Recent Progress and Open Challenges in the Theoretical Description of Hot QCD Matter

A View from the Lattice

Marco Panero

Instituto de Física Teórica, Universidad Autónoma de Madrid & CSIC

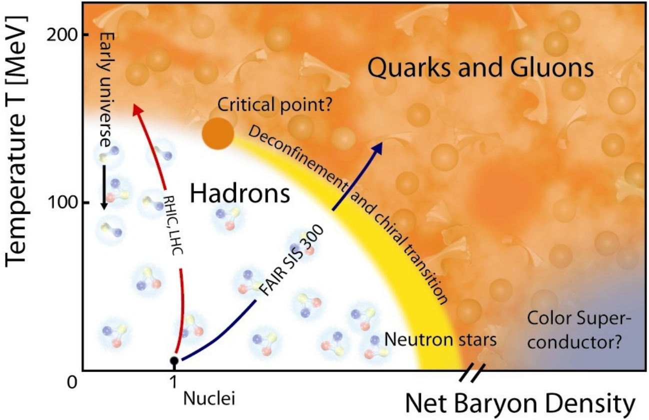
Torino, 19 September 2014

Outline

Outline

What?

Strong nuclear interactions under extreme conditions



[N. Cabibbo and G. Parisi, 1975], **[J. C. Collins and M. J. Perry, 1975]**

Why?

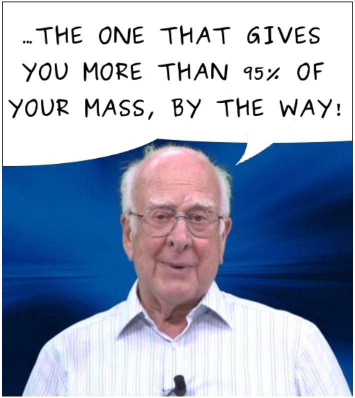
Adapted from **[Z. Weiner, 2010]**

Why?

- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim 200$ MeV realized in nature until about 10^{-6} s after the Big Bang; cooling rate of early Universe depends on QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, ... **[E. Shuryak, 2009]**
- Very rich physics, involving several non-trivial theoretical problems
- The focus of a *large*, *successful* and *long-lasting* experimental programme (BNL, LHC, GSI, JINR) through heavy-ion collisions

Why?

- An important test of QCD, one of the building blocks of the Standard Model



- Temperatures $\gtrsim 200$ MeV realized in nature until about 10^{-6} s after the Big Bang; cooling rate of early Universe depends on QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars

Why?

- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim 200$ MeV realized in nature until about 10^{-6} s after the Big Bang; cooling rate of early Universe depends on QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, . . . **[E. Shuryak, 2009]**
- Very rich physics, involving several non-trivial theoretical problems
- The focus of a *large*, *successful* and *long-lasting* experimental programme (BNL, LHC, GSI, JINR) through heavy-ion collisions

Why?

- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim 200$ MeV realized in nature until about 10^{-6} s after the Big Bang; cooling rate of early Universe depends on QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, . . . **[E. Shuryak, 2009]**
- Very rich physics, involving several non-trivial theoretical problems
- The focus of a *large*, *successful* and *long-lasting* experimental programme (BNL, LHC, GSI, JINR) through heavy-ion collisions

Why?

- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim 200$ MeV realized in nature until about 10^{-6} s after the Big Bang; cooling rate of early Universe depends on QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, . . . **[E. Shuryak, 2009]**
- Very rich physics, involving several non-trivial theoretical problems
- The focus of a *large*, *successful* and *long-lasting* experimental programme (BNL, LHC, GSI, JINR) through heavy-ion collisions

Why?

- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim 200$ MeV realized in nature until about 10^{-6} s after the Big Bang; cooling rate of early Universe depends on QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
  - Simultaneously relativistic and strongly coupled
  - A liquid that cools into a gas
  - "The most perfect" liquid—shear viscosity close to its quantum limit **[G. Policastro, D. T. Son and A. O. Starinets, 2001]**
  - Fast thermalization—close to limit set by causality
  - It creates its own vacuum state to exist
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, ... **[E. Shuryak, 2009]**
- Very rich physics, involving several non-trivial theoretical problems
- The focus of a *large*, *successful* and *long-lasting* experimental programme (BNL, LHC, GSI, JINR) through heavy-ion collisions

Why?

- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim 200$ MeV realized in nature until about 10^{-6} s after the Big Bang; cooling rate of early Universe depends on QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
  - Simultaneously relativistic and strongly coupled
  - A liquid that cools into a gas
  - "The most perfect" liquid—shear viscosity close to its quantum limit **[G. Policastro, D. T. Son and A. O. Starinets, 2001]**
  - Fast thermalization—close to limit set by causality
  - It creates its own vacuum state to exist
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, . . . **[E. Shuryak, 2009]**
- Very rich physics, involving several non-trivial theoretical problems
- The focus of a *large*, *successful* and *long-lasting* experimental programme (BNL, LHC, GSI, JINR) through heavy-ion collisions

Why?

- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim 200$ MeV realized in nature until about 10^{-6} s after the Big Bang; cooling rate of early Universe depends on QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
  - Simultaneously relativistic and strongly coupled
  - A liquid that cools into a gas
  - "The most perfect" liquid—shear viscosity close to its quantum limit **[G. Policastro, D. T. Son and A. O. Starinets, 2001]**
  - Fast thermalization—close to limit set by causality
  - It creates its own vacuum state to exist
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, ... **[E. Shuryak, 2009]**
- Very rich physics, involving several non-trivial theoretical problems
- The focus of a *large*, *successful* and *long-lasting* experimental programme (BNL, LHC, GSI, JINR) through heavy-ion collisions

Why?

- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim 200$ MeV realized in nature until about 10^{-6} s after the Big Bang; cooling rate of early Universe depends on QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
  - Simultaneously relativistic and strongly coupled
  - A liquid that cools into a gas
  - "The most perfect" liquid—shear viscosity close to its quantum limit **[G. Policastro, D. T. Son and A. O. Starinets, 2001]**
  - Fast thermalization—close to limit set by causality
  - It creates its own vacuum state to exist
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, . . . **[E. Shuryak, 2009]**
- Very rich physics, involving several non-trivial theoretical problems
- The focus of a *large*, *successful* and *long-lasting* experimental programme (BNL, LHC, GSI, JINR) through heavy-ion collisions

Why?

- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim$ 200 MeV realized in nature until about 10^{-6} s after the Big Bang; cooling rate of early Universe depends on QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
  - Simultaneously relativistic and strongly coupled
  - A liquid that cools into a gas
  - "The most perfect" liquid—shear viscosity close to its quantum limit **[G. Policastro, D. T. Son and A. O. Starinets, 2001]**
  - Fast thermalization—close to limit set by causality
  - It creates its own vacuum state to exist
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, . . . **[E. Shuryak, 2009]**
- Very rich physics, involving several non-trivial theoretical problems
- The focus of a *large*, *successful* and *long-lasting* experimental programme (BNL, LHC, GSI, JINR) through heavy-ion collisions

Why?

- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim 200$ MeV realized in nature until about 10^{-6} s after the Big Bang; cooling rate of early Universe depends on QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, . . . **[E. Shuryak, 2009]**
- Very rich physics, involving several non-trivial theoretical problems
- The focus of a *large*, *successful* and *long-lasting* experimental programme (BNL, LHC, GSI, JINR) through heavy-ion collisions

Why?

- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim 200$ MeV realized in nature until about 10^{-6} s after the Big Bang; cooling rate of early Universe depends on QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, . . . **[E. Shuryak, 2009]**
- Very rich physics, involving several non-trivial theoretical problems
- The focus of a *large*, *successful* and *long-lasting* experimental programme (BNL, LHC, GSI, JINR) through heavy-ion collisions

Why?

- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim 200$ MeV realized in nature until about 10^{-6} s after the Big Bang; cooling rate of early Universe depends on QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, ... **[E. Shuryak, 2009]**
- Very rich physics, involving several non-trivial theoretical problems ⇒ Calls for a combination of different tools: weak-coupling, lattice, holography, effective theories, phenomenological models, ...
- The focus of a *large*, *successful* and *long-lasting* experimental programme (BNL, LHC, GSI, JINR) through heavy-ion collisions

Why?

- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim$ 200 MeV realized in nature until about 10^{-6} s after the Big Bang; cooling rate of early Universe depends on QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, ... **[E. Shuryak, 2009]**
- Very rich physics, involving several non-trivial theoretical problems ⇒ Calls for a combination of different tools: weak-coupling, lattice, holography, effective theories, phenomenological models, ...—*Inspiring*
- The focus of a *large*, *successful* and *long-lasting* experimental programme (BNL, LHC, GSI, JINR) through heavy-ion collisions

Why?

- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim$ 200 MeV realized in nature until about 10^{-6} s after the Big Bang; cooling rate of early Universe depends on QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, ... **[E. Shuryak, 2009]**
- Very rich physics, involving several non-trivial theoretical problems ⇒ Calls for a combination of different tools: weak-coupling, lattice, holography, effective theories, phenomenological models, ...—*Inspiring*
- The focus of a *large*, *successful* and *long-lasting* experimental programme (BNL, LHC, GSI, JINR) through heavy-ion collisions

Why?

- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim 200$ MeV realized in nature until about 10^{-6} s after the Big Bang; cooling rate of early Universe depends on QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, . . . **[E. Shuryak, 2009]**
- Very rich physics, involving several non-trivial theoretical problems ⇒ Calls for a combination of different tools: weak-coupling, lattice, holography, effective theories, phenomenological models, . . . —*Inspiring*
- The focus of a *large*, *successful* and *long-lasting* experimental programme (BNL, LHC, GSI, JINR) through heavy-ion collisions: The quark-gluon plasma is here to stay

How?

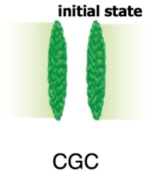
A minimal summary of experimental QGP production — The stages

1. Heavy nuclei (Au, Pb) accelerated to ultra-relativistic energies; initial, "cold nuclear matter" conditions modelled as a color-glass-condensate (CGC)
2. Collision (central/peripheral); formation of a "glasma"
3. Thermalization: the QGP is formed
4. Hydrodynamic expansion governed by EoS and transport coefficients
5. Hadronization
6. Freeze-out; flight to detectors

How?

A minimal summary of experimental QGP production — The stages

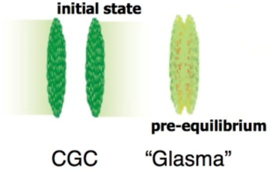
1. Heavy nuclei (Au, Pb) accelerated to ultra-relativistic energies; initial, "cold nuclear matter" conditions modelled as a color-glass-condensate (CGC)
2. Collision (central/peripheral); formation of a "glasma"
3. Thermalization: the QGP is formed
4. Hydrodynamic expansion governed by EoS and transport coefficients
5. Hadronization
6. Freeze-out; flight to detectors



How?

A minimal summary of experimental QGP production — The stages

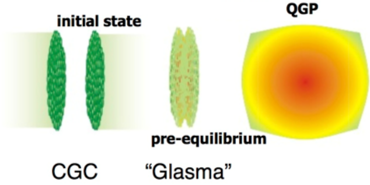
1. Heavy nuclei (Au, Pb) accelerated to ultra-relativistic energies; initial, “cold nuclear matter” conditions modelled as a color-glass-condensate (CGC)
2. Collision (central/peripheral); formation of a “glasma”
3. Thermalization: the QGP is formed
4. Hydrodynamic expansion governed by EoS and transport coefficients
5. Hadronization
6. Freeze-out; flight to detectors



How?

A minimal summary of experimental QGP production — The stages

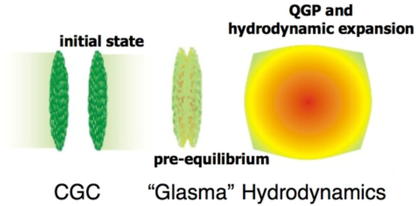
1. Heavy nuclei (Au, Pb) accelerated to ultra-relativistic energies; initial, "cold nuclear matter" conditions modelled as a color-glass-condensate (CGC)
2. Collision (central/peripheral); formation of a "glasma"
3. Thermalization: the QGP is formed
4. Hydrodynamic expansion governed by EoS and transport coefficients
5. Hadronization
6. Freeze-out; flight to detectors



How?

A minimal summary of experimental QGP production — The stages

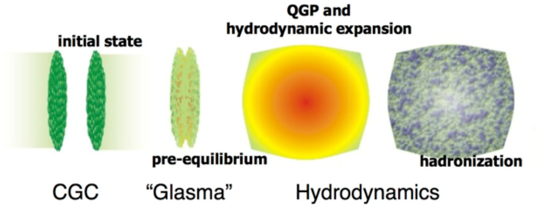
1. Heavy nuclei (Au, Pb) accelerated to ultra-relativistic energies; initial, "cold nuclear matter" conditions modelled as a color-glass-condensate (CGC)
2. Collision (central/peripheral); formation of a "glasma"
3. Thermalization: the QGP is formed
4. Hydrodynamic expansion governed by EoS and transport coefficients
5. Hadronization
6. Freeze-out; flight to detectors



How?

A minimal summary of experimental QGP production — The stages

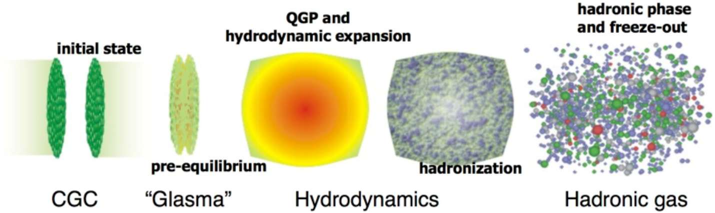
1. Heavy nuclei (Au, Pb) accelerated to ultra-relativistic energies; initial, “cold nuclear matter” conditions modelled as a color-glass-condensate (CGC)
2. Collision (central/peripheral); formation of a “glasma”
3. Thermalization: the QGP is formed
4. Hydrodynamic expansion governed by EoS and transport coefficients
5. Hadronization
6. Freeze-out; flight to detectors



How?

A minimal summary of experimental QGP production — The stages

1. Heavy nuclei (Au, Pb) accelerated to ultra-relativistic energies; initial, "cold nuclear matter" conditions modelled as a color-glass-condensate (CGC)
2. Collision (central/peripheral); formation of a "glasma"
3. Thermalization: the QGP is formed
4. Hydrodynamic expansion governed by EoS and transport coefficients
5. Hadronization
6. Freeze-out; flight to detectors



How?

A minimal summary of experimental QGP production — Observations and challenges for theorists

- Elliptic flow **[ALICE Collaboration, 2010]**
- Photon and dilepton spectra **[PHENIX Collaboration, 2010]**
- Quarkonium melting **[CMS Collaboration, 2012]**
- Strangeness enhancement **[STAR Collaboration, 2009]**
- Jet quenching **[CMS Collaboration, 2012]**

How?

A minimal summary of experimental QGP production — Observations and challenges for theorists

- Elliptic flow **[ALICE Collaboration, 2010]**
- Photon and dilepton spectra **[PHENIX Collaboration, 2010]**
- Quarkonium melting **[CMS Collaboration, 2012]**
- Strangeness enhancement **[STAR Collaboration, 2009]**
- Jet quenching **[CMS Collaboration, 2012]**

How?

A minimal summary of experimental QGP production — Observations and challenges for theorists

- Elliptic flow **[ALICE Collaboration, 2010]**
- Photon and dilepton spectra **[PHENIX Collaboration, 2010]**
- Quarkonium melting **[CMS Collaboration, 2012]**
- Strangeness enhancement **[STAR Collaboration, 2009]**
- Jet quenching **[CMS Collaboration, 2012]**

How?

A minimal summary of experimental QGP production — Observations and challenges for theorists

- Elliptic flow **[ALICE Collaboration, 2010]**
- Photon and dilepton spectra **[PHENIX Collaboration, 2010]**
- Quarkonium melting **[CMS Collaboration, 2012]**
- Strangeness enhancement **[STAR Collaboration, 2009]**
- Jet quenching **[CMS Collaboration, 2012]**

How?

A minimal summary of experimental QGP production — Observations and challenges for theorists

- Elliptic flow **[ALICE Collaboration, 2010]**
- Photon and dilepton spectra **[PHENIX Collaboration, 2010]**
- Quarkonium melting **[CMS Collaboration, 2012]**
- Strangeness enhancement **[STAR Collaboration, 2009]**
- Jet quenching **[CMS Collaboration, 2012]**

How?

A minimal summary of experimental QGP production — Observations and challenges for theorists

- Elliptic flow **[ALICE Collaboration, 2010]**
- Photon and dilepton spectra **[PHENIX Collaboration, 2010]**
- Quarkonium melting **[CMS Collaboration, 2012]**
- Strangeness enhancement **[STAR Collaboration, 2009]**
- Jet quenching **[CMS Collaboration, 2012]**

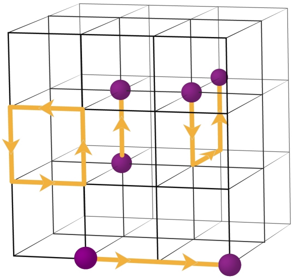
Outline

Basic ideas

Regularize the QCD path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**

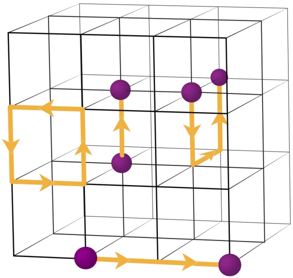
Basic ideas

Regularize the QCD path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**



Basic ideas

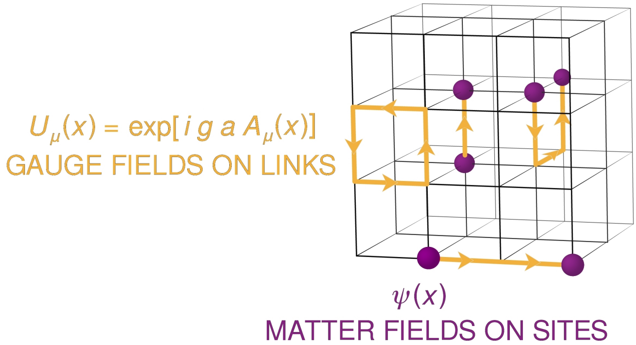
Regularize the QCD path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**



$\psi(x)$
MATTER FIELDS ON SITES

Basic ideas

Regularize the QCD path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**



Basic ideas

Regularize the QCD path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**

Define gauge and matter fields on lattice elements, build a gauge-invariant lattice action and observables

$$S = -\frac{1}{g^2}\sum_{\square}\mathrm{Tr}(U_\square + U_\square^\dagger) + \sum_{x,y,f} a^4 \bar{\psi}_f(x) M^f_{x,y} \psi_f(y)$$

$$M^f_{x,y} = m\delta_{x,y} - \frac{1}{2a}\sum_{\mu}\left[(r-\gamma_\mu)U_\mu(x)\delta_{x+a\hat{\mu},y} + (r+\gamma_\mu)U^\dagger_\mu(y)\delta_{x-a\hat{\mu},y}\right]$$

Basic ideas

Regularize the QCD path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**

Define gauge and matter fields on lattice elements, build a gauge-invariant lattice action and observables

Continuum QCD action recovered for $a \to 0$

Basic ideas

Regularize the QCD path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**

Define gauge and matter fields on lattice elements, build a gauge-invariant lattice action and observables

Continuum QCD action recovered for $a \to 0$

A gauge-invariant, non-perturbative regularization

Basic ideas

Regularize the QCD path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**

Define gauge and matter fields on lattice elements, build a gauge-invariant lattice action and observables

Continuum QCD action recovered for $a \to 0$

A gauge-invariant, non-perturbative regularization

Suitable for numerical simulation: Sample configuration space according to a *statistical weight* proportional to $\exp(-S)$, compute expectation values

$$\langle \mathcal{O} \rangle = \frac{\int \prod \mathrm{d}\psi(x)\mathrm{d}\bar{\psi}(x) \prod \mathrm{d}U_\mu(x) \mathcal{O} \exp(-S)}{\int \prod \mathrm{d}\psi(x)\mathrm{d}\bar{\psi}(x) \prod \mathrm{d}U_\mu(x) \exp(-S)}$$

Basic ideas

Regularize the QCD path integrals by discretizing the theory on a Euclidean lattice of spacing a **[K. G. Wilson, 1974]**

Define gauge and matter fields on lattice elements, build a gauge-invariant lattice action and observables

Continuum QCD action recovered for $a \to 0$

A gauge-invariant, non-perturbative regularization

Suitable for numerical simulation: Sample configuration space according to a *statistical weight* proportional to $\exp(-S)$, compute expectation values

Note: Importance sampling made possible by *real positive* statistical weight

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"
- "The results depend on the *details* of your discretization"
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"
- "It is *dumb*: you only get *numbers*, you don't *learn* anything"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"
- "The results depend on the *details* of your discretization"
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"
- "It is *dumb*: you only get *numbers*, you don't *learn* anything"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"

- "The results depend on the *details* of your discretization"
- "You can never recover the correct rotational and translational *symmetries* of the original

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
  - In the physical, large-volume and continuum limits, it is *the* mathematically rigorous non-perturbative definition of QCD
- "The results depend on the *details* of your discretization"
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"
- "It is *dumb*: you only get *numbers*, you don't *learn* anything"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"
- "It is *dumb*: you only get *numbers*, you don't *learn* anything"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
  - The *intermediate* results do depend on the discretization details, those extrapolated to the *continuum limit* do not
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"
- "It is *dumb*: you only get *numbers*, you don't *learn* anything"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"
- "It is *dumb*: you only get *numbers*, you don't *learn* anything"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"— *False*
  - When the lattice spacing $a \to 0$ the continuum theory, with its full symmetries, emerges as *a good low-energy effective theory* of the lattice theory
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"
- "It is *dumb*: you only get *numbers*, you don't *learn* anything"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"— *False*
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"
- "It is *dumb*: you only get *numbers*, you don't *learn* anything"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"— *False*
- "You always have undesired additional quark species (*doublers*)"— *False*
  - They are easily removed e.g. by adding a Wilson term (or in more sophisticated ways)
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"
- "It is *dumb*: you only get *numbers*, you don't *learn* anything"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"— *False*
- "You always have undesired additional quark species (*doublers*)"— *False*
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"
- "It is *dumb*: you only get *numbers*, you don't *learn* anything"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"— *False*
- "You always have undesired additional quark species (*doublers*)"— *False*
- "It only works / it is only defined at *strong* coupling"— *False*
    - It is defined at any value of the coupling; the continuum limit $a \to 0$ is taken at *weak* coupling
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"
- "It is *dumb*: you only get *numbers*, you don't *learn* anything"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"— *False*
- "You always have undesired additional quark species (*doublers*)"— *False*
- "It only works / it is only defined at *strong* coupling"— *False*
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"
- "It is *dumb*: you only get *numbers*, you don't *learn* anything"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"— *False*
- "You always have undesired additional quark species (*doublers*)"— *False*
- "It only works / it is only defined at *strong* coupling"— *False*
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"— *False*
  - Moore's law and algorithmic progress came to the rescue: for standard lattice QCD computations, quenched calculations are now *obsolete*
- "It is *dumb*: you only get *numbers*, you don't *learn* anything"

Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"— *False*
- "You always have undesired additional quark species (*doublers*)"— *False*
- "It only works / it is only defined at *strong* coupling"— *False*
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"— *False*
- "It is *dumb*: you only get *numbers*, you don't *learn* anything"

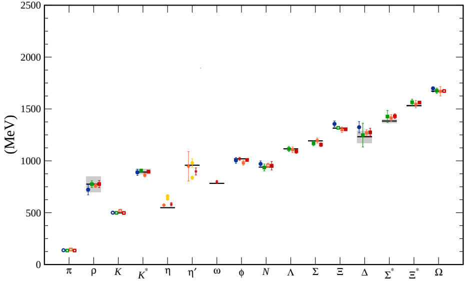
Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"— *False*
- "The results depend on the *details* of your discretization"— *False*
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"— *False*
- "You always have undesired additional quark species (*doublers*)"— *False*
- "It only works / it is only defined at *strong* coupling"— *False*
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"— *False*
- "It is *dumb*: you only get *numbers*, you don't *learn* anything"
  - I'll try to prove you wrong in the rest of this talk

Examples of applications

Some lattice QCD results at zero temperature

- Hadron spectrum



Adapted from **[A. Kronfeld, 2012]**

- π-, K-, D- and B-meson matrix elements
- Light quark masses (combining lattice QCD and chiral extrapolation)
- α_s (combining lattice QCD and perturbation theory)
- $K \to \pi\pi$ transitions
- Nonleptonic B and D decays
- and more

Examples of applications

Some lattice QCD results at zero temperature

- Hadron spectrum
- π-, K-, D- and B-meson matrix elements
- Light quark masses (combining lattice QCD and chiral extrapolation)
- α_s (combining lattice QCD and perturbation theory)
- $K \to \pi\pi$ transitions
- Nonleptonic B and D decays
- and more ...

Examples of applications

Some lattice QCD results at zero temperature

- Hadron spectrum
- π-, K-, D- and B-meson matrix elements
- Light quark masses (combining lattice QCD and chiral extrapolation)
- α_s (combining lattice QCD and perturbation theory)
- $K \to \pi\pi$ transitions
- Nonleptonic B and D decays
- and more ...

Examples of applications

Some lattice QCD results at zero temperature

- Hadron spectrum
- π-, K-, D- and B-meson matrix elements
- Light quark masses (combining lattice QCD and chiral extrapolation)
- α_s (combining lattice QCD and perturbation theory)
- $K \to \pi\pi$ transitions
- Nonleptonic B and D decays
- and more ...

Examples of applications

Some lattice QCD results at zero temperature

- Hadron spectrum
- π-, K-, D- and B-meson matrix elements
- Light quark masses (combining lattice QCD and chiral extrapolation)
- α_s (combining lattice QCD and perturbation theory)
- $K \to \pi\pi$ transitions
- Nonleptonic B and D decays
- and more ...

Examples of applications

Some lattice QCD results at zero temperature

- Hadron spectrum
- π-, K-, D- and B-meson matrix elements
- Light quark masses (combining lattice QCD and chiral extrapolation)
- α_s (combining lattice QCD and perturbation theory)
- $K \to \pi\pi$ transitions
- Nonleptonic B and D decays
- and more ...

Examples of applications

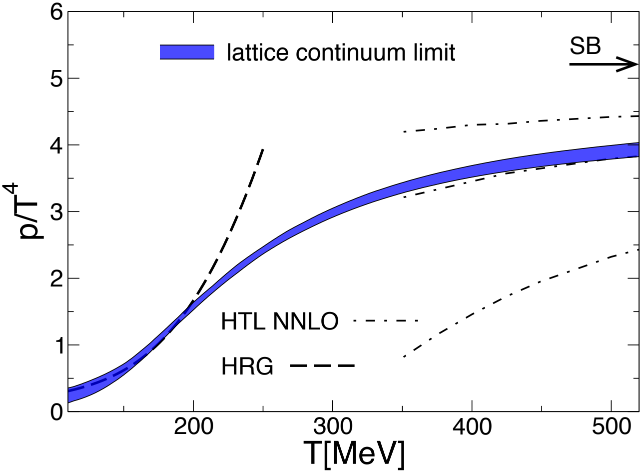
Some lattice QCD results at zero temperature

- Hadron spectrum
- π-, K-, D- and B-meson matrix elements
- Light quark masses (combining lattice QCD and chiral extrapolation)
- α_s (combining lattice QCD and perturbation theory)
- $K \to \pi\pi$ transitions
- Nonleptonic B and D decays
- and more . . .

Outline

Thermodynamics

- Equation of state in QCD



[S. Borsányi et al., 2013]

See also **[A. Bazavov et al., 2012]** and **[T. Bhattacharya et al., 2014]**

Thermodynamics

- Equation of state in QCD and in QCD-like theories: the large-N limit

Thermodynamics

- Equation of state in QCD and in QCD-like theories: the large-N limit

QCD at large N: Why bother?

The large-N limit of QCD (at fixed $\lambda = g^2 N$) has interesting implications **[G. 't Hooft, 1974]**

It plays a crucial rôle in the holographic gauge/string duality **[J. Maldacena, 1998]**

Important applications at finite temperature **[J. Casalderrey-Solana et al., 2014]**

What can the lattice say? **[M. P., 2012]**, **[B. Lucini and M. P., 2013]**

Thermodynamics

- Equation of state in QCD and in QCD-like theories: the large-N limit

QCD at large N: Why bother?

The large-N limit of QCD (at fixed $\lambda = g^2 N$) has interesting implications **[G. 't Hooft, 1974]**

It plays a crucial rôle in the holographic gauge/string duality **[J. Maldacena, 1998]**

Important applications at finite temperature **[J. Casalderrey-Solana et al., 2014]**

What can the lattice say? **[M. P., 2012]**, **[B. Lucini and M. P., 2013]**

Thermodynamics

- Equation of state in QCD and in QCD-like theories: the large-N limit

QCD at large N: Why bother?

The large-N limit of QCD (at fixed $\lambda = g^2 N$) has interesting implications **[G. 't Hooft, 1974]**

It plays a crucial rôle in the holographic gauge/string duality **[J. Maldacena, 1998]**

$$\lambda = \frac{R^4}{l_s^4} \qquad \frac{\lambda}{N} = 4\pi g_s$$

Important applications at finite temperature **[J. Casalderrey-Solana et al., 2014]**

What can the lattice say? **[M. P., 2012], [B. Lucini and M. P., 2013]**

Thermodynamics

- Equation of state in QCD and in QCD-like theories: the large-N limit

QCD at large N: Why bother?

The large-N limit of QCD (at fixed $\lambda = g^2 N$) has interesting implications **[G. 't Hooft, 1974]**

It plays a crucial rôle in the holographic gauge/string duality **[J. Maldacena, 1998]**: *classical* string limit

Important applications at finite temperature **[J. Casalderrey-Solana et al., 2014]**

What can the lattice say? **[M. P., 2012]**, **[B. Lucini and M. P., 2013]**

Thermodynamics

- Equation of state in QCD and in QCD-like theories: the large-N limit

QCD at large N: Why bother?

The large-N limit of QCD (at fixed $\lambda = g^2 N$) has interesting implications **[G. 't Hooft, 1974]**

It plays a crucial rôle in the holographic gauge/string duality **[J. Maldacena, 1998]**: *classical* string limit

Important applications at finite temperature **[J. Casalderrey-Solana et al., 2014]**

What can the lattice say? **[M. P., 2012]**, **[B. Lucini and M. P., 2013]**

Thermodynamics

- Equation of state in QCD and in QCD-like theories: the large-N limit

QCD at large N: Why bother?

The large-N limit of QCD (at fixed $\lambda = g^2 N$) has interesting implications **[G. 't Hooft, 1974]**

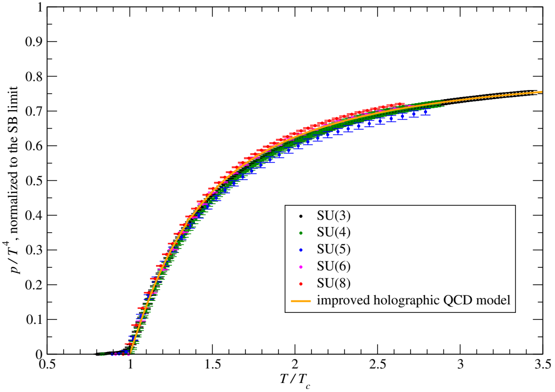
It plays a crucial rôle in the holographic gauge/string duality **[J. Maldacena, 1998]**: *classical* string limit

Important applications at finite temperature **[J. Casalderrey-Solana et al., 2014]**

What can the lattice say? **[M. P., 2012]**, **[B. Lucini and M. P., 2013]**

Thermodynamics

- Equation of state in QCD and in QCD-like theories: the large-N limit



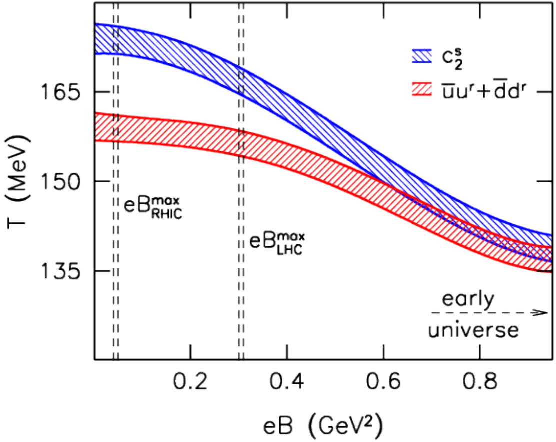
[M. P., 2009]

See also:

- Polyakov loops **[A. Mykkänen, K. Rummukainen and M. P., 2012]** (relevant for phenomenological models **[C. Ratti, M. A. Thaler and W. Wiese, 2006]**, **[H. Hansen et al., 2007]**)
- EoS in G_2 gauge theory **[M. Caselle et al., 2014]**
- EoS in 2+1 dimensions **[M. Caselle et al., 2011]**, **[M. Caselle et al., 2012]**

Thermodynamics

- Equation of state in QCD and in QCD-like theories
- Dependence on (electro)magnetic fields: relevant
  for electro-weak phase transition in early Universe / peripheral heavy-ion collisions / magnetars



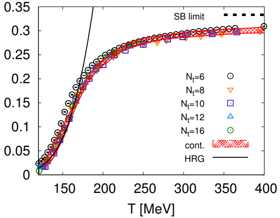
[G. S. Bali et al., 2011]

Thermodynamics

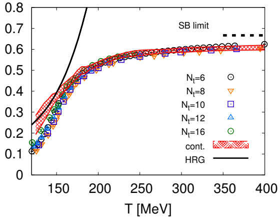
- Equation of state in QCD and in QCD-like theories
- Dependence on (electro)magnetic fields
- Freeze-out conditions from fluctuations of conserved charges (baryon number B, electric charge Q, strangeness S) **[F. Karsch, 2012]**

$$\chi_{ijk}^{BQS} = \frac{1}{VT^3} \frac{\partial^i}{\partial(\mu_B/T)^i} \frac{\partial^j}{\partial(\mu_Q/T)^j} \frac{\partial^k}{\partial(\mu_S/T)^k} \ln Z$$

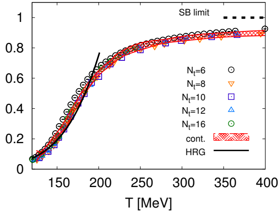
$\chi_2^B \sim \langle(\delta B)^2\rangle$



$\chi_2^Q \sim \langle(\delta Q)^2\rangle$



$\chi_2^S \sim \langle(\delta S)^2\rangle$



[S. Borsányi et al., 2011]

See also **[A. Bazavov et al., 2012]**
Consistent determination of freeze-out conditions: $T_{\rm fr} = 144(10)$ MeV, $\mu_{\rm fr}^B = 102(6)$ MeV at RHIC (STAR, $\sqrt{s} = 39$ GeV) **[S. Borsányi et al., 2014]**
Comparison with hadron resonance gas model **[P. Alba et al., 2014]** and with statistical hadronization model **[A. Andronic, P. Braun-Munzinger and J. Stachel, 2009]**

Quarkonium melting

A QGP "thermometer" **[T. Matsui and H. Satz, 1986]**

Quarkonium melting

A QGP "thermometer" **[T. Matsui and H. Satz, 1986]**

General strategy for the lattice computation:

- Heavy quarks are, well, heavy
- Compute correlation functions of sources with desired quantum numbers; $G_{\mathrm{E}}(\tau) \simeq \int_{-2M}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \exp(-\omega\tau)\rho(\omega)$
- Invert to extract spectral function $\rho(\omega)$

Quarkonium melting

A QGP "thermometer" **[T. Matsui and H. Satz, 1986]**

General strategy for the lattice computation:

- Heavy quarks are, well, heavy
- Compute correlation functions of sources with desired quantum numbers; $G_E(\tau) \simeq \int_{-2M}^{\infty} \frac{d\omega}{2\pi} \exp(-\omega\tau)\rho(\omega)$
- Invert to extract spectral function $\rho(\omega)$

Quarkonium melting

A QGP "thermometer" **[T. Matsui and H. Satz, 1986]**

General strategy for the lattice computation:

- Heavy quarks are, well, heavy $\Rightarrow$ Can be treated non-relativistically (NRQCD)
- Compute correlation functions of sources with desired quantum numbers; $G_E(\tau) \simeq \int_{-2M}^{\infty} \frac{d\omega}{2\pi} \exp(-\omega\tau)\rho(\omega)$
- Invert to extract spectral function $\rho(\omega)$

Quarkonium melting

A QGP "thermometer" **[T. Matsui and H. Satz, 1986]**

General strategy for the lattice computation:

- Heavy quarks are, well, heavy $\Rightarrow$ Can be treated non-relativistically (NRQCD)
- Compute correlation functions of sources with desired quantum numbers; $G_{\mathrm{E}}(\tau) \simeq \int_{-2M}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \exp(-\omega\tau)\rho(\omega)$
- Invert to extract spectral function $\rho(\omega)$

Quarkonium melting

A QGP "thermometer" **[T. Matsui and H. Satz, 1986]**

General strategy for the lattice computation:

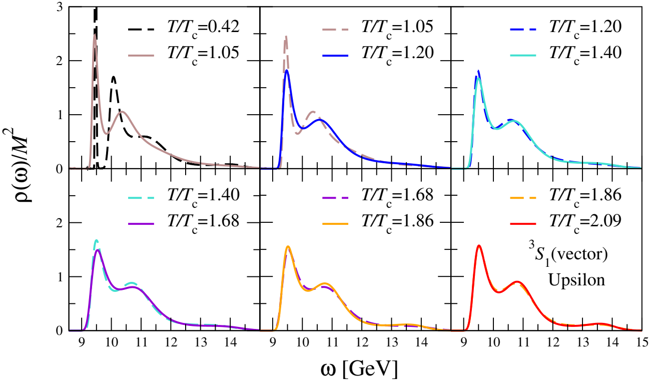
- Heavy quarks are, well, heavy $\Rightarrow$ Can be treated non-relativistically (NRQCD)
- Compute correlation functions of sources with desired quantum numbers; $G_{\mathrm{E}}(\tau) \simeq \int_{-2M}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \exp(-\omega\tau)\rho(\omega)$
- Invert to extract spectral function $\rho(\omega)$

Quarkonium melting

A QGP "thermometer" **[T. Matsui and H. Satz, 1986]**

General strategy for the lattice computation:

- Heavy quarks are, well, heavy ⇒ Can be treated non-relativistically (NRQCD)
- Compute correlation functions of sources with desired quantum numbers; $G_{\mathrm{E}}(\tau) \simeq \int_{-2M}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \exp(-\omega\tau)\rho(\omega)$
- Invert to extract spectral function $\rho(\omega)$



Bottomonium excitation melting **[G. Aarts et al., 2011]**

Transport coefficients

Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges **[H. B. Meyer, 2011]**

Transport coefficients

Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges **[H. B. Meyer, 2011]**

Example: Shear (η) and bulk (ζ) viscosities [P. Romatschke, 2010]

$$T^{\mu\nu} = (\epsilon + p)u^{\mu}u^{\nu} + pg^{\mu\nu} - \mathbb{P}^{\mu i}\mathbb{P}^{\nu j}\left[\eta\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}g_{ij}\partial_k u^k\right) + \zeta g_{ij}\partial_k u^k\right]$$

Transport coefficients

Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges **[H. B. Meyer, 2011]**

Difficult to access on Euclidean lattice $\Rightarrow$ Indirectly reconstructed from Kubo formulae

Transport coefficients

Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges **[H. B. Meyer, 2011]**

Difficult to access on Euclidean lattice ⇒ Indirectly reconstructed from Kubo formulae

Example: Shear viscosity

$$\eta = \pi \lim_{\omega \to 0} \lim_{\mathbf{k} \to 0} \frac{\rho(\omega, \mathbf{k})}{\omega}$$

with ρ the spectral function, related to a suitable (e.g. $T^{\mu\nu}$) Euclidean correlator via

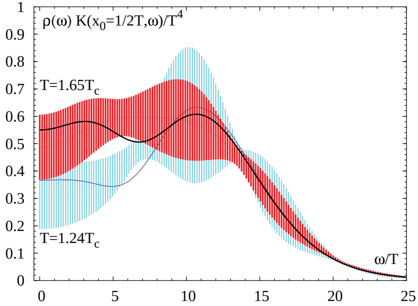
$$G_{\mathrm{E}}(t, \mathbf{k}) = \int_0^\infty \mathrm{d}\omega \, \rho(\omega, \mathbf{k}) \frac{\cosh\left[\omega\left(t - \frac{1}{2T}\right)\right]}{\sinh\left(\frac{\omega}{2T}\right)}$$

Transport coefficients

Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges **[H. B. Meyer, 2011]**

Difficult to access on Euclidean lattice ⇒ Indirectly reconstructed from Kubo formulae

Numerically *very* challenging



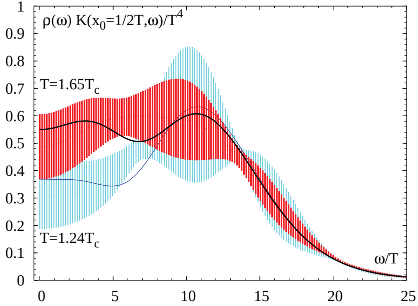
[H. B. Meyer, 2007]

Transport coefficients

Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges **[H. B. Meyer, 2011]**

Difficult to access on Euclidean lattice ⇒ Indirectly reconstructed from Kubo formulae

Numerically *very* challenging



[H. B. Meyer, 2007]

Analytical guidance e.g. from holography? **[G. S. Bali et al., in progress]**

Jet quenching

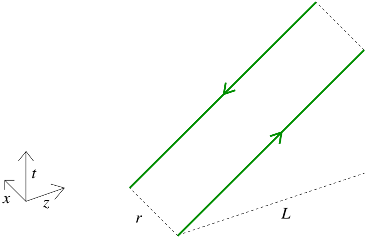
A hard quark moving through the QGP: average momentum broadening described by jet quenching parameter $\hat{q}$

$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{L} = \int \frac{\mathrm{d}^2 p_\perp}{(2\pi)^2} p_\perp^2 \, C(p_\perp)$$

Jet quenching

A hard quark moving through the QGP: average momentum broadening described by jet quenching parameter $\hat{q}$

$C(p_\perp)$, the differential parton-plasma constituents collision rate, related to two-point correlator of *light-cone Wilson lines*

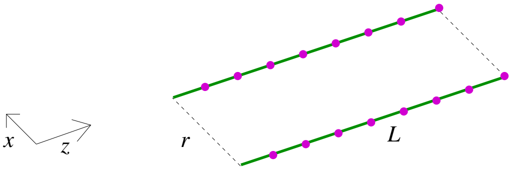


Jet quenching

A hard quark moving through the QGP: average momentum broadening described by jet quenching parameter $\hat{q}$

$C(p_\perp)$, the differential parton-plasma constituents collision rate, related to two-point correlator of *light-cone Wilson lines*

Our strategy **[M. P., K. Rummukainen and A. Schäfer, 2014]**: Compute non-perturbative *soft* contribution to $\hat{q}$ from a *dimensionally reduced effective theory* on the lattice—*exact* for soft modes **[S. Caron-Huot, 2009]**, **[M. Laine, 2012]**



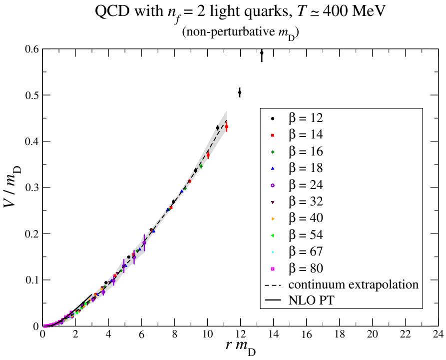
Jet quenching

A hard quark moving through the QGP: average momentum broadening described by jet quenching parameter $\hat{q}$

$C(p_\perp)$, the differential parton-plasma constituents collision rate, related to two-point correlator of *light-cone Wilson lines*

Our strategy **[M. P., K. Rummukainen and A. Schäfer, 2014]**: Compute non-perturbative *soft* contribution to $\hat{q}$ from a *dimensionally reduced effective theory* on the lattice—*exact* for soft modes **[S. Caron-Huot, 2009]**, **[M. Laine, 2012]**

Direct access to collision kernel in coordinate space



Jet quenching

A hard quark moving through the QGP: average momentum broadening described by jet quenching parameter $\hat{q}$

$C(p_\perp)$, the differential parton-plasma constituents collision rate, related to two-point correlator of *light-cone Wilson lines*

Our strategy **[M. P., K. Rummukainen and A. Schäfer, 2014]**: Compute non-perturbative *soft* contribution to $\hat{q}$ from a *dimensionally reduced effective theory* on the lattice—*exact* for soft modes **[S. Caron-Huot, 2009]**, **[M. Laine, 2012]**

Direct access to collision kernel in coordinate space

Evidence for *rather large* non-perturbative effects: $\hat{q} \simeq 6$ GeV^2/fm at RHIC

Jet quenching

A hard quark moving through the QGP: average momentum broadening described by jet quenching parameter $\hat{q}$

$C(p_\perp)$, the differential parton-plasma constituents collision rate, related to two-point correlator of *light-cone Wilson lines*

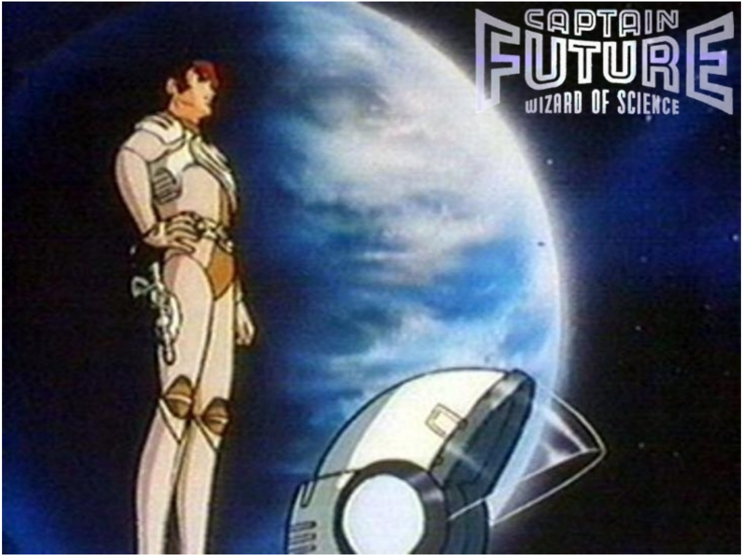
Our strategy **[M. P., K. Rummukainen and A. Schäfer, 2014]**: Compute non-perturbative *soft* contribution to $\hat{q}$ from a *dimensionally reduced effective theory* on the lattice—*exact* for soft modes **[S. Caron-Huot, 2009]**, **[M. Laine, 2012]**

Direct access to collision kernel in coordinate space

Evidence for *rather large* non-perturbative effects: $\hat{q} \simeq 6$ GeV^2/fm at RHIC

Direct relation to non-perturbative contribution to screening masses **[B. Brandt et al., 2014]**

Outline


CAPTAIN
FUTURE
WIZARD OF SCIENCE

Photon production rates

As electromagnetic probes, photon (and dilepton) rates provide important information on early stages of the nuclear collision **[Ghiglieri et al., 2013]**

$$\frac{\mathrm{d}\Gamma_\gamma}{\mathrm{d}^3\mathbf{k}} = -\frac{1}{(2\pi)^3 2|\mathbf{k}|} W^<(k^0 = k)$$

with $W^<(K)$ the photon polarization

$$W^<(K) = \int \mathrm{d}^4 X \exp\left(iK \cdot X\right) \mathrm{Tr} \rho J^\mu(0) J_\mu(X)$$

where J is the electromagnetic current and ρ the density operator

Photon production rates

As electromagnetic probes, photon (and dilepton) rates provide important information on early stages of the nuclear collision **[Ghiglieri et al., 2013]**

$$\frac{\mathrm{d}\Gamma_\gamma}{\mathrm{d}^3\mathbf{k}} = -\frac{1}{(2\pi)^3 2|\mathbf{k}|} W^<(k^0 = k)$$

with $W^<(K)$ the photon polarization

$$W^<(K) = \int \mathrm{d}^4 X \exp\left(iK \cdot X\right) \mathrm{Tr} \rho J^\mu(0) J_\mu(X)$$

where J is the electromagnetic current and ρ the density operator

Like for the $\hat{q}$ computation, soft QCD contributions can be computed in a dimensionally reduced effective theory on the lattice

Real-time $Q\bar{Q}$ potential

The *real-time* static quark-antiquark potential $V(r,t)$ in hot QCD is generically *complex* **[M. Laine et al., 2007]**, **[A. Beraudo, J.-P. Blaizot and C. Ratti, 2008] [N. Brambilla et al., 2008] [T. Hayata, K. Nawa and T. Hatsuda, 2012]**

Real-time $Q\bar{Q}$ potential

The *real-time* static quark-antiquark potential $V(r, t)$ in hot QCD is generically *complex* **[M. Laine et al., 2007]**, **[A. Beraudo, J.-P. Blaizot and C. Ratti, 2008] [N. Brambilla et al., 2008] [T. Hayata, K. Nawa and T. Hatsuda, 2012]**

- Real part: Debye screening
- Imaginary part: Landau damping

Real-time $Q\bar{Q}$ potential

The *real-time* static quark-antiquark potential $V(r, t)$ in hot QCD is generically *complex* **[M. Laine et al., 2007]**, **[A. Beraudo, J.-P. Blaizot and C. Ratti, 2008]** **[N. Brambilla et al., 2008]** **[T. Hayata, K. Nawa and T. Hatsuda, 2012]**

- Real part: Debye screening
- Imaginary part: Landau damping

Real-time $Q\bar{Q}$ potential

The *real-time* static quark-antiquark potential $V(r,t)$ in hot QCD is generically *complex* **[M. Laine et al., 2007]**, **[A. Beraudo, J.-P. Blaizot and C. Ratti, 2008] [N. Brambilla et al., 2008] [T. Hayata, K. Nawa and T. Hatsuda, 2012]**

Outline of lattice strategy:

- Compute Euclidean thermal Wilson loops $W_{\mathrm{E}}(r,\tau)$
- Extract spectral function $\rho(r,\omega)$ by inverting

$$W_{\mathrm{E}}(r,\tau) = \int \mathrm{d}\omega \exp(-\omega\tau)\rho(r,\omega)$$

- Compute the real-time potential via

$$V(r,t) = \frac{\int \mathrm{d}\omega\,\omega\, \exp(-i\omega t)\rho(r,\omega)}{\int \mathrm{d}\omega\, \exp(-i\omega t)\rho(r,\omega)}$$

Real-time $Q\bar{Q}$ potential

The *real-time* static quark-antiquark potential $V(r,t)$ in hot QCD is generically *complex* **[M. Laine et al., 2007]**, **[A. Beraudo, J.-P. Blaizot and C. Ratti, 2008] [N. Brambilla et al., 2008] [T. Hayata, K. Nawa and T. Hatsuda, 2012]**

Outline of lattice strategy:

- Compute Euclidean thermal Wilson loops $W_{\mathrm{E}}(r,\tau)$
- Extract spectral function $\rho(r,\omega)$ by inverting

$$W_{\mathrm{E}}(r,\tau) = \int \mathrm{d}\omega \exp(-\omega\tau)\rho(r,\omega)$$

- Compute the real-time potential via

$$V(r,t) = \frac{\int \mathrm{d}\omega\, \omega \exp(-i\omega t)\rho(r,\omega)}{\int \mathrm{d}\omega \exp(-i\omega t)\rho(r,\omega)}$$

Real-time $Q\bar{Q}$ potential

The *real-time* static quark-antiquark potential $V(r,t)$ in hot QCD is generically *complex* **[M. Laine et al., 2007]**, **[A. Beraudo, J.-P. Blaizot and C. Ratti, 2008] [N. Brambilla et al., 2008] [T. Hayata, K. Nawa and T. Hatsuda, 2012]**

Outline of lattice strategy:

- Compute Euclidean thermal Wilson loops $W_{\mathrm{E}}(r,\tau)$
- Extract spectral function $\rho(r,\omega)$ by inverting

$$W_{\mathrm{E}}(r,\tau) = \int \mathrm{d}\omega \exp(-\omega\tau)\rho(r,\omega)$$

- Compute the real-time potential via

$$V(r,t) = \frac{\int \mathrm{d}\omega\, \omega \, \exp(-i\omega t)\rho(r,\omega)}{\int \mathrm{d}\omega \, \exp(-i\omega t)\rho(r,\omega)}$$

Real-time $Q\bar{Q}$ potential

The *real-time* static quark-antiquark potential $V(r,t)$ in hot QCD is generically *complex* **[M. Laine et al., 2007]**, **[A. Beraudo, J.-P. Blaizot and C. Ratti, 2008] [N. Brambilla et al., 2008] [T. Hayata, K. Nawa and T. Hatsuda, 2012]**

Outline of lattice strategy:

- Compute Euclidean thermal Wilson loops $W_{\mathrm{E}}(r,\tau)$
- Extract spectral function $\rho(r,\omega)$ by inverting

$$W_{\mathrm{E}}(r,\tau) = \int \mathrm{d}\omega \exp(-\omega\tau)\rho(r,\omega)$$

- Compute the real-time potential via

$$V(r,t) = \frac{\int \mathrm{d}\omega\, \omega \, \exp(-i\omega t)\rho(r,\omega)}{\int \mathrm{d}\omega \, \exp(-i\omega t)\rho(r,\omega)}$$

Real-time $Q\bar{Q}$ potential

The *real-time* static quark-antiquark potential $V(r,t)$ in hot QCD is generically *complex* **[M. Laine et al., 2007]**, **[A. Beraudo, J.-P. Blaizot and C. Ratti, 2008] [N. Brambilla et al., 2008] [T. Hayata, K. Nawa and T. Hatsuda, 2012]**

Outline of lattice strategy:

- Compute Euclidean thermal Wilson loops $W_{\mathrm{E}}(r,\tau)$
- Extract spectral function $\rho(r,\omega)$ by inverting

$$W_{\mathrm{E}}(r,\tau) = \int \mathrm{d}\omega \exp(-\omega\tau)\rho(r,\omega)$$

- Compute the real-time potential via

$$V(r,t) = \frac{\int \mathrm{d}\omega\, \omega \exp(-i\omega t)\rho(r,\omega)}{\int \mathrm{d}\omega \exp(-i\omega t)\rho(r,\omega)}$$

Results exist for $\mathrm{SU}(3)$ Yang-Mills **[A. Rothkopf, T. Hatsuda and S. Sasaki, 2012]**, similar studies in full QCD ongoing **[A. Bazavov, Y. Burnier and P. Petreczky, 2014]**

Marching towards lattice QCD at finite density

Sign problem: At finite quark chemical potential μ, fermionic determinant *complex*

Marching towards lattice QCD at finite density

Sign problem: At finite quark chemical potential μ, fermionic determinant *complex* $\Rightarrow$ Goodbye configuration importance sampling

Marching towards lattice QCD at finite density

Sign problem: At finite quark chemical potential μ, fermionic determinant *complex* $\Rightarrow$ Goodbye configuration importance sampling

Some traditional workarounds:

- QCD, but not really μ
- μ, but not really QCD

Marching towards lattice QCD at finite density

Sign problem: At finite quark chemical potential μ, fermionic determinant *complex* $\Rightarrow$ Goodbye configuration importance sampling

Some traditional workarounds:

- QCD, but not really μ: expansions around $\mu = 0$, imaginary chemical potential, isospin chemical potential, . . .
- μ, but not really QCD

Marching towards lattice QCD at finite density

Sign problem: At finite quark chemical potential μ, fermionic determinant *complex* $\Rightarrow$ Goodbye configuration importance sampling

Some traditional workarounds:

- QCD, but not really μ: expansions around $\mu = 0$, imaginary chemical potential, isospin chemical potential, ...
- μ, but not really QCD: $\mathrm{SU}(2)$-QCD, QCD with adjoint quarks, G_2-QCD, ...

Marching towards lattice QCD at finite density

Sign problem: At finite quark chemical potential μ, fermionic determinant *complex* $\Rightarrow$ Goodbye configuration importance sampling

Some traditional workarounds:

- QCD, but not really μ
- μ, but not really QCD

Some potentially promising new routes:

- Dualities and worm algorithms
- Large-N orbifold dualities
- Density of states' method

Marching towards lattice QCD at finite density

Sign problem: At finite quark chemical potential μ, fermionic determinant *complex* $\Rightarrow$ Goodbye configuration importance sampling

Some traditional workarounds:

- QCD, but not really μ
- μ, but not really QCD

Some potentially promising new routes:

- Dualities and worm algorithms—still mostly limited to Abelian models **[Y. Delgado Mercado, C. Gattringer and A. Schmidt, 2013]**, **[S. Chandrasekharan and A. Li, 2012]**; see also **[M. P., 2005]**
- Large-N orbifold dualities
- Density of states' method

Marching towards lattice QCD at finite density

Sign problem: At finite quark chemical potential μ, fermionic determinant *complex* $\Rightarrow$ Goodbye configuration importance sampling

Some traditional workarounds:

- QCD, but not really μ
- μ, but not really QCD

Some potentially promising new routes:

- Dualities and worm algorithms
- Large-N orbifold dualities **[A. Cherman, M. Hanada and D. Robles-Llana, 2011]**; see also **[P. Kovtun, M. Ünsal and L. G. Yaffe, 2005]**
- Density of states' method

Marching towards lattice QCD at finite density

Sign problem: At finite quark chemical potential μ, fermionic determinant *complex* $\Rightarrow$ Goodbye configuration importance sampling

Some traditional workarounds:

- QCD, but not really μ
- μ, but not really QCD

Some potentially promising new routes:

- Dualities and worm algorithms
- Large-N orbifold dualities
- Density of states' method **[K. Langfeld, B. Lucini and A. Rago, 2012]**

Marching towards lattice QCD at finite density

Sign problem: At finite quark chemical potential μ, fermionic determinant *complex* $\Rightarrow$ Goodbye configuration importance sampling

Some traditional workarounds:

- QCD, but not really μ
- μ, but not really QCD

Some potentially promising new routes:

- Dualities and worm algorithms
- Large-N orbifold dualities
- Density of states' method

A word of caution: might be a NP-hard problem **[M. Troyer and U.-J. Wiese, 2005]**

Closing words: Getting social

You are welcome to download an interactive, electronic version of this presentation from

[http://www.stp.dias.ie/∼panero/torino2014.pdf]

that you are encouraged to **[share]**, **[discuss]** and **[give feedback]** about.

Thanks for your attention!