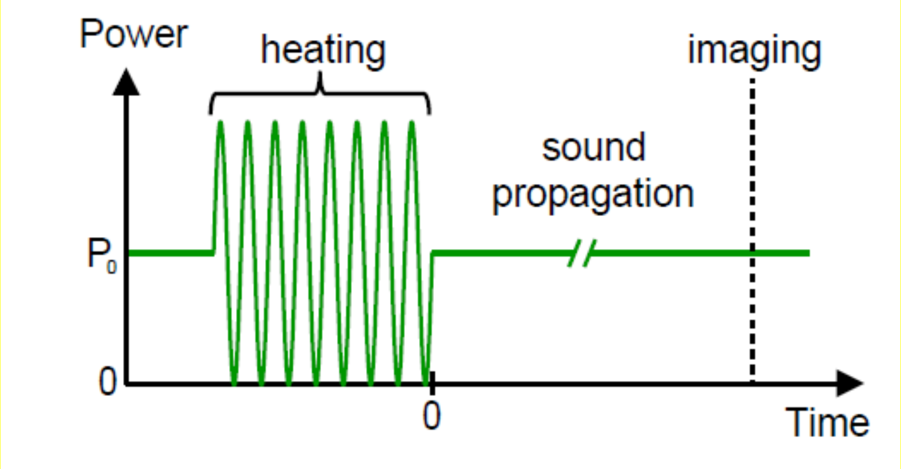
By measuring velocity of the signal at different times (different pulse positions) one extracts behavior as a function of T / T_F^{1D}. In fact T is fixed but T_F decreases as the perturbation moves to the periphery (lower density)

Velocity of **first sound** of radially trapped unitary Fermi gas agres with adiabatic law at all temperatures also in the superfluid phase

$$\boxed{mc^2 = \frac{7}{5}\frac{P_1}{n_1}}$$



To excite **second sound** one keeps the repulsive (green) laser power constant with the exception of a short time modulation producing local heating in the center of the trap

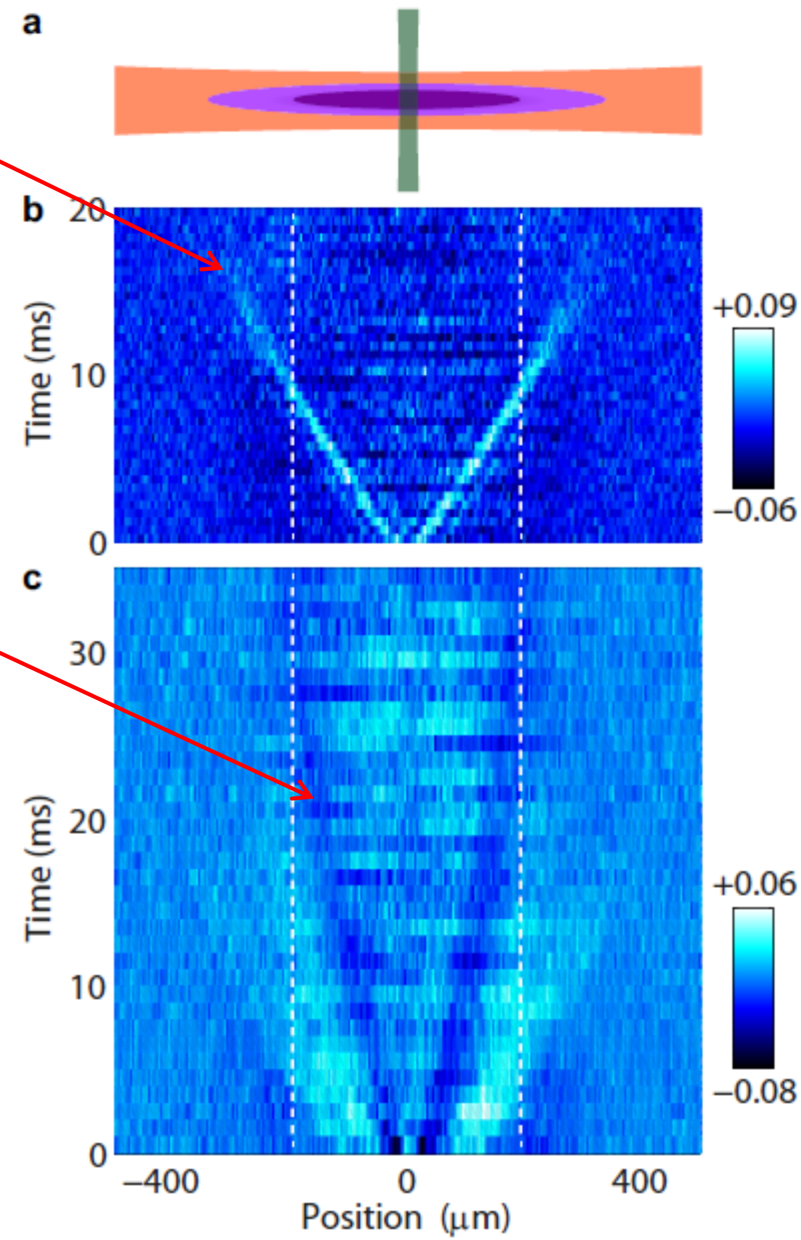


The average laser power is kept constant to limit the excitation of pressure waves (first sound)

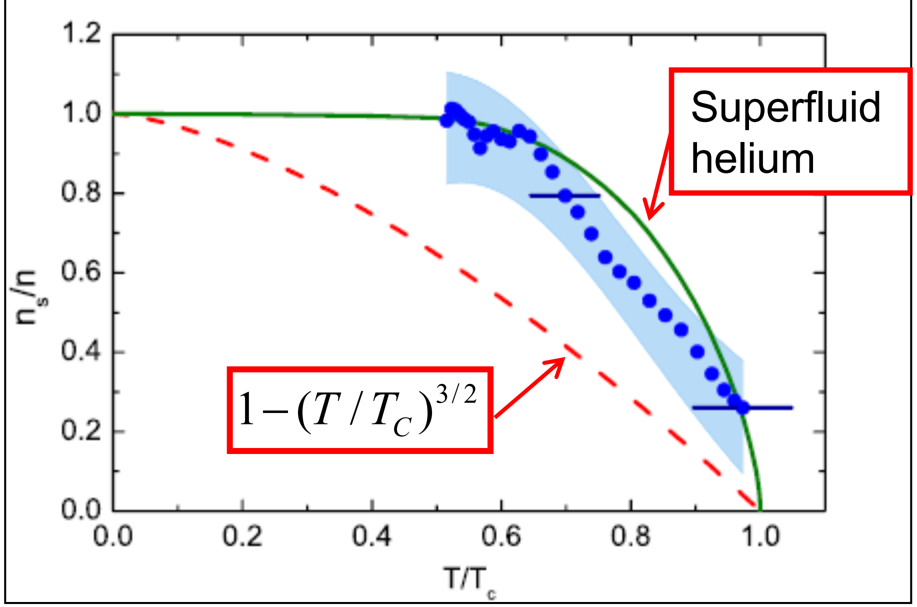
First sound
propagates also beyond the boundary between the superfluid and the normal parts

Second sound
propagates only within the region of co-existence of the super and normal fluids.

Second sound is basically an isobaric wave
Signal is visibile because of small, but **finite thermal expansion**.



From measurement of second sound velocity in cigar geometry and 3D reconstruction one determines **superfluid density**

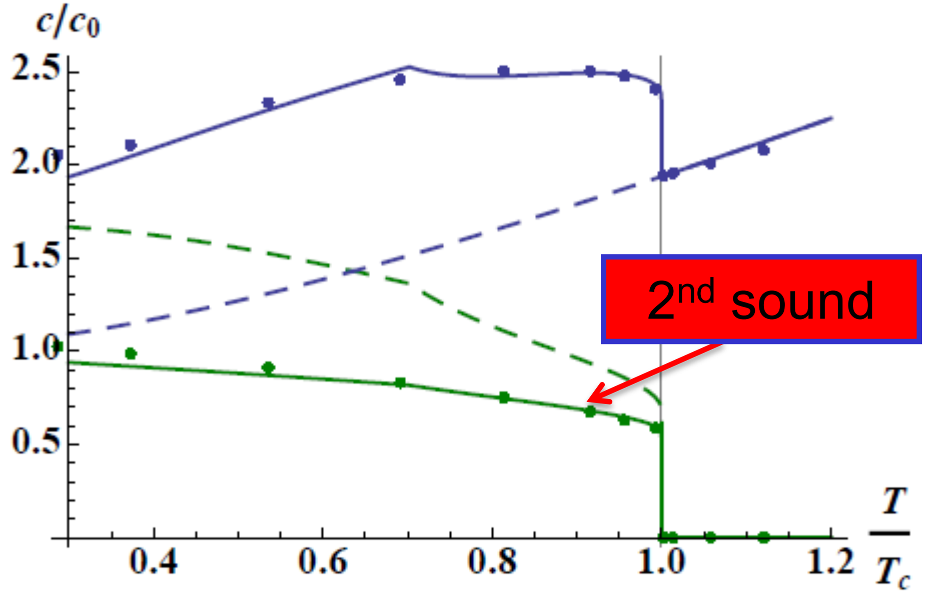


- Superfluid fraction of **unitary Fermi gas** similar to the one of **superfluid helium**
- Very **different** behavior compared to dilute **BEC gas**
- Superfluid density **differs** significantly from condensate **fracton of pairs** (about 0.5 at T=0, Astrakharchik et al 2005)
- New benchmark for **many-body calculations**

What happens to second sound in 2D Bose gases ?

Key features in 2D

a) Absence of Bose-Einstein Condensation (Hohenberg-Mermin-Wagner theorem)
b) Superfluid density and second sound velocity have jump at the **BKT** transition



Ozawa and S.S. PRL 2014

Future experiements on second sound can provide unique information on T-dependence of superfluid density in 2D

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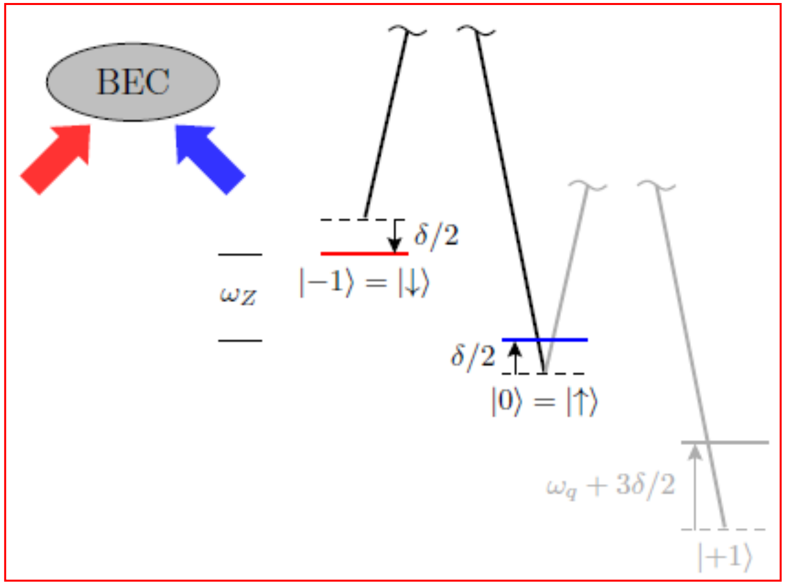
Simplest realization of (1D) spin-orbit coupling in s=1/2 Bose-Einstein condensates (Spielman team at Nist, 2009)

Two detuned ($\Delta\omega_L$) and polarized laser beams + non linear Zeeman field (ω_Z) provide Raman transitions between **two** spin states, giving rise to the single particle spin-orbit Hamitonian

$$h_0 = \frac{1}{2}[(p_x - k_0\sigma_z)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z$$



p_x is canonical momentum

$v_x = p_x - k_0\sigma_z$ is physical velocity

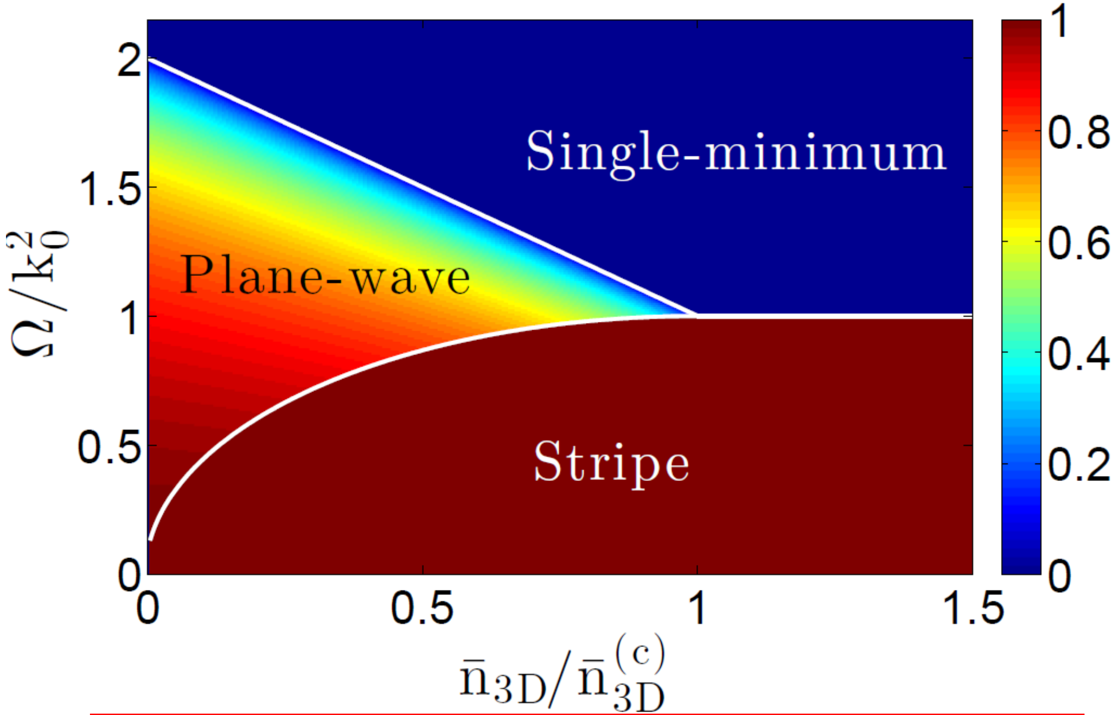
k_0 is laser wave vector difference

Ω is strength of Raman coupling

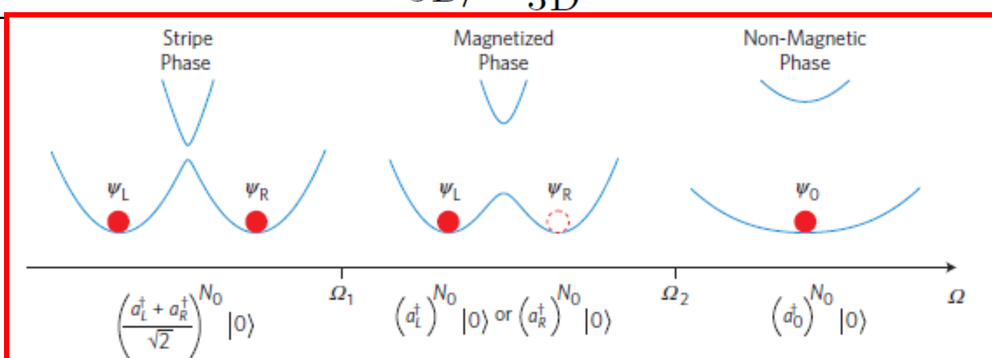
$\delta = \Delta\omega_L - \omega_Z$ is effective Zeeman field

Quantum phases predicted in the presence of interactions

$$H = \sum_i h_0(i) + \sum_{\alpha,\beta} \frac{1}{2} \int d\vec{r}\, g_{\alpha\beta} n_\alpha n_\beta$$



Ho and Zhang, 2011, Yun Li et al. 2013



Hamiltonian $$h_0 = \frac{1}{2}[(p_x - k_0\sigma_z)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z$$

- is **translationally invariant** $\boxed{[h_0, p_x] = 0}$
- breaks **parity** and **time reversal** symmetry

  breaks **Galilean** invariance

CONSEQUENCES

- **Translational** invariance: $\Rightarrow$ uniform ground state unless crystalline order is formed spontaneously (**stripes**)

- **Violation** of **parity** and **time** reversal symmetry $\Rightarrow$ breaking of symmetry $\omega(q) = \omega(-q)$ in excitation spectrum. Emergence of **rotons**

  (theory: Martone et al. PRA 2012; exp: Shuai Chen, arXiv:1408.1755)

- **Violation** of **Galilean** invariance: $\Rightarrow$ breakdown of Landau criterion for superfluid velocity, new dynamical

  (exp: Zhang et. al. PRL 2012, theory: Ozawa et al. (PRL 2013))

Novel dynamic behavior of Spin-orbit coupled BECs

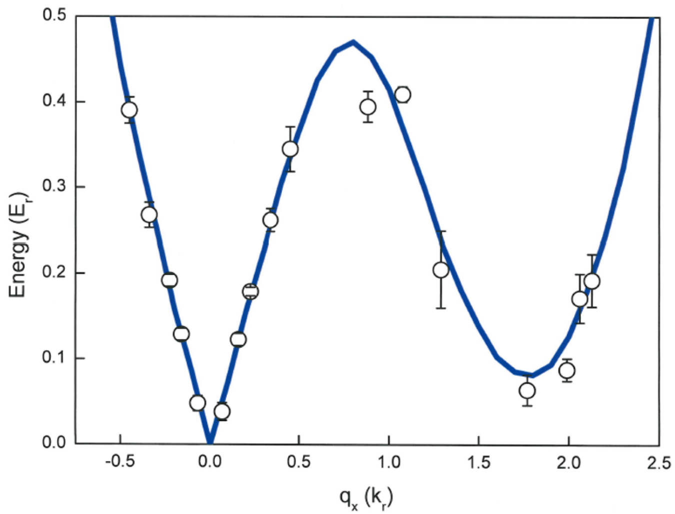
- Ocurrence of roton minimum

- Double gapless band in the striped phase
  (effect of supersolidity)

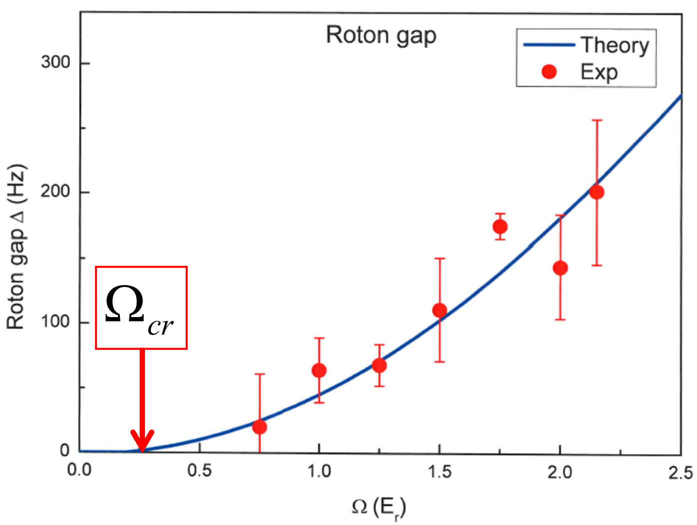
Phonon-maxon-roton in the plane wave phase of a 87Rb spin-orbit condensate

Theory: Martone et al., PRA 2012
Exp: Shuai Chen et al. arXiv:1408.1755

see also Khamehchi et al: arXiv: 1409.5387



$\omega(q) \neq \omega(-q)$ as a consequence of violation of **parity** and **time reversal** symmetry



Roton gap decreases as Raman coupling is lowered: **onset of crystallization** (**striped** phase)

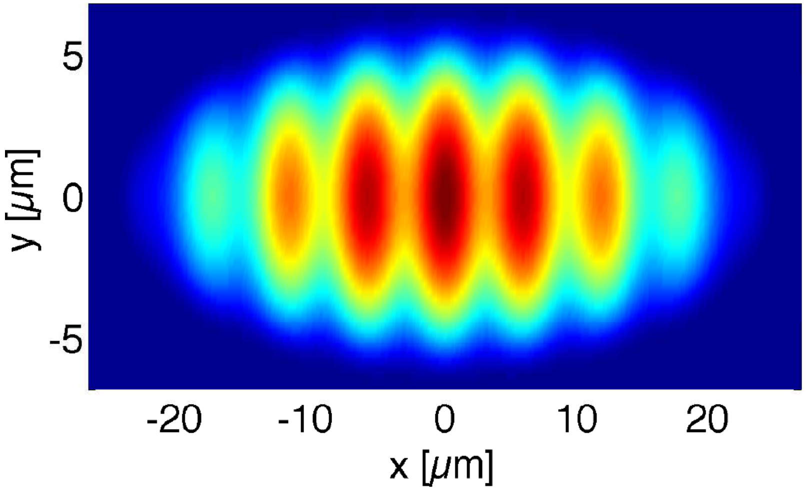
Superstripes in spin-orbited coupled BECs

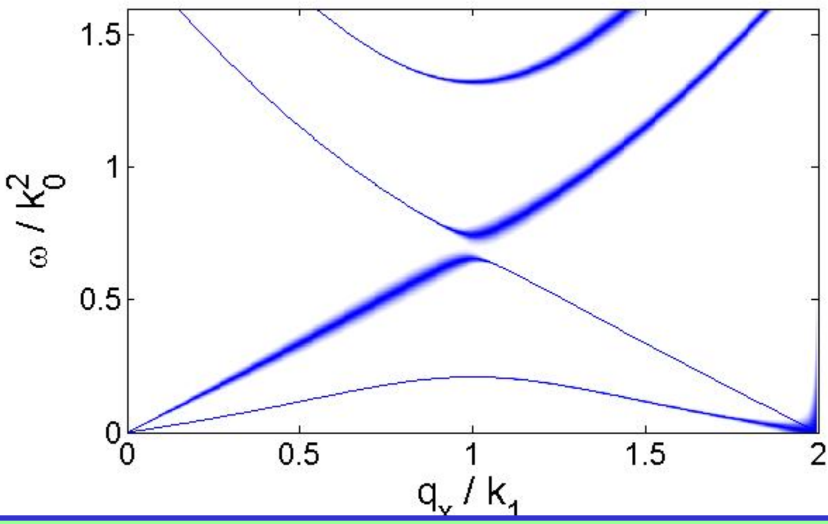
(Yun Li et al. Trento, PRL 2013)

At small Raman coupling one predicts stripe phase (spontaneous breaking of translational symmetry):

i) Emergence of density fringes

ii) Two gapless bands in excitation spectrum





Typical features of supersolidity

Superfluidity in ultracold atomic gases

- **Quantized vortices**
- Quenching of **moment of inertia**
- **Josephson** oscillations
- Absence of **viscosity** and **Landau critical velocity**
- **BKT transition** in 2D Bose gases
- **Lambda transition** in resonant Fermi gas
- **First** and **second sound**
- **Superfluidity in Spin-orbit coupled BEC's**

The Trento BEC team

http://bec.science.unitn.it/

COLD ATOMS MEET HIGH ENERGY PHYSICS

Workshop at ECT* Trento (June 22-25, 2015)

Organizers: Massimo Inguscio (LENS Florence and INRIM Torino), Guido Martinelli (SISSA Trieste) and Sandro Stringari (Trento)

Main topics include: Sponatenously broken symmetries, abelian and non abelian gauge fields, supersymmetries, Fulde-Ferrel-Larchin-Ochinokov phase, Superfluidity in strongly interacting Fermi systems, High density QCD and bosonic superfluidity, quantum hydrodynamics, Kibble-Zurek mechanism, SU(N) configurations, quantum simulation of quark confinement, magnetic monopoles, Majorana Fermions, role of extra dimensions, lattice QCD, black holes, Hawking radiation, Higgs excitations in cold atoms, AdS/CFT correspondence, Efimov states, instantons.