Quantum Simulation of Abelian and non-Abelian Gauge Theories

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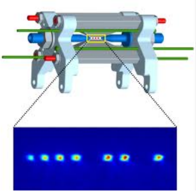
January 16, 2015





Outline

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The first "digital computer" in Babylonia about 2400 b.c.

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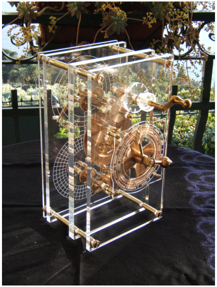
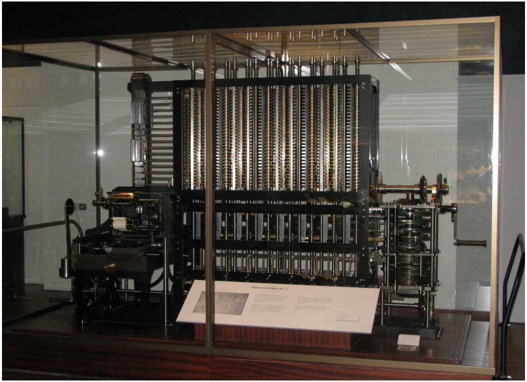
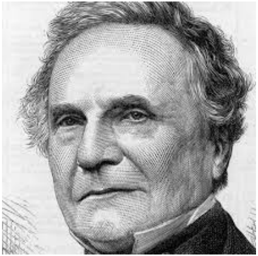
The first "analog computer": Antikythera for determining the position of celestial bodies, Crete, about 100 b.c.



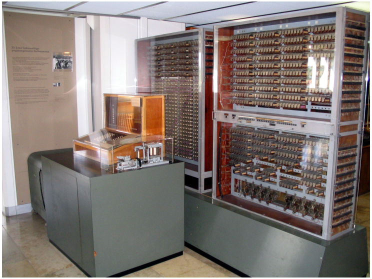
The first programmable computer:
mechanical "difference engine"
Charles Babbage (1791-1871)





was realized by his son after Babagge's death.

Konrad Zuse's (1910-1992) relay-driven computer Z3



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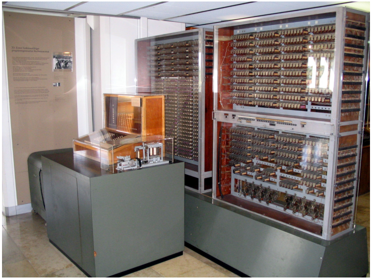
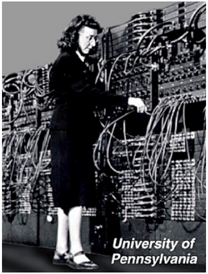
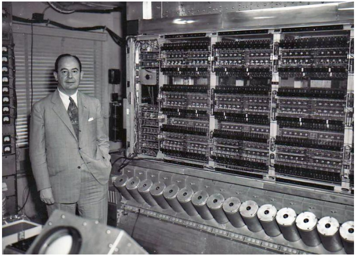
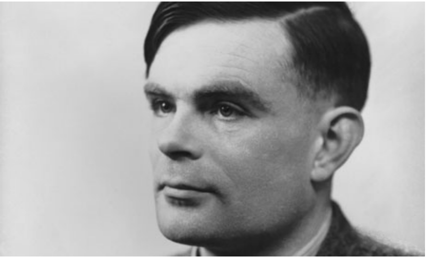


From the vacuum-tube ENIAC to the IBM Blue Gene

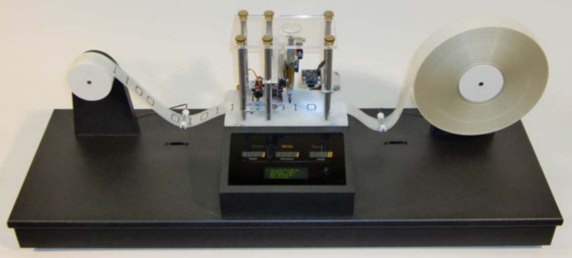


Pioneers of theoretical computer science: John von Neumann (1903-1992) and Alan Turing (1912-1954)





Model of a universal Turing machine



RSA encryption: multiplication is easy, factorization is hard.

RSA decryption challenge in 1991:
factorize the following 174-digit number with 576 bits

$$
\begin{aligned}
RSA576 \quad = \quad & 18819881292060796383869723946165043980716356 \\
& 33794173827007633564229888597152346654853190 \\
& 60606504743045317388011303396716199692321205 \\
& 734031879550656996221305168759307650257059
\end{aligned}
$$

RSA encryption: multiplication is easy, factorization is hard.

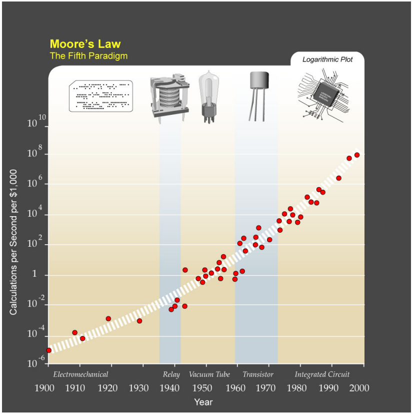
RSA decryption challenge in 1991:
factorize the following 174-digit number with 576 bits

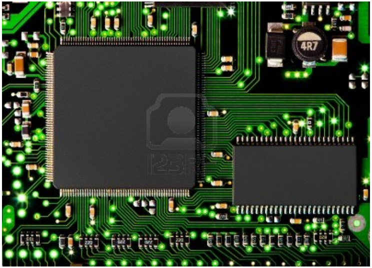
$$
\begin{aligned}
RSA576 &= 1881988129206079638386972394616504398071635\,6 \\
&\ 337941738270076335642298885971523466548531906 \\
&\ 060650474304531738801130339671619969232120 5 \\
&\ 73403187955065699622130516875930765025705 9 \\
&= 398075086424064937397125500550386491199064 36 \\
&\ 23425267084063851895759463889572617685833 17 \\
&* 472772146107435302536223071973048224632914 69 \\
&\ 53020971164598521711305207112563635903975 27
\end{aligned}
$$

This problem was solved only in 2003 by two mathematicians in Bonn using very large computer resources.

Only in 2009, when the challenge was no longer active, the 232-digit number RSA768 with 768 bits has finally been factorized.

Moore's law: "Every two years the number of transistors per area increases by a factor of 2."





Modern micro chips consist of several billions of transistors, each about 10^{-8} m in size. This is already close to the quantum mechanical limit set by the size of individual atoms.

Outline

Richard Feynman's vision of 1982

"I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

Deutsch's universal quantum computer could use Shor's algorithm to solve the factorization problem.

David Deutsch

Peter Shor

Deutsch's universal quantum computer could use Shor's algorithm to solve the factorization problem.

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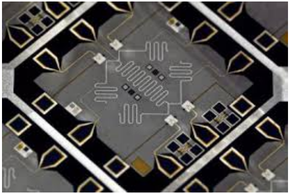
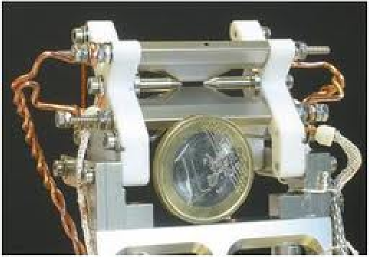
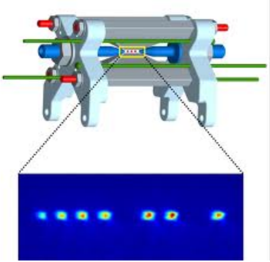
Until today, only $15 = 3 \cdot 5$ has been correctly factorized by a quantum computer, at least in about 50 % of all trials.

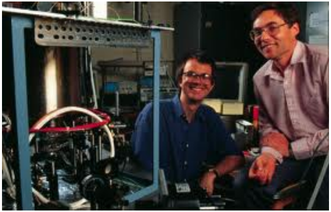


Ion traps as a digital quantum computer?

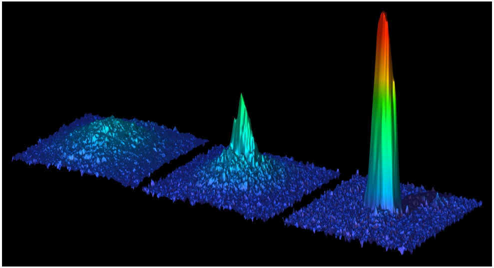
 

Franklin Medal 2010: I. Cirac, D. Wineland, P. Zoller

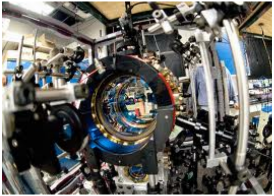
Bose-Einstein condensation in ultra-cold atomic gases

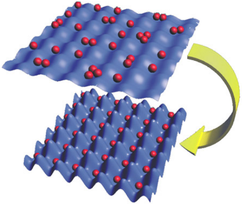
 

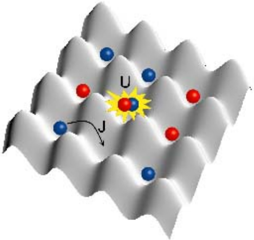
Eric Cornell, Carl Wieman, Wolfgang Ketterle, 1995



Ultra-cold atoms in optical lattices as an analog quantum simulator for the bosonic Hubbard model







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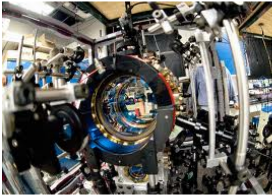
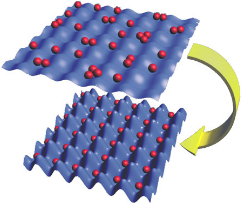
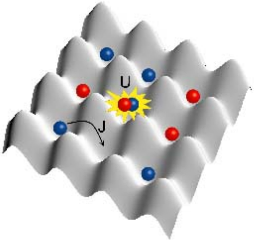
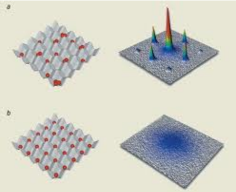


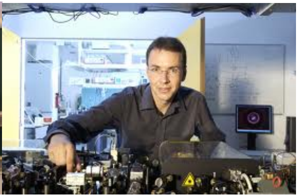




Transition from a superfluid to a Mott insulator



Theodor Hänsch

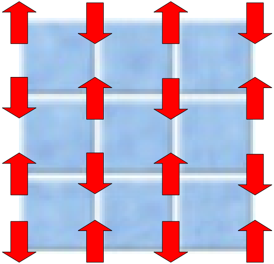


Immanuel Bloch

Can one understand high-T_c superconductivity in this way?

Outline

The spin $\frac{1}{2}$ quantum Heisenberg model



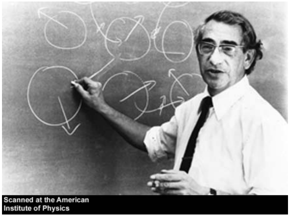
Quantum spins $[S_x^a, S_y^b] = i\delta_{xy}\varepsilon_{abc}S_x^c$ and their Hamiltonian

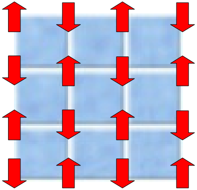
$$H = J\sum_{\langle xy\rangle}\vec{S}_x\cdot\vec{S}_y$$

Partition function at inverse temperature $\beta = 1/T$

$$Z = \mathrm{Tr}\exp(-\beta H)$$

The Hubbard Model for doped antiferromagnets





$$H = -t \sum_{\langle xy \rangle} (c_x^\dagger c_y + c_y^\dagger c_x) + U \sum_x (c_x^\dagger c_x - 1)^2, \quad c_x = \begin{pmatrix} c_{x\uparrow} \\ c_{x\downarrow} \end{pmatrix}$$

Sign problem of fermionic path integrals

$$Z_f = \mathrm{Tr}\exp(-\beta H) = \sum_{[n]} \mathrm{Sign}[n] \exp(-S[n]) \ , \quad \mathrm{Sign}[n] = \pm 1$$

Average sign is exponentially small

$$\langle \mathrm{Sign} \rangle = \frac{\sum_{[n]} \mathrm{Sign}[n] \exp(-S[n])}{\sum_{[n]} \exp(-S[n])} = \frac{Z_f}{Z_b} = \exp(-\beta V \Delta f)$$

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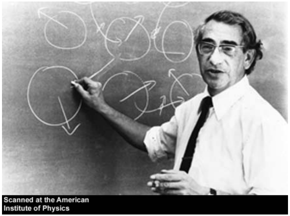

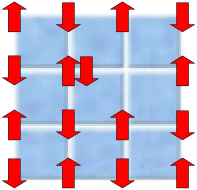

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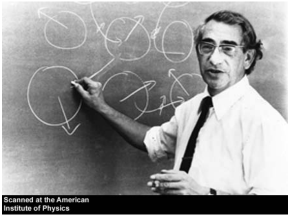

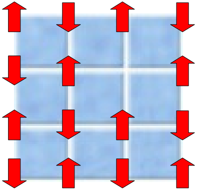

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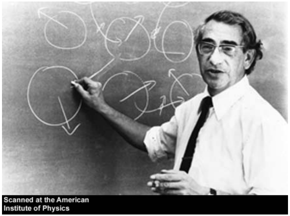

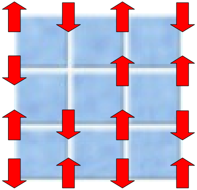

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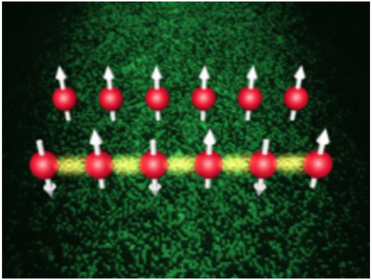
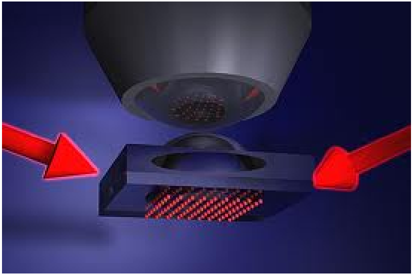
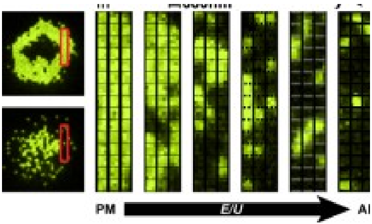
Optical lattice quantum simulation of quantum spin systems



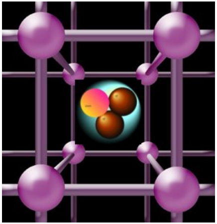




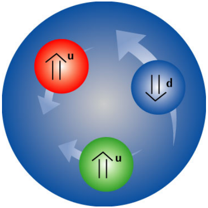
J. Simon, W. S. Bakir, R. Ma, M. E. Tal, P. M. Preis, M. Greiner, Nature 472 (2011) 307.

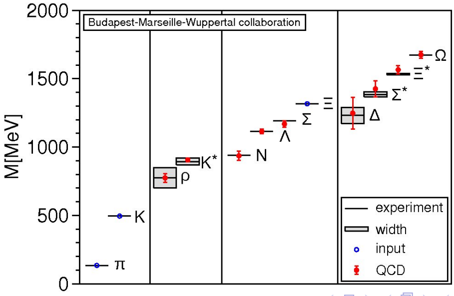
Outline

Kenneth Wilson's lattice QCD describes confinement of quarks and gluons inside protons und neutrons

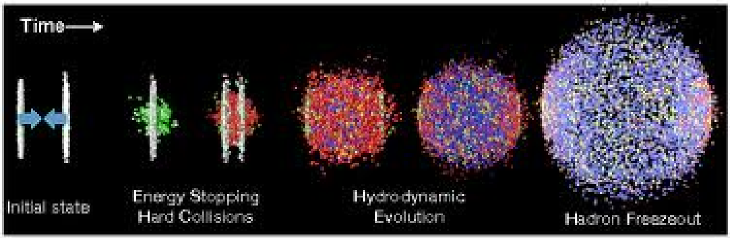


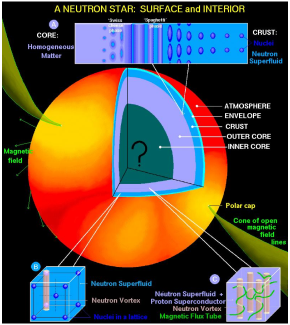
and confirms the experimentally measured mass spectrum

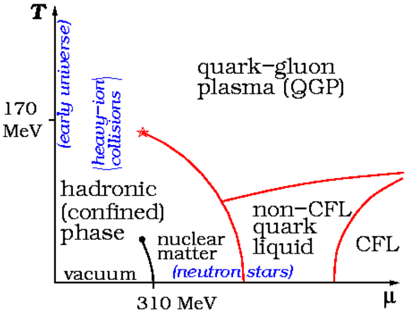




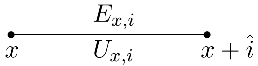
Can heavy-ion collision physics or nuclear astrophysics benefit from quantum simulations in the long run?







Hamiltonian formulation of Wilson's $U(1)$ lattice gauge theory



$$U = \exp(iA), \ U^\dagger = \exp(-iA) \in U(1)$$

Electric field operator E

$$E = -i\partial_A, \ [E, U] = U, \ [E, U^\dagger] = -U^\dagger, \ [U, U^\dagger] = 0$$

Generator of $U(1)$ gauge transformations

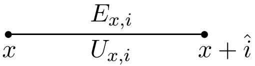
$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

$U(1)$ gauge invariant Hamiltonian

$$H = \frac{g^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2g^2} \sum_{x,i\neq j} (U_{x,i} U_{x+\hat{i},j} U^\dagger_{x+\hat{j},i} U^\dagger_{x,j} + \text{h.c.})$$

operates in an infinite-dimensional Hilbert space per link

$U(1)$ quantum links from spins $\frac{1}{2}$

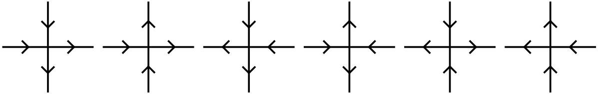


$$U = S_1 + iS_2 = S_+, \ U^\dagger = S_1 - iS_2 = S_-$$

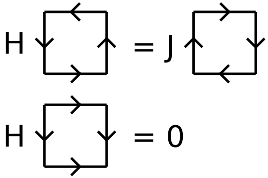
Electric flux operator E

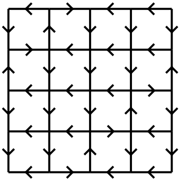
$$E = S_3, \ [E, U] = U, \ [E, U^\dagger] = -U^\dagger, \ [U, U^\dagger] = 2E$$

Gauss law



Ring-exchange plaquette Hamiltonian



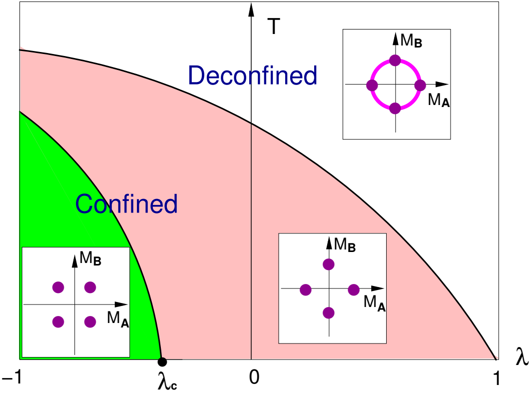


D. Horn, Phys. Lett. B100 (1981) 149
P. Orland, D. Rohrlich, Nucl. Phys. B338 (1990) 647
S. Chandrasekharan, UJW, Nucl. Phys. B492 (1997) 455

Hamiltonian with Rokhsar-Kivelson term

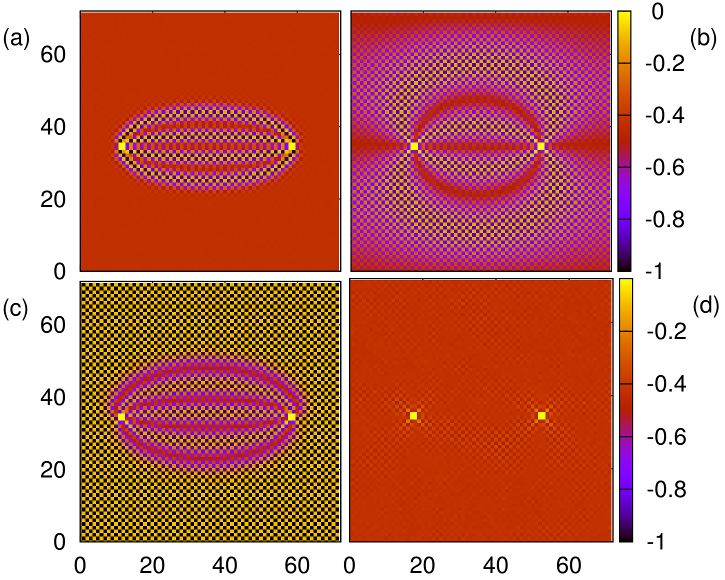
$$H = -J \left[\sum_{\square} (U_{\square} + U_{\square}^{\dagger}) - \lambda \sum_{\square} (U_{\square} + U_{\square}^{\dagger})^2 \right]$$

Phase diagram



D. Banerjee, F.-J. Jiang, P. Widmer, UJW, JSTAT (2013) P12010.

Energy density of charge-anti-charge pair $Q = \pm 2$



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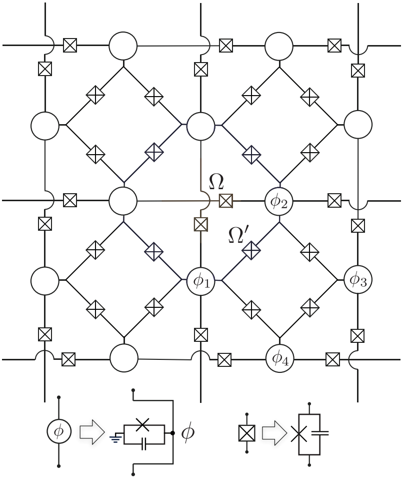
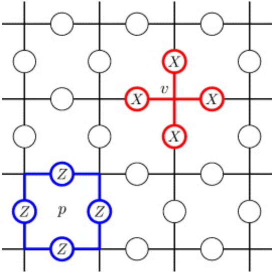
Outline

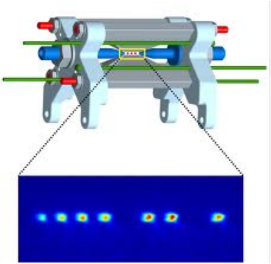
"String theory on a chip" with superconducting circuits



D. Marcos, P. Rabl, E. Rico, P. Zoller,
Phys. Rev. Lett. 111 (2013) 110504 (2013).
D. Marcos, P. Widmer, E. Rico, M. Hafezi, P. Rabl, UJW, P. Zoller,
arXiv:1407.6066.

Digital quantum simulation of Kitaev's toric code (a $\mathbb{Z}(2)$ quantum link model) with trapped ions

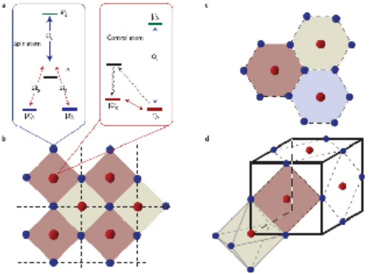


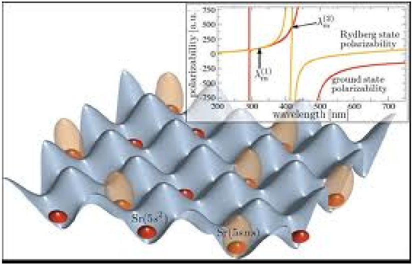


- Precisely controllable many-body quantum device, which can execute a prescribed sequence of quantum gate operations.
- State of simulated system encoded as quantum information.
- Dynamics is represented by a sequence of quantum gates, following a stroboscopic Trotter decomposition.

A. Y. Kitaev, Ann. Phys. 303 (2003) 2.
B. P. Lanyon, C. Hempel, D. Nigg, M. Müller, R. Gerritsma, F. Zähringer, P. Schindler, J. T. Barreiro, M. Rambach, G. Kirchmair, M. Hennrich, P. Zoller, R. Blatt, C. F. Roos, Science 334 (2011) 6052.

$U(1)$ quantum link models can also be simulated with Rydberg atoms in an optical lattice





- Lasers can excite atoms to high-lying Rydberg states.
- Rydberg atoms are large and have collective interactions.
- Ensemble Rydberg atoms represent qubits at link centers.
- Control atoms at lattice sites ensure the Gauss' law.

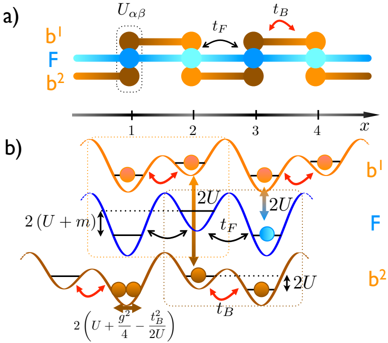
M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, P. Zoller, Phys. Rev. Lett. 102 (2009) 170502.
H. Weimer, M. Müller, I. Lesanovsky, P. Zoller, H. P. Büchler, Nat. Phys. 6 (2010) 382.
L. Tagliacozzo, A. Celi, P. Orland, M. Lewenstein, Nature Communications 4 (2013) 2615.
L. Tagliacozzo, A. Celi, A. Zamora, M. Lewenstein, Ann. Phys. 330 (2013) 160.

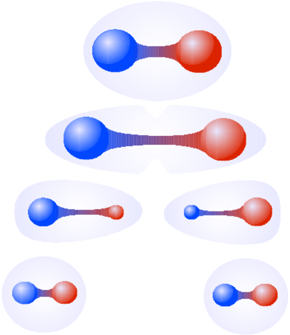
Hamiltonian for staggered fermions and $U(1)$ quantum links

$$H = -t \sum_x \left[\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \text{h.c.} \right] + m \sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} \sum_x E_{x,x+1}^2$$

$$U_{x,x+1} = b_x b_{x+1}^\dagger, \; E_{x,x+1} = \frac{1}{2} \left(b_{x+1}^\dagger b_{x+1} - b_x^\dagger b_x \right)$$

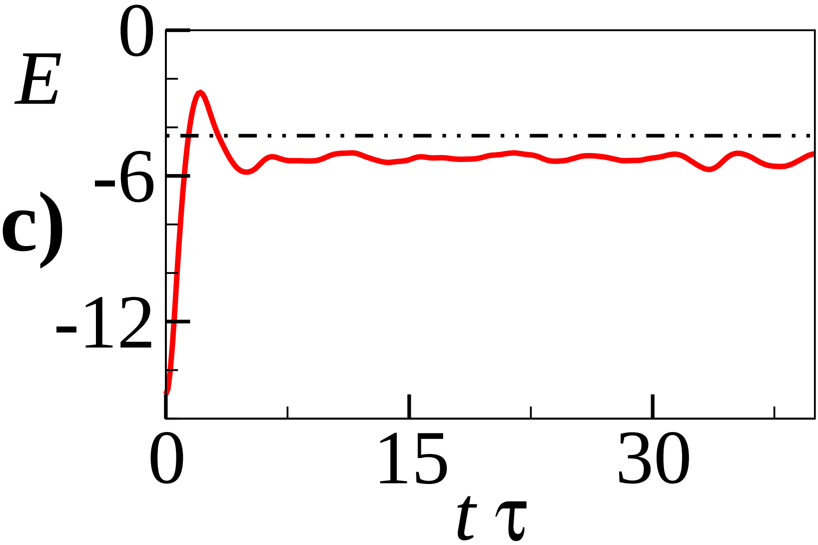
Optical lattice with Bose-Fermi mixture of ultra-cold atoms





D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 109 (2012) 175302.

Quantum simulation of the real-time evolution of string breaking



Outline

$U(N)$ quantum link operators

$$U^{ij} = S_1^{ij} + iS_2^{ij},\ U^{ij\dagger} = S_1^{ij} - iS_2^{ij},\ i,j \in \{1,2,\dots,N\},\ [U^{ij},(U^\dagger)^{kl}] \neq 0$$

$SU(N)_L \times SU(N)_R$ gauge transformations of a quantum link

$$[L^a, L^b] = if_{abc}L^c,\ [R^a, R^b] = if_{abc}R^c,\ a,b,c \in \{1,2,\dots,N^2-1\}$$

$$[L^a, R^b] = [L^a, E] = [R^a, E] = 0$$

Infinitesimal gauge transformations of a quantum link

$$[L^a, U] = -\lambda^a U,\ [R^a, U] = U\lambda^a,\ [E, U] = U$$

Algebraic structures of $U(N)$ quantum link models

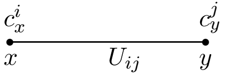
$U^{ij},\ L^a,\ R^a, E,\ 2N^2 + 2(N^2-1) + 1 = 4N^2 - 1$ $SU(2N)$ generators

R. Brower, S. Chandrasekharan, UJW, Phys. Rev. D60 (1999) 094502

Fermionic rishons at the two ends of a link

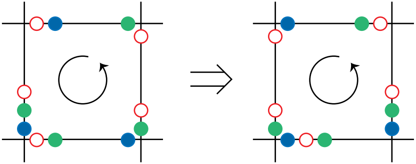
$$\{c_x^i, c_y^{j\dagger}\} = \delta_{xy}\delta_{ij},\ \{c_x^i, c_y^j\} = \{c_x^{i\dagger}, c_y^{j\dagger}\} = 0$$

Rishon representation of link algebra

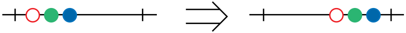


$$U_{xy}^{ij} = c_x^i c_y^{j\dagger},\ L_{xy}^a = c_x^{i\dagger}\lambda_{ij}^a c_x^j,\ R_{xy}^a = c_y^{i\dagger}\lambda_{ij}^a c_y^j,\ E_{xy} = \frac{1}{2}(c_y^{i\dagger}c_y^i - c_x^{i\dagger}c_x^i)$$

Can a "rishon abacus" implemented with ultra-cold atoms be used as a quantum simulator?

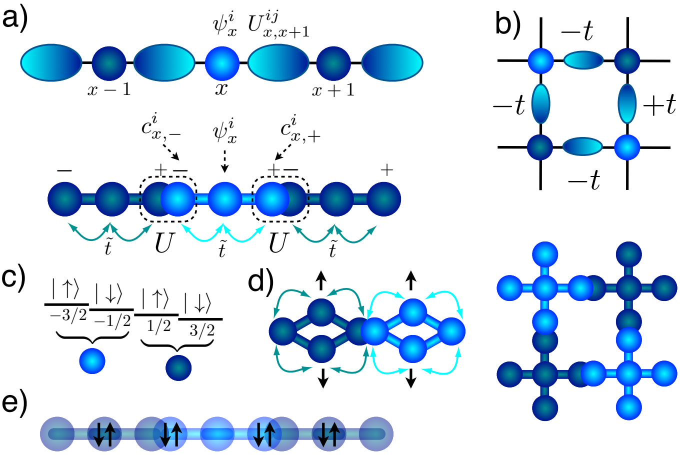


Tr Up



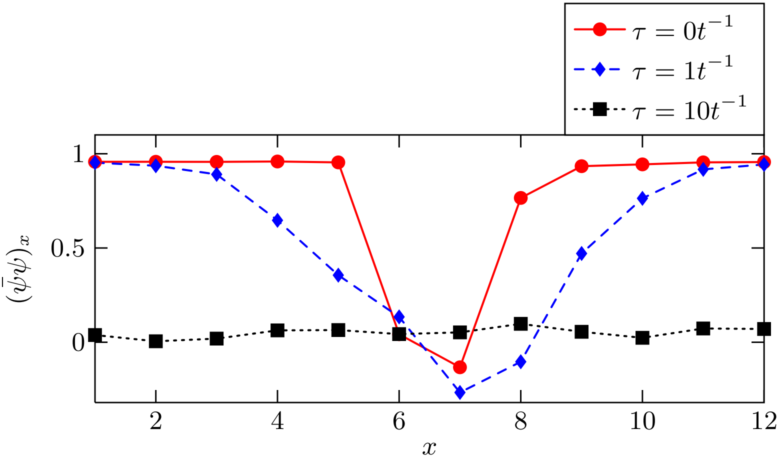
det $U_{x,\mu}$

Optical lattice with ultra-cold alkaline-earth atoms (^{87}Sr or ^{173}Yb) with color encoded in nuclear spin

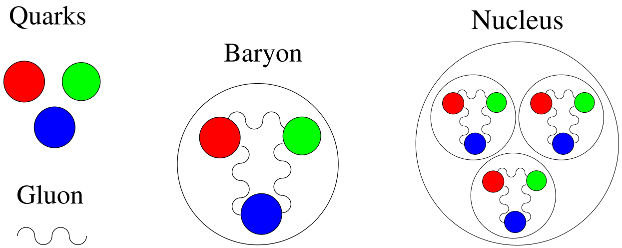


D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 110 (2013) 125303

Expansion of a “fireball” mimicking a hot quark-gluon plasma



Nuclear Physics from $SU(3)$ QCD



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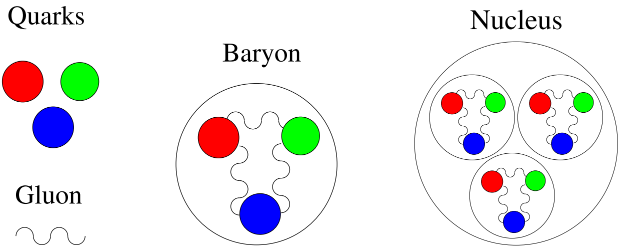
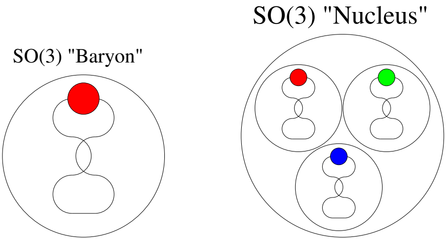


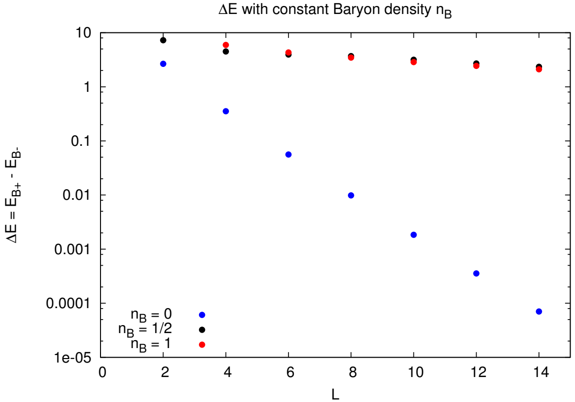
"Nuclear Physics" in an $SO(3)$ lattice gauge theory?



1-d $SO(3)$ quantum link model with adjoint triplet-fermions

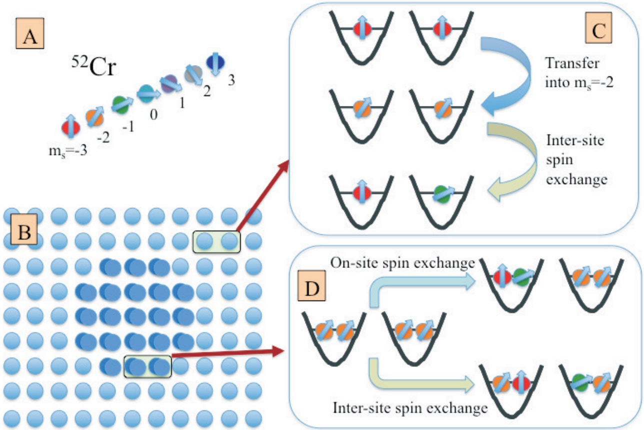
$$H = -t \sum_x \left[\psi_x^{i\dagger} O_{x,x+1}^{ij} \psi_{x+1}^j + \text{h.c.} \right] + m \sum_x (-1)^x \psi_x^{i\dagger} \psi_x^i$$

Restoration of chiral symmetry at baryon density $n_B \geq \frac{1}{2}$



M. Dalmonte, E. Rico, D. Banerjee, M. Bögli, P. Stebler, UJW, P. Zoller, in preparation

Implementation with magnetic atoms (e.g. Cr), whose dipolar interactions allow spin-spin interactions without superexchange



A. de Paz, A. Sharma, A. Chotia, E. Marechal, J. H. Huckans, P. Pedri, L. Santos, O. Gorceix, L. Vernac, and B. Laburthe-Tolra, Phys. Rev. Lett. 111 (2013) 185305.

Analog quantum simulator proposals

H. P. Büchler, M. Hermele, S. D. Huber, M. P. A. Fisher, P. Zoller, Phys. Rev. Lett. 95 (2005) 040402.
E. Zohar, B. Reznik, Phys. Rev. Lett. 107 (2011) 275301.
E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 109 (2012) 125302; Phys. Rev. Lett. 110 (2013) 055302; Phys. Rev. Lett. 110 (2013) 125304.
D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 109 (2012) 175302.
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Digital quantum simulator proposals

M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, P. Zoller, Phys. Rev. Lett. 102 (2009) 170502; Nat. Phys. 6 (2010) 382.
L. Tagliacozzo, A. Celi, P. Orland, M. Lewenstein, Nature Communications 4 (2013) 2615.
L. Tagliacozzo, A. Celi, A. Zamora, M. Lewenstein, Ann. Phys. 330 (2013) 160.

Review on quantum simulators for lattice gauge theories

UJW, Annalen der Physik 525 (2013) 777, arXiv:1305.1602.

Outline

Conclusions

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• The path towards quantum simulation of QCD will be a long one.

However, with a lot of interesting physics along the way.